אוניברסיטת בר-אילן

הפקולטה להנדסה

BAR-ILAN UNIVERSITY

Engineering Faculty

תורת הגרפים ושימושיה 83-652

תשע"ז סמסטר ב' מועד ב'

שם הקורס	תורת הגרפים ושימושיה		
מספר הקורס	83-652		
שם המרצה	פרופ' שמואל וימר		
	תשע"ז	סמסטר ב'	מועד ב'
משך הבחינה	שלש שעות		
חומר עזר	כל חומר אסור בשימוש. **יש לצרף את שאלוני הבחינה למחברת.**		
	יש לענות על כל השאלות. כל תשובה יש לנמק ולהסביר הייטב. סה"כ הנקודות האפשריות 110. ציון הבחינה לא יעלה על 100. **יש לכתוב בעט בלבד. כתיבה בעפרון לא תיבדק.**		

בהצלחה!

<u>שאלה 1</u> (40 נק') (השאלה הוכחה בכתה)

Show that for a connected nontrivial graph with $2k$ odd vertices, the minimum number of pairwise edge disjoint trails covering the edges is $\max\{1, k\}$.

Hints:
1. Show that for $k = 0$ at least one trail must exist.
2. If there are odd vertices, match pairs of these by a new edge.

<u>שאלה 3</u> (35 נק')

Denote by $\alpha(G)$ the size of the largest independent set of G. Show that the vertices of a graph $G(V, E)$ can be covered by no more than $\alpha(G)$ vertex-disjoint paths.

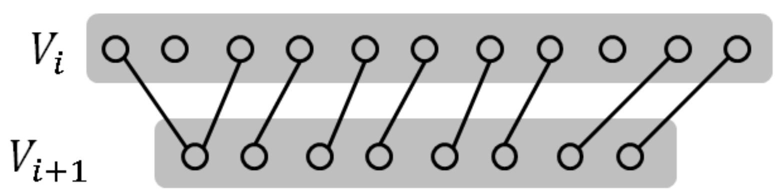
Hints:
1. Let V_1 be a maximum independent set of G, and let V_{i+1} be the maximum independent set of $G - V_1 - V_2 - \cdots - V_i$.
2. Show that $G[V_i, V_{i+1}]$ is bipartite.
3. Use König's theorem to show that V_{i+1} has a matching into V_i covering V_{i+1}.

<u>פתרון:</u>

Proof: Let V_1 be a maximum independent set of G, and let V_{i+1} be the maximum independent set of $G - V_1 - V_2 - \cdots - V_i$.

There is $|V_{i+1}| \leq |V_i|$ by construction.

Since V_i and V_{i+1} are independent sets, for any $e(x, y) \in G[V_i \cup V_{i+1}]$ there is $x \in V_i$ and $y \in V_{i+1}$. Hence $G[V_i \cup V_{i+1}]$ is bipartite, denoted $G[V_i, V_{i+1}]$.



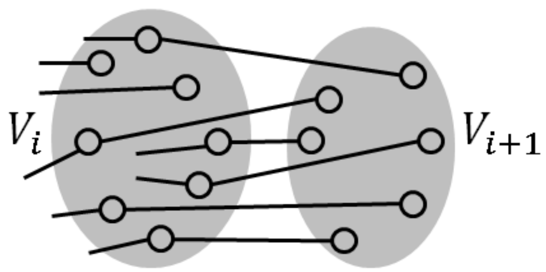
We show that the minimum vertex cover satisfies $\beta(G[V_i, V_{i+1}]) = |V_{i+1}|$.

Firstly, for any $v \in V_i$ there is $d(v) \leq 1$. Otherwise V_i would not be maximal by its choice since it could be enlarged by replacing $v \in V_i$ by few vertices of V_{i+1}.

Hence, the minimum vertex cover may consist of V_{i+1} vertices alone.

It must include all the vertices of V_{i+1}, as otherwise V_i could be enlarged, hence not maximal by its choice. Consequently $\beta(G[V_i, V_{i+1}]) = |V_{i+1}|$.

By König's theorem there is $\beta(G[V_i \cup V_{i+1}]) = \alpha'(G[V_i \cup V_{i+1}])$. Hence there is a matching F_{i+1} of V_{i+1} into V_i.



$F_2 \cup F_3 \cup \cdots$ consists of $|V_2|$ vertex-disjoint paths covering $V(G)$, except $|V_1| - |V_2|$ vertices of V_1.

Taking these as one-point paths, we obtain $|V_1| = \alpha(G)$ vertex-disjoint paths covering $V(G)$. ■