בס״ד

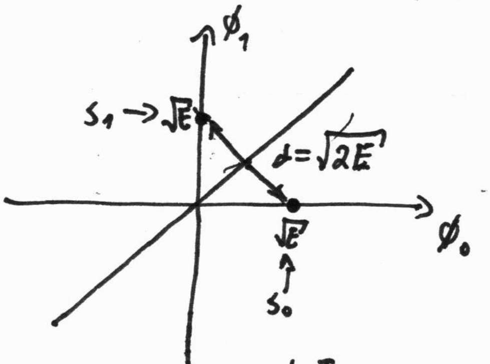
מועד א' 2010

2.א. $\phi_0 = \frac{s_0}{\|s_0\|}$ $\quad \|s_0\|^2 = \int_0^T s_0^2(t)\,dt = \int_0^{T/4} \frac{2E}{T}dt + \int_{3T/4}^{T} \frac{2E}{T}dt = \frac{2E}{T}\left[\frac{T}{4}+\frac{T}{4}\right] = E$

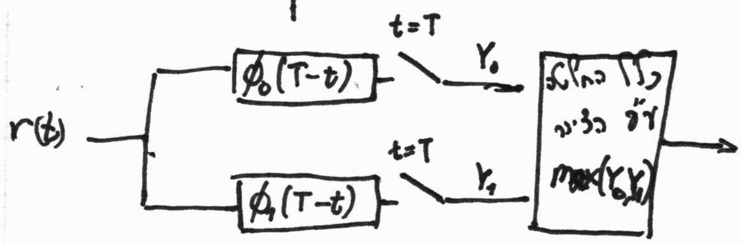
$$\Rightarrow \phi_0 = \begin{cases} \sqrt{\frac{2}{T}} & , 0<t<\frac{T}{4} \vee \frac{3T}{4}<t<T \\ 0 & , o.w. \end{cases}$$

$\Psi_1 = s_1 - \phi_0\langle s_1,\phi_0\rangle = s_1$ (רואים בציור ← בזמן $\int$)

$$\phi_1 = \frac{\Psi_1}{\|\Psi_1\|} = \frac{s_1}{\|s_1\|} = \frac{s_1}{\sqrt{E}} = \begin{cases} \sqrt{\frac{3}{T}} & , \frac{T}{2}<t<\frac{3T}{4} \\ \sqrt{\frac{1}{T}} & , \frac{T}{4}<t<\frac{T}{2} \\ 0 & , o.w. \end{cases}$$



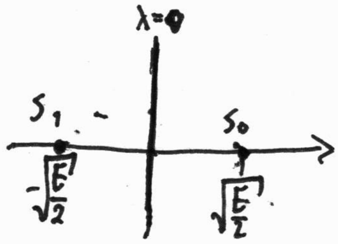
ב.



$$P_E = Q\left(\sqrt{\frac{E}{2}}\Big/\sqrt{\frac{N_0}{2}}\right) = Q\left(\sqrt{\frac{E}{N_0}}\right)$$

ג. $h(t) = \frac{\phi_0(T-t)-\phi_1(T-t)}{\sqrt{2}} \Longrightarrow$ גם $\lambda=0$

ד. כמו קודם $d = 2\sqrt{\frac{E}{2}} = \sqrt{2E}$ $\Leftarrow$



$$a^2 = \frac{N_0}{2}\cdot\|h(t)\| = \frac{N_0}{2}\cdot\left(\frac{1+1}{\sqrt{2}^2}\right) = \frac{N_0}{2}$$

$$P_E = Q\left(\frac{d}{2\sigma}\right) = Q\left(\sqrt{\frac{E}{N_0}}\right)$$

ה. ~~עושים את המוצאים לפי הנוסחה~~ $\cancel{Y_i(t=T) = \int_0^T h(\tau)\, s_i(T-(t-\tau))\,d\tau}$

$\cancel{\Rightarrow Y_i(t=T) = \int_0^T h(\tau)\, s_i(\tau)\,d\tau}$

ראה נספח

המשך - ב ה:

$$\cancel{Y_0(t=T)=\int_0^{T/4}\frac{A}{\sqrt{T}}\cdot\sqrt{\frac{2E}{T}}\,d\tau=\frac{A}{T}\sqrt{2E}\cdot\frac{T}{4}=\frac{A\sqrt{2E}}{4}}$$

$$\cancel{Y_1(t=T)=\int_{T/4}^{T/2}\frac{1}{\sqrt{T}}\cdot\sqrt{\frac{2E}{T}}\,d\tau=\frac{\sqrt{2E}}{T}\cdot\frac{T}{4}=\frac{\sqrt{2E}}{4}}$$

$$Y_0(t=T)=\int_0^{T/4}\underbrace{\frac{A}{\sqrt{T}}}_{h(T-\tau)}\underbrace{\sqrt{\frac{2E}{T}}}_{s_0(\tau)}\,d\tau=\frac{A\sqrt{2E}}{4}$$

$$Y_1(t=T)=\int_{T/4}^{T/2}\frac{1}{\sqrt{T}}\cdot\sqrt{\frac{E}{T}}\,d\tau=\frac{\sqrt{E}}{4}$$

המשך - ראה בנוסחת!

$$\cancel{\lambda=\frac{Y_0+Y_1}{2}=\frac{\sqrt{E}}{8}\left(\sqrt{2}A+\sqrt{3}\right)}$$ ~~במקרה הגבולי $\lambda=0$~~

$$\cancel{\Rightarrow\frac{\sqrt{E}}{8}\left(\sqrt{2}A+\sqrt{3}\right)=0\Rightarrow A=-\sqrt{\frac{3}{2}}}$$

1.א.

$$P_E=Q\left(\left\|\sqrt{\frac{E}{i^2N_0}}\right\|\right)=Q\left(\sqrt{\sum_0^M\frac{E}{i^2N_0}}\right)=Q\left(\sqrt{\frac{E}{N_0}\sum_0^M\frac{1}{i^2}}\right)$$

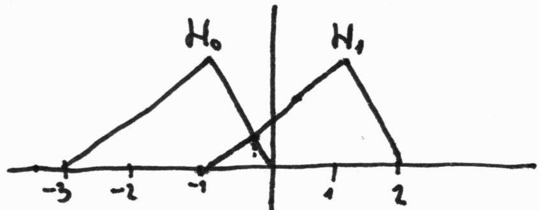
$$P_E=Q\left(\frac{d}{2\sigma}\right)$$

2.

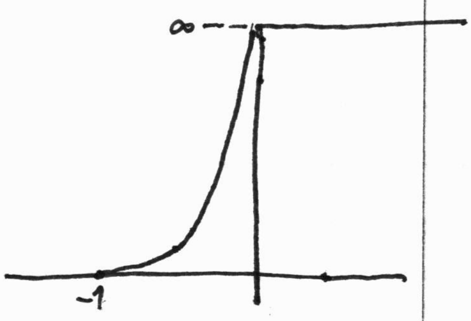
$$\lim_{M\to\infty}Q\left(\sqrt{\frac{E}{N_0}\sum_0^M\frac{1}{i^2}}\right)\underset{\text{דרך הנוסחה}}{=}Q\left(\sqrt{\frac{E\pi^2}{6N_0}}\right)$$

3.א.

$$f_r(r|H_1)=f_n(r-1)$$
$$f_r(r|H_0)=f_n(r+1)$$



$$\frac{f_r(r|H_1)}{f_r(r|H_0)}=\frac{f_n(r-1)}{f_n(r+1)}=\begin{cases}\frac{0}{f_n(r+1)}=0 & r<-1\\[4pt] \frac{1}{2}\cdot\frac{2+(r-1)}{1-(r+1)}=\frac{1+r}{-2r} & -1<r<0\\[4pt] \frac{f_n(r-1)}{0}=\infty & r>0\end{cases}$$



ב. ניתן לראות דרך הגרף שקיימת נקודת חיתוך שהיא T.

$r<-1:\ 0<\lambda\Rightarrow H_0$

$r>0:\ \infty>\lambda\Rightarrow H_1$

$$-1<r<0:\ \frac{1+r}{-2r}\underset{0}{\overset{1}{\gtrless}}\lambda\Rightarrow r\gtrless-\frac{1}{1+2\lambda}=T\Rightarrow -1<T<0$$

ג.

$$P_{FA}=P(H_1|H_0)=\int_T^\infty f_n(r+1)\,dr=\int_T^{\overset{\text{הגבול העליון של }H_0\to}{0}}\frac{2}{3}(-r)\,dr=-\frac{2}{3}\cdot-\frac{T^2}{2}=\boxed{\frac{T^2}{3}}$$

$$P_D=P(H_1|H_1)=\int_T^\infty f_n(r-1)\,dr=\int_T^1\frac{1}{3}(1+r)\,dr+\int_1^2\frac{2}{3}(2-r)\,dr=\frac{1}{3}\left[1-T+\frac{1}{2}-\frac{T^2}{2}\right]+\frac{2}{3}\left[4-2-2+\frac{1}{2}\right]$$

$$=\frac{1}{2}-\frac{T}{3}-\frac{T^2}{6}+\frac{1}{3}=\frac{5}{6}-\frac{T}{3}-\frac{T^2}{6}=\boxed{-\frac{1}{6}\left[T^2+2T-5\right]}$$

$$T=0 \Rightarrow 0 < P_{FA} < \frac{1}{3} \Leftarrow T=-1$$ ד.

$$T=0 \Rightarrow \frac{5}{6} < P_D < 1 \Leftarrow T=-1$$

$$P_{FA} = \frac{T^2}{3} = 0 \Rightarrow T=0$$ ה.

$$P_{FA} = \frac{T^2}{3} = \frac{1}{12} \Rightarrow T^2 = \frac{1}{4} \Rightarrow T = \pm\frac{1}{2} \underset{-1<T<0}{\Rightarrow} T = -\frac{1}{2}$$ ו.

$$T = -\frac{1}{1+2\lambda} = -\frac{1}{2} \Rightarrow 2 = 1+2\lambda \Rightarrow \lambda = \frac{1}{2} = \frac{P_0}{1-P_0 = P_1} \Rightarrow \boxed{P_0 = \frac{1}{3},\ P_1 = \frac{2}{3}}$$

2. ה. ראינו כי:

$$y_0(t=T)=\frac{A\sqrt{2E}}{4} \qquad y_1(t=T)=\frac{\sqrt{E}}{4}$$

$$E_y=\frac{y_0+y_1}{2}=\frac{\sqrt{E}}{8}\left(\sqrt{2}A-1\right)$$

$$V_y^2=\frac{N_0}{2}\cdot\|h(t)\|^2=\frac{N_0}{2}\left[\int_{\frac{T}{2}}^{\frac{3T}{4}}\frac{1}{T}dt+\int_{\frac{3T}{4}}^{T}\frac{A^2}{T}dt\right]=\frac{N_0}{8}\left(1+A^2\right)$$

לכן:

$$P_E=Q\left(\frac{d}{2\sigma}\right)=Q\left(\frac{E_y}{2}\cdot\frac{1}{\sqrt{\frac{N_0}{8}\left(1+A^2\right)}}\right)=Q\left(\sqrt{\frac{E\left(\sqrt{2}A-1\right)^2}{8N_0\left(A^2+1\right)}}\right)$$

כדי שהשגיאה תשאף למינימום נרצה שאין בשורש יהיה מקסימלי

לכן נגזור ונשווה ל-0

$$\frac{\cancel{E}}{\cancel{8}N_0}\left[\frac{\cancel{2}\left(\sqrt{2}A-1\right)\cdot\sqrt{2}\left(A^2+1\right)-\cancel{2}A\left(\sqrt{2}A-1\right)^2}{\cancel{\left(A^2+1\right)^2}}\right]=0$$

$$\cancel{\sqrt{2}}\left(\cancel{\sqrt{2}A-1}\right)\left(A^2+1\right)-A\left(\sqrt{2}A-1\right)^{\cancel{2}}=0$$

$$\cancel{A^2}+1-\cancel{A^2}+\frac{A}{\sqrt{2}}=0 \Rightarrow \boxed{A=-\sqrt{2}}$$