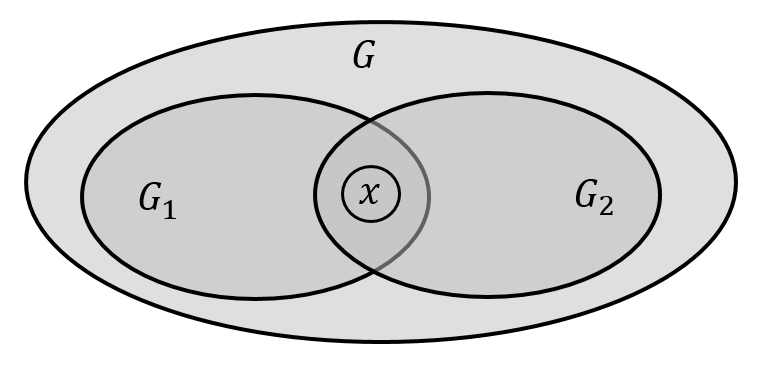
**Question 1** (Connectivity 60 Pts).

Prove that a connected bipartite –regular graph is -connected.

Hints:

1. Prove by contradiction, consider the graph structure below.
2. Then consider the degree of in one of the subgraph and recall that any subgraph is still bipartite.



**Question 2** (Coloring 60 Pts).

Let be critical (every is –colorable). Prove that is at least –edge-connected (deletion of less than edges leaves connected).

Hint:

1. Prove by contradiction, separate into two pieces by edge deletion.
2. Color each piece and then match the colors of the two pieces such that is colored properly.

**Proof 1**: Assume in contrary that is not -connected, hence it is -connected.

Therefore, can be decomposed into two subgraphs and sharing a single vertex , namely, , , and .

Since and is a disconnecting vertex, it must have neighbors in both and , hence .

Since is -connected, there is no edge connecting to , hence .

Since is bipartite, let , where and are in the two color classes of , namely all connect only vertices of to . Hence .

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It follows that the right hand side is divisible by whereas the left hand sides does not, hence a contradiction.

**Proof 2**: Assume in contrary that the deletion of edges ,…, separates into two components and , and let be the smallest such number.

It follows that , the end vertices of belong to and .

Since is critical, and are colorable. So let ,…, and ,…, be the color classes of and , respectively, i.e., and .

We would like to match some with some , such that for none of ,…, the same color appears on its two end vertices, thus ensuring proper coloring in contradiction of its criticality.

Since , there is an among ,…, not connected to by any edge of ,…,.

We can certainly select this such that either or should be incident with some edge of ,…,. So let us call this color class .

We are left with a matching problem of color classes with color classes and . The above process can be repeated until all ,…, are consumed.