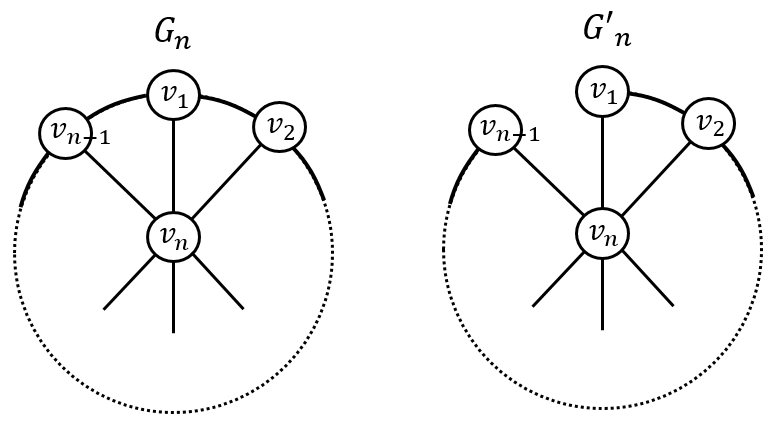
**Question 1** (Chromatic polynomial 40 Pts).

Consider the following -vertex wheel graphs and , where and is obtained by removing an edge of .



Prove the following:

1. (5Pts)
2. (10Pts)
3. (15Pts) for (use induction).

Let .

1. (5Pts) How many vertices and edges has?
2. (5Pts) Is bipartite?

**Question 2** (Graph connectivity 40 Pts).

1. (25 Pts.) Let be connected such that its longest paths contain vertices. Let and be such longest paths in and . Prove that and must have a common vertex.
2. (15 Pts.) Let be connected and an edge of . Suppose that every spanning tree of contains . Show that ( deleted) is disconnected.

**Question 3** (Cliques and independent sets 40 Pts).

Let be such that . Show that either ( vertex clique) or ( vertex independent set).

**Proof 3**: By induction on .

For or the assertion is obvious.

Suppose that . Let . There is

.

There is

.

Substitution yields

Hence either

1. or (b) .

If case (a) holds, let be induced the neighbors of . By induction hypothesis either or .

For we are done.

For , the vertex set spans in hence .

Same conclusions follow if case (b) holds.