Stable matching for channel access control in cognitive radio systems

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***Abstract*—In this paper we propose a game theoretic approach to the allocation of channels to multiple cognitive users who share a set of frequencies. The famous Gale-Shapley stable matching algorithm is utilized to compute the channel allocations. We analyze the stable matching performance for the case of cognitive resource allocation and prove that in contrast to the general case, in the cognitive resource allocation problem there is a unique stable matching. We then show that the stable matching has performance very close to the optimal centralized allocation. It always achieves at least half of the total rate of the centralized allocation and under Rayleigh fading it achieves about 96% of the total centralized rate. Comparisons to random channel allocations are also discussed.**

***Index Terms*—Spectrum optimization, distributed coordination, game theory, cognitive radio, stable matching.**

I. Introduction

Cognitive radio is a radio system operating over multiple frequency selective wireless channels in which users can change their transmission or reception parameters to communicate efficiently by avoiding interference with licensed or unlicensed users. To allow multiple users to share the frequency band, users have to sense the radio spectrum to control their own transmission based on the quality of the channels and the activity in these channels. Such systems can be implemented by dividing the bandwidth into N orthogonal sub-bands using Orthogonal Frequency Division Multiplexing (OFDM). The diversity of channel realizations is advantageous if the assignment of sub-bands to the users is done efficiently with a minimum amount of coordination.

Sub-carrier allocation for centrally managed systems was addressed extensively in the last decade because of the high demand for efficient spectrum utilization in wireless and wireline communication systems. The main issue for OFDMA systems is joint power and sub-carrier allocation in the downlink direction [1], [2], [3]; and sub-carrier assignment in the uplink direction [4], [5], [6], [7], [8]. The optimal sub-carrier assignment can be computed using the well-known Hungarian method for solving assignment problems [9].

Yin and Liu [10] considered a downlink OFDMA where the base station allocates sub-carriers, power and data rate per sub-carrier for each user to maximize the overall transmit data rate subject to a total power constraint and rate constraints for each user, assuming a flat channel response for each user. They proposed a suboptimal two-step algorithm where power is first allocated and then the Hungarian method is used to assign sub-carriers to users.

Jang et al. [11] introduced a transmit power adaptation method that maximizes the total data rate of multiuser OFDM systems in a downlink transmission, where each sub-carrier is assigned to the user with the best channel gain for that sub-carrier and the transmit power is distributed over the sub-carriers by a water-filling policy. The Hungarian method for solving the assignment problem has been used extensively as an optimization method for solving other resource allocation problems. Zhu et al. [12] applied it to simplify the computation of a suboptimal solution of the Nash bargaining solution under total power constraint. Wong et al. [13],[1] and Pietrzyk and Jannsen [14] applied the Hungarian method to assign sub-carriers to users based on Quality of Service (QoS) requirements while minimizing the total transmitted power. The same problem has been addressed in optimizing resources in PON systems [15].

In contrast to cellular and optical systems which have centralized access management, cognitive radio systems are inherently distributed. Therefore, there is no way to use an optimal centralized strategy for channel allocation. When no controller exists (the case for cognitive radio) distributed allocation protocols may be the best candidate system and are the topic of this paper. The simplest approach is to use random channel allocation. This type of allocation is very simple to implement through standard random access techniques, and its convergence time is very fast as shown below. However, for a large number of users, we also show that the performance of the random allocation is significantly worse than the best centralized strategy (the relative loss is $1/\log N$, where N is the number of channels).

Since the loss of random channel allocation is unacceptable, cognitive approaches that take channel quality functions into account are needed. An example of a simple approach of this kind is 'stable' allocation. By the Gale-Shapley Stable Marriage Theorem [16] a stable allocation always exists. The theorem is very general and states that whenever we have two sets of N men and N women, where every man and woman has his or her own preference regarding the opposite sex players, we can always find a stable matching; i.e., we cannot find a man and a woman who prefer each other over their partners in the matching. In the general case, there are many stable matchings, their number can be quite large, and the set of stable matchings has a (set theoretic) lattice structure. However, we show below that in the spectrum allocation problem there is always a unique stable matching (almost surely with respect to channel quality distribution).

The main advantage of the stable matching is that it can be computed using the Gale-Shapley algorithm, which is decentralized by nature. Another advantage is of course stability, which is desirable in a non-regulated scenario. We prove that the worst case total rate of the stable allocation is one half that of the optimal centralized allocation, which is significantly better than the random allocation. We also show in simulation that for independent Rayleigh fading channels, the expected total rate of the stable allocation is much better than the above theorem and exceeds 96% of the expected total rate of the optimal allocation, regardless of N. A full statistical analysis of the Rayleigh case is beyond the scope of this paper.

We also consider the time delay until the desired allocation is reached. For random allocations we prove that this time delay is $O(\log N)$. We show that the worst case time delay before reaching the stable allocation is $O(N^2)$, and simulations show that on average this time delay is $O(N)$. Therefore, if the dynamics is sufficiently slow, reaching the stable allocation is preferable, especially for large values of N. However for large N and very fast dynamics random allocation might be superior due to the very short time until convergence.

The structure of the paper is as follows: In Section II we describe the basic model, and define the properties of stable matchings. In Section III we prove that in the special case of spectral allocation the stable matching is unique. In Section IV the lower bound on the achievable rate for stable matching as compared to optimal allocation is given. In Sections V and VI we address the complexity of the stable and random allocation algorithms and their relaxation time. Simulation results are given in Section VII.

II. Model Formulation

Assume that N users have access to N wireless channels (the results of this paper can be generalized to the case where we have unequal number of users and channels). Assume that each user has N channel utility functions representing the transmission quality on each channel. We assume that these utilities are i.i.d. continuous random variables. A simple example of these channel utility functions are the ergodic capacities of each user on each channel. We will denote the utility of channel j when used by user i by $u_{i,j}$ and define the utility matrix as $\mathbf{U} = (u_{i,j})$. We assume that the u_{ij} are i.i.d. with a continuous probability distribution. Therefore, at any given time the N^2 channel utility functions are almost surely all different; hence we assume this in what follows. Although the analysis can be done for arbitrary utility functions we assume that $u_{i,j}$ represents the rate that user i can achieve when using channel j.

Assume that each user is capable of transmitting on a single channel at a time, but can sense all activity on the N channels. Since we assume the dynamics is slow, we can optimize the allocation of channels to users. To that end we need some definitions:

Definition II.1. *A spectral matching between users and channels is a permutation $P : [N]\to[N]$ where $[N] = \{1, ..., N\}$.*

The optimal centralized channel allocation problem is now formalized as follows:
Find a permutation $P : [N]\to[N]$ such that

$$P = \arg\max_{P\in S_N} \sum u_{i,P(i)} \tag{1}$$

where S_N is the permutation group on [N]. Although the problem is discrete and the size of S_N is $N!$ the solution has complexity $O(N^3)$ using the Hungarian method. We also define the total utility of a matching P by

$$u(P) = \sum u_{i,P(i)} \tag{2}$$

Before continuing with the channel allocation problem, we describe the Gale-Shapley theorem. Assume that we have two sets A, B of men and women each of size N. For each $a \in A$ there is a one-to-one function $f_a(b) : B\to[N]$ which ranks the preferences of a. Similarly, for each $b \in B$ there is a one-to-one function $g_b(a) : A\to[N]$ which ranks the preferences of b where a higher value means a higher preference. A matching is a one-to-one function from A to B.

Definition II.2. *A matching $S : A\to B$ is stable iff for every $a \in A$ and $b \in B$ satisfying $S(a) \neq b$ either $f_a(S(a)) > f_a(b)$ or $g_b(S^{-1}(b)) > g_b(a)$.*

More explicitly a matching is stable, if for any pair a and $b \neq S(a)$ either a prefers $S(a)$ over b or b prefers its own partner over a. The Gale-Shapley theorem states that for any preference functions there is always a stable matching. As mentioned before, generally the stable matching is not unique.

As described in the Introduction, when N is large or when there is no centralized controller we would like to find a distributed low complexity solution based on stable matching. Note that in this case the preferences of the users and the preferences of the channels are defined by the matrix $\mathbf{U}$ of user-channel utility. Hence, we obtain that in our case stability is defined as follows:

Definition II.3. *A spectral matching S is stable iff for every $i, j \in [N]$ satisfying $S(i) \neq j$ either $u_{i,S(i)} > u_{i,j}$ or $u_{S^{-1}(j),j} > u_{i,j}$.*

In the remainder of the paper we examine the properties of stable matchings as candidates for channel allocation strategies.

III. Uniqueness of the stable matching

We now analyze stable matching for the cognitive radio spectral allocation problem.

Proposition III.1. *Let $\mathbf{U}$ be an N by N matrix whose entries are all different. If the preferences of all users and channels are determined by the matrix $\mathbf{U}$ then there is a unique stable matching.*

Proof: We prove the Proposition by induction on N; the basis $N = 1$ is trivial. Let $u_{i,j}$ be the maximal entry of the utility matrix $\mathbf{U}$, and let $\mathbf{U}'$ be the matrix we get by deleting row i and column j of the matrix $\mathbf{U}$. If S is a stable matching for $\mathbf{U}$ then clearly $S(i) = j$, and in addition $S \setminus \{(i,j)\}$ must be a stable matching for $\mathbf{U}'$. By induction there exists a unique stable matching S' for the smaller matrix $\mathbf{U}'$, and from the above remarks we can conclude that $S := S' \cup \{(i,j)\}$ is a unique stable matching for $\mathbf{U}$. ■

The proof is constructive and shows that the unique stable matching is the result of the centralized greedy algorithm which chooses the best user-channel pair each time, deletes the corresponding row and column, and continues recursively. This provides a centralized $O(N^2 \log N)$ complexity algorithm for finding the unique stable matching.

Definition III.1. *Given a utility matrix* $\mathbf{U}$ *whose entries are all different, let* $S_{\mathbf{U}}$ *be the stable matching determined by* $\mathbf{U}$.

IV. The achievable rate of the stable matching

Since the stable matching is unique we may define the *stable utility* to be the total utility of the stable matching. We now inquire how the stable utility fares compared to the *optimal utility*, i.e. the total utility of an optimal matching. The next proposition shows that in the worst case the ratio is 2.

Proposition IV.1. *Let* $\mathbf{U}$ *be an* N *by* N *matrix whose entries are all different and non-negative, and let* $S = S_{\mathbf{U}}$ *denote the stable matching. Then for any matching* P *we have* $u(P) < 2u(S_{\mathbf{U}})$*, where the function* $u(P)$ *is defined in (2).*

Proof: Assume without loss of generality that the stable matching is the identity, i.e. $\forall i \in [N] : \; S_{\mathbf{U}}(i) = i$, and define for every $k \in [N]$ $v_k := u_{k,k}$. We can also assume that $\forall k < l : v_k > v_l$. Note that v_k is the maximal entry in the submatrix $(u_{i,j})_{i,j \geq k}$ (for any $k \in [N]$).

Let P be any matching, and assume $w_1 > w_2 > \ldots > w_N$ are such that $\{w_l \mid l \in [N]\} = \{u_{i,P(i)} \mid i \in [N]\}$. Now fix some $k \in [N]$. The entries of $\mathbf{U}$ which are outside the submatrix $(u_{i,j})_{i,j \geq k}$ can be covered by $(k-1)$ rows and $(k-1)$ columns. Since each row and column contains at most one of the w_l, we can conclude from the maximality of v_k in $(u_{i,j})_{i,j \geq k}$ that it is smaller than at most $2(k-1)$ of the utilities w_l; hence $v_k > w_{2k-1}$. It follows that

$$\begin{aligned} u(P) &= w_1 + w_2 + w_3 + w_4 + \ldots + w_N < \\ &< v_1 + v_1 + v_2 + v_2 + \ldots + v_{\lceil N/2 \rceil} \leq \\ &\leq 2(\sum_{k=1}^{N} v_k) = 2u(S_{\mathbf{U}}) \end{aligned} \tag{3}$$

as required. ■

The following example shows that the worst case can actually occur.

Example IV.2. *Let* $N = 2M$*, and assume that for some small positive* Δ *and* $\epsilon(i,j)$ *satisfying* $\forall i,j : |\epsilon(i,j)| < \Delta/2M$ *the utilities are given by*

$$u_{i,j} = \begin{cases} 1 + \Delta + \epsilon(i,j) \; : \; i,j \in \{1, 2, \ldots, M\} \\ |\epsilon(i,j)| \; : \; i,j \in \{M+1, M+2, \ldots, 2M\} \\ 1 + \epsilon(i,j) \; : \; \text{otherwise} \end{cases}$$

Then the stable matching $S_{\mathbf{U}}$ *satisfies* $1 \leq i \leq M \Leftrightarrow 1 \leq S_{\mathbf{U}}(i) \leq M$*; hence its total utility is bounded as follows:* $u(S_{\mathbf{U}}) \in [M(1+\Delta) - \Delta, M(1+\Delta) + \Delta]$*. On the other hand any optimal matching* P *will satisfy* $1 \leq i \leq M \Leftrightarrow M+1 \leq P(i) \leq 2M$*; hence the optimal utility is in the interval* $[2M - \Delta, 2M + \Delta]$*. When* $\Delta \to 0$ *the ratio between the stable and optimal utilities approaches 2.*

V. Distributed implementation and complexity of the stable matching allocation

The main advantage of the stable matching approach over finding the optimal matching, in the context of ad-hoc networks, is that the implementation of the Gale-Shapley algorithm is decentralized by its very nature. Specifically, the following is guaranteed to converge to the stable matching: We initialize by declaring each user to be *roaming*, and at every time slot we have two steps. First, each roaming user attempts to transmit on his best channel out of those he has not yet tried, and each non-roaming user attempts to transmit on the same channel as on the previous time slot. Second, on each channel j the best user out of the set U_j of users attempting to transmit on j is declared to be non-roaming (in case U_j is nonempty), while all other users in U_j are declared to be roaming (the details of the distributed implementation of this step are omitted).

Finally, the complexity of finding the stable matching is significantly lower than $O(N^3)$, as we describe below. For any utility matrix $\mathbf{U}$ whose entries are all different we denote by $t_{\mathbf{U}}$ the number of time slots required for the Gale-Shapley algorithm described above to reach the stable matching $S_{\mathbf{U}}$. Clearly $t_{\mathbf{U}}$ depends solely on the relative order of $\mathbf{U}$'s entries, and therefore does not depend on the specific statistics of the utilities. First we deal with the worst case: this is known to be $O(N^2)$ for arbitrary preference lists. The next example shows that the worst case is still $O(N^2)$ even in the special case that all preference lists come from matrix $\mathbf{U}$.

Example V.1. *Define the utility matrix* $\mathbf{U}$ *by*

$$u_{i,j} := \begin{cases} N(N+1) - Ni - j \; : \; i \geq j \\ N(N+1) - (N+1)i + j \; : \; i < j \end{cases}$$

Then the stable matching is $S_{\mathbf{U}} = Id$*, and it is easy to show that user* i *attempts to transmit on channel* i *only after* $1 + 1 + 2 + 3 + \ldots + (i-1)$ *time slots. Hence* $t_{\mathbf{U}} = 1 + 1 + 2 + 3 + \ldots + (N-1) = 1 + N(N-1)/2$.

We give the details for $N = 5$ *below. The utility matrix is shown in Table I with the maximal utility for every user highlighted in bold.*

$\mathbf{U}$	ch-1	ch-2	ch-3	ch-4	ch-5
user-1	**24**	23	22	21	20
user-2	**19**	18	17	16	15
user-3	13	**14**	12	11	10
user-4	7	8	**9**	6	5
user-5	1	2	3	**4**	0

TABLE I
Utility matrix for $N = 5$

Table II shows the transmission attempts made by users before the stable matching is reached.

Time	ch-1	ch-2	ch-3	ch-4	ch-5
t=1	**1,2**	3	4	5	
t=2	1	**2,3**	4	5	
t=3	**1,3**	2	4	5	
t=4	1	2	**3,4**	5	
t=5	1	**2,4**	3	5	
t=6	**1,4**	2	3	5	
t=7	1	2	3	**4,5**	
t=8	1	2	**3,5**	4	
t=9	1	**2,5**	3	4	
t=10	**1,5**	2	3	4	
t=11	1	2	3	4	5

TABLE II
GALE-SHAPLEY ALGORITHM AT WORK

The total utility of the stable matching in this example is $u(S_{\mathbf{U}}) = 24 + 18 + 12 + 6 + 0 = \mathbf{60}$.

VI. RELAXATION TIME OF THE RANDOM ALLOCATION SCHEME

In this section we show that the expected time required for a random allocation scheme to stabilize is $O(\log N)$. In this case we compute this time when we have K users and $N \geq K$ channels. The random allocation scheme works as follows: Declare all K users to be *roaming* and all N channels to be *free*. At every time slot each non-roaming user stays on his channel, and each roaming user attempts to transmit on a random free channel. Such an access mechanism is easily achieved using standard random access techniques, assuming that the users are cognitive, and know the channel state of each of the N channels. If there is no collision then the user becomes non-roaming and the channel becomes busy. We now examine the expected delay until the system stabilizes. We denote this expected delay by $T_{K,N}$.

Proposition VI.1. *There is some constant C s.t. for every $0 \leq K \leq N$ we have*

$$T_{K,N} \leq C\ln(K+1) \tag{4}$$

Proof:

The proposition is proved by induction on K; the case $K = 0$ is trivial (since $T_{0,N} = 0$).

For every $0 \leq i \leq K$ let $q_{K,N}(i)$ denote the probability that, at time $t = 1$, exactly i of the users stabilize (i.e. become non-roaming). Then we have

$$T_{K,N} = 1 + \sum_{i=0}^{K} q_{K,N}(i) T_{K-i,N-i} \tag{5}$$

Let $q_0 := q_{K,N}(0)$. By the induction hypothesis we obtain

$$\begin{aligned} T_{K,N} &= \frac{1}{1-q_0} + \sum_{i=1}^{K} \frac{q_{K,N}(i)}{1-q_0} T_{K-i,N-i} \leq \\ &\leq \frac{1}{1-q_0} + \sum_{i=1}^{K} \frac{q_{K,N}(i)}{1-q_0} C\ln(K-i+1) \end{aligned} \tag{6}$$

By concavity of the function $C\ln(x)$ and since $\sum_{i=1}^{K} \frac{q_{K,N}(i)}{1-q_0} = 1$ we get

$$T_{K,N} \leq \frac{1}{1-q_0} + C\ln\Big[\sum_{i=1}^{K} \frac{q_{K,N}(i)}{1-q_0}(K-i+1)\Big] \tag{7}$$

Now let $\hat{i}$ denote the expected number of users that stabilize at time $t = 1$. Then by using symmetry between the K users and $K \leq N$ we obtain

$$\begin{aligned} \sum_{i=0}^{K} i \cdot q_{K,N}(i) &= \hat{i} = K \cdot \mathrm{Prob}(\text{user 1 stabilizes}) = \\ &= K \cdot \frac{N(N-1)^{K-1}}{N^K} = \\ &= K(1-\frac{1}{N})^{K-1} > Ke^{-1} \end{aligned} \tag{8}$$

From (8) we can conclude that $\sum_{i=1}^{K} \frac{q_{K,N}(i)}{1-q_0}(K-i+1) = K+1-\frac{1}{1-q_0}\sum_{i=0}^{K} i \cdot q_{K,N}(i) < K+1-\frac{Ke^{-1}}{1-q_0}$, and in particular $K+1-\frac{Ke^{-1}}{1-q_0}$ is positive. Therefore (7) gives us

$$T_{K,N} \leq \frac{1}{1-q_0} + C\ln\big(K+1-\frac{Ke^{-1}}{1-q_0}\big) \tag{9}$$

In order to show $T_{K,N} \leq C\ln(K+1)$ it suffices to show

$$\begin{aligned} \frac{1}{1-q_0} &\leq C\ln(K+1) - C\ln\big(K+1-\frac{Ke^{-1}}{1-q_0}\big) = \\ &= -C\ln\big(1-\frac{Ke^{-1}}{(1-q_0)(K+1)}\big) \end{aligned} \tag{10}$$

Since $-\ln(1-x) \geq x$ it suffices to show $\frac{1}{1-q_0} \leq C\frac{Ke^{-1}}{(1-q_0)(K+1)}$, and this indeed holds for all $K \geq 1$ for the constant $C = 2e$.

■

VII. SIMULATION RESULTS

As we saw in Example V.1, the delay until the stable matching is reached is $O(N^2)$ in the worst case. However, due to the dynamic nature of the wireless channel, we are actually interested in the *expected* value of $t_{\mathbf{U}}$ (where the order of $\mathbf{U}$'s entries is chosen at random out of the $(N^2)!$ possibilities). Figure 1 below shows the result of running the Gale-Shapley algorithm on random square matrices. For purposes of comparison we also show the expected convergence time of the Gale-Shapley algorithm for the general Stable Marriage Problem; i.e. for $2N$ random preference lists. The expected

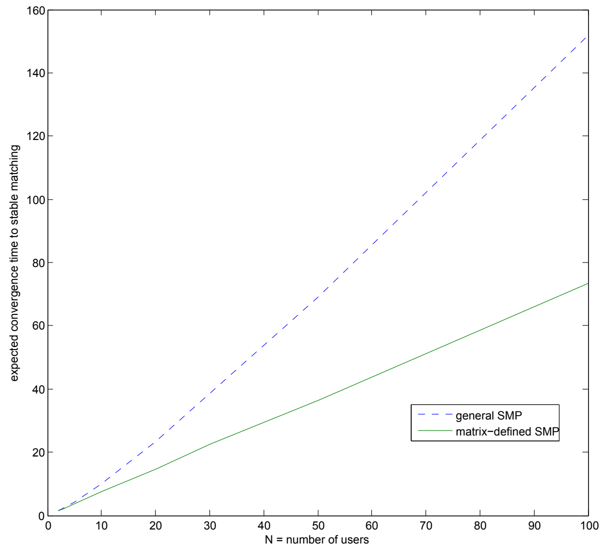

Fig. 1. Expected convergence time to stable matching

value of $t_{\mathbf{U}}$ is approximately αN for some constant $\alpha \approx 0.73$.

In contrast to the delay, the stable utility depends on the channel statistics. Figure 2 below shows the result of simulating N Rayleigh fading channels, and comparing the expected stable rate to the optimal centralized rate. We also

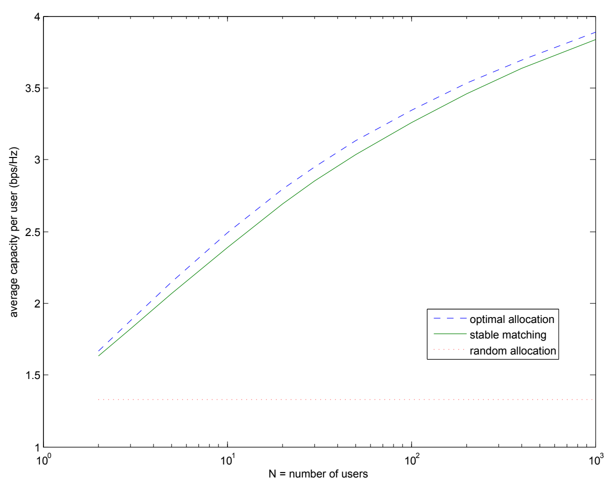

Fig. 2. Rates of optimal, stable and random allocations for Rayleigh fading channels

include for reference the expected rate of a random matching, which is defined by:

$$R_{i,random} = E\left[\log_2\left(1 + \frac{|h_{ii}|P_i}{\sigma^2}\right)\right] \tag{11}$$

where, P_i is the power used by user i, σ^2 is the noise variance and h_{ii} is the fading coefficient. Note that the stable rate is significantly higher than the rate of the random allocation: the ratio is already ~ 1.2 for $N = 2$, and rises to ~ 2.4 when $N = 80$. Note also that this ratio seems to be roughly linear in $\log(N)$.

In Figure 3 we show the ratio between the expected stable rate and the expected optimal rate. This ratio is always above 0.96; i.e. we lose at most 4% by using stable matching.

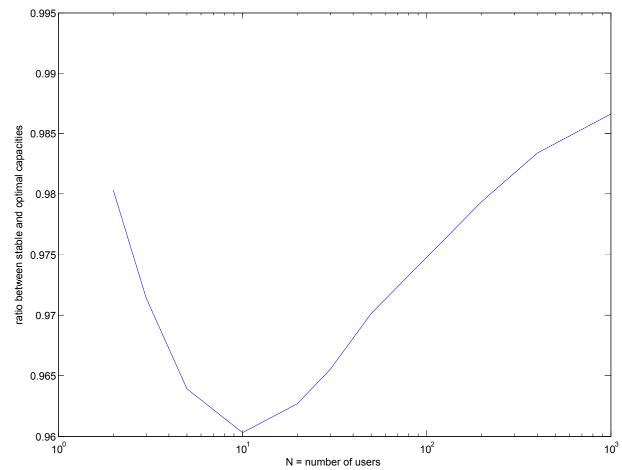

Fig. 3. Ratio of stable to optimal rate for Rayleigh fading channels

VIII. Conclusions

In this paper we analyzed stable matching for frequency allocation in cognitive radio systems. We showed that the stable matching achieves at least half of the centralized aggregate rate. Furthermore, we showed that on Rayleigh fading channels the loss is on the order of 4%. We analyzed the convergence time, and showed that with some additional cognitive mechanisms the stabilization time is linear in N.

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