Simultaneous Connectivity in Heterogeneous Cognitive Radio Networks

Michal Yemini*, Anelia Somekh-Baruch*, Reuven Cohen+ and Amir Leshem*
*Faculty of Engineering, Bar-Ilan University, Ramat-Gan, Israel
Email:michal.yeminy@biu.ac.il, somekha@biu.ac.il, leshema@eng.biu.ac.il.
+Department of Mathematics, Bar-Ilan University, Ramat-Gan, Israel
Email:reuven@math.biu.ac.il.

***Abstract*—In this paper we analyze the connectivity of cognitive radio ad-hoc networks. Contrary to previous works, we pursue the connectivity of *both* the primary and secondary networks, a state we call "simultaneous connectivity". We determine that if the networks are simultaneously connected then their infinite connected components are unique. In addition, we characterize the region of densities in which both the primary and secondary networks have a unique infinite connected component.**

I. Introduction

Connectivity is an essential property for the operation of networks. The analysis of the connectivity of mobile ad-hoc networks can be challenging, for these networks do not have fixed structures. A natural method for analyzing the connectivity of mobile ad-hoc networks is continuum percolation [1]–[6]. It is assumed that the nodes of the network are distributed according to a Poisson point process (PPP) and that two nodes are connected if the distance between them does not exceed a certain value. A network is said to be connected (or percolated) under continuum percolation models if there exists an unbounded (or infitine) connected component in the network. There is a prolific literature on the connectivity of large-scale homogeneous networks including [1], [2], [7]–[10]. These works assume one network scenario in which all the nodes belong to a single network. A realization of such a network is depicted in Fig. 1.

In recent years, radio communication applications became ubiquitous in communication systems. These applications spectrum demands exhaust the limited and valuable free spectrum resources. New regulations aim at balancing the growing radio communication demands with the limited free spectrum resources. To accomplish this challenging goal, a new concept dubbed Cognitive Radio has emerged. Cognitive radio networks improve spectrum utilization by giving the opportunity to cognitive (secondary) users to transmit while limiting their interference on non-cognitive (primary) users in the network.

The connectivity of non-cooperative ad-hoc cognitive radio networks with heterogeneous nodes is considered in several works, among them are [4]–[6], [11]–[13]. However, these works analyze models in which the secondary nodes are components of a *multi-hop* network, whereas the primary nodes are only a part of a *single-hop* network. These limited models do not capture the complex connectivity demands of cognitive radio networks with multi-hop primary network.

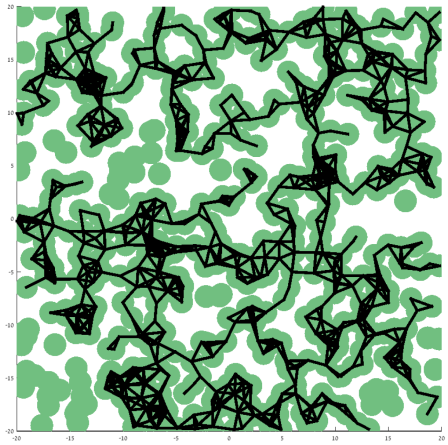

Fig. 1. A realization of a homogeneous network with parameters $\lambda = 40\ \mathrm{km}^{-2}$ and $\rho = D_t = 210$ m. The black lines indicate the disks that compose the largest connected component.

In this paper we pursue the connectivity of *both* primary and secondary networks of a cognitive radio ad-hoc network. This is the first work that discusses this setup, we hope that our analysis provides motivations and insights into generalizations of applications of stochastic geometry such as routing, medium-access control and interference analysis in wireless networks. We call the state in which both of these networks are connected "*simultaneous connectivity*". We assume that the nodes of the primary and secondary networks are distributed according to two independent two-dimensional PPPs with densities λ_p and λ_s, respectively. Every primary node has a transmission range D_t, and every secondary node has a transmission range d_t. We assume that a secondary node is not active, unless its distance to each primary node exceeds a length of D_f. We call this model the *heterogeneous model*; Figs. 1-3 depict a realization for such a model: Fig. 1 depicts the primary network, Fig. 2 includes the guard zone of each primary user and active and passive secondary nodes, and Fig. 3 depicts the active nodes of the secondary network.

Define the simultaneous connectivity region to be the set of all 5-tuples $(D_t\ , d_t\ , D_f\ , \lambda_p\ , \lambda_s)$ for which there is at least one unbounded connected component in both the primary and

978-1-5090-1806-2/16/$31.00 ©2016 IEEE

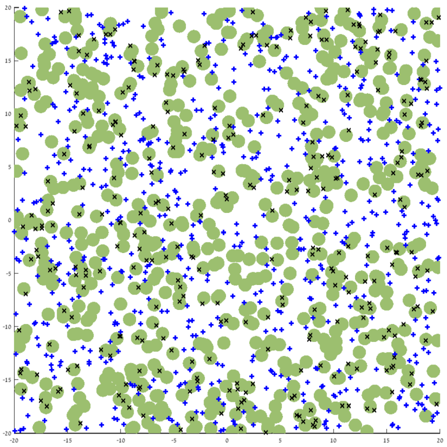

Fig. 2. The guard zone of the primary nodes which appear in Fig. 1 and a realization of the active and passive secondary nodes. We set $\lambda_s = 60\text{km}^{-2}$, $\lambda_p = 40\text{km}^{-2}$, $D_f = 60$ m. Green disks depict the guard zone of the primary nodes, blue $+$ indicates active secondary nodes and black x indicates passive secondary nodes.

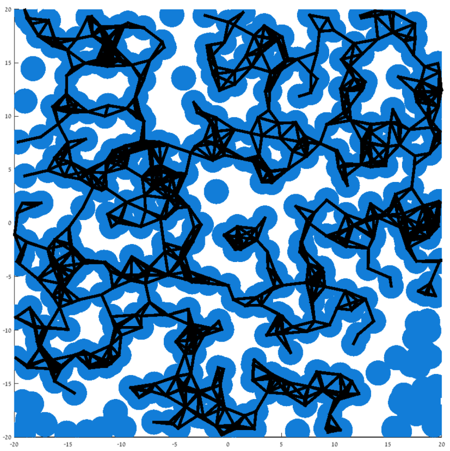

Fig. 3. The active secondary nodes of the realization which appears in Fig. 2 and the parameter $d_t = 230$ m. The black lines indicate the disks that compose the largest connected component of the secondary network.

secondary networks. In this paper we analyze the relationship between the densities λ_p and λ_s of the two dimensional PPPs and the radii D_t,d_t and D_f. Our analysis provides motivations and insights into generalizations of applications of stochastic geometry such as routing [14]–[17], medium-access control [18]–[22] and interference analysis in wireless networks [23]–[27]. We note that there are many additional works regarding stochastic geometry and its applications in the analysis and design of wireless ad-hoc networks, see for example [25], [28]–[31]. However, due to space limitation we only cite several articles.

II. System Model and Definitions

In this section we present the heterogeneous model. We also state fundamental definitions and results of Percolation Theory which we apply in our analysis of the connectivity of the heterogeneous model.

A. The Heterogeneous Model

In this model the primary nodes are distributed according to a two-dimensional PPP with density λ_p. We assume that the transmission range of primary nodes , D_t, is fixed. Similarly, the nodes of the secondary network are distributed according to a PPP with density λ_s .This PPP is independent of the PPP of the primary network. We also assume that the transmission range of secondary nodes, d_t, is fixed. We next provide several definitions corresponding to the heterogeneous model.

Definition 2.1: There is a communication opportunity from node x_i to node x_j in the primary network if $\|x_i - x_j\|_2 \leq D_t$, where $\|\cdot\|_2$ denotes the L_2 norm.

Definition 2.2: There is a communication opportunity from node z_i to node z_j in the secondary network if the following conditions hold:

1) $\|z_i - z_j\|_2 \leq d_t$,
2) there is no primary node x such that $\|x - z_i\|_2 \leq D_f$,
3) there is no primary node x such that $\|x - z_j\|_2 \leq D_f$.

Consequently, D_f is the radius of the guard zone of a primary node and transmitting/receiving secondary nodes.

We only discuss bidirectional links; that is, we say that there is a link between the nodes z_i and z_j if there exists a communication opportunity from node z_i to node z_j and vice versa.

Definition 2.3: Let X_p be the set of nodes of the primary network. The connected component of node $x \in X_p$ consists of all nodes in X_p for which there exists a path to x in X_p such that every two consecutive nodes in the path have a communication opportunity. Additionally, an unbounded connected component of the primary network is a connected component of the primary network which consists of an infinite number of nodes.

The definition of a connected component in the secondary network is similar.

Definition 2.4: The simultaneous connectivity region $\mathcal{C}$ consists of all 5-tuples $(D_t\, , d_t\, , D_f\, , \lambda_p\, , \lambda_s)$ such that both the primary and secondary networks include a.s. at least one unbounded connected component.

The connectivity of the primary and secondary networks can be studied by representing the two networks by the two independent Boolean models (see the following section). Nevertheless, our connectivity definitions differ from those of a simple Boolean model (see [32]).

We now provide some definitions which are required for the analysis of the heterogeneous model.

B. The Gilbert Disk (Boolean) Model

The Gilbert disk model scatters points in $\mathbb{R}^2$ according to a PPP. Each point in the PPP is assumed to have a fixed radius. In the following we present several definitions related to the Gilbert disk model.

Definition 2.5 (Point Process): Let $\mathcal{B}^2$ be the σ-algebra of Borel sets in $\mathbb{R}^2$, and let N be the set of all simple counting measures on $\mathcal{B}^2$. Let $\mathcal{N}$ be the σ-algebra which is generated by the sets

$$\{n \in N : n(A) = k\}, \tag{1}$$

where $A \in \mathcal{B}^2$, and k is an integer. A point process X is a measurable mapping from a probability space $(\Omega\,,\mathcal{F}\,,P)$ into $(N\,,\mathcal{N})$. The distribution of X is denoted by μ and is defined by $\mu(G) = P(X^{-1}(G))$, for all $G \in \mathcal{N}$. Hereafter, for convenience we refer to $(N\,,\mathcal{N})$ as $(\Omega\,,\mathcal{F})$.

Definition 2.6 (Gilbert Disk (Boolean) Model): Suppose that X is a point process. A Gilbert disk (Boolean) model is composed of point process X and a fixed radius ρ such that each point $x \in X$ is a center of a disk with a fixed radius ρ. Note that this model is equivalent to a Boolean model with fixed radii. As mentioned before, in this paper we assume that X is a PPP with density λ. We denote this Poisson Gilbert disk (Boolean) model by $(X\,,\rho\,,\lambda)$.

We represent the heterogeneous network by the following Gilbert disk models $(X_p, D_t/2, \lambda_p)$ and $(X_s, d_t/2, \lambda_s)$, where $(\Omega_p\,,\mathcal{F}_p\,,P_p)$ and $(\Omega_s\,,\mathcal{F}_s\,,P_s)$ are the probability spaces of two independent PPPs X_p and X_s, respectively. Further, D_t and d_t are the transmission radii in the primary and secondary networks, respectively.

C. Occupied Components

Define $O(z\,,r) \triangleq \{x \in \mathbb{R}^2 : \|x - z\|_2 \leq r\}$. Every Poisson Boolean model $(X\,,\rho\,,\lambda)$ partitions $\mathbb{R}^2$ into two regions, the *occupied region*, which we denote by

$$\mathcal{O} \triangleq \bigcup_{x \in X} O(x\,,\rho), \tag{2}$$

and the *vacant region*. The occupied region consists of the points in $\mathbb{R}^2$ that are covered by at least one disk, whereas the vacant region consists of all points in $\mathbb{R}^2$ that are not covered by any disk.

Two nodes $x_1\,,x_2 \in X$ are connected if $O(x_1\,,\rho) \cap O(x_2\,,\rho) \neq \emptyset$ (however, in the secondary network we consider only active nodes). The connected components in the occupied region are called *occupied components*, while the connected components in the vacant region are called *vacant components*.

We remark that by definition of the occupied components in the Boolean model, two nodes are connected if the distance between them does not exceed 2ρ. Therefore, we represent each network by a model in which ρ is half of the transmission radius. Further, an occupied component/region in the secondary network consists only of active secondary nodes of the secondary network.

D. The Critical Probability

We next define the critical probability of the Gilbert disk model.

Definition 2.7 (Critical Probability): Let $d(A) \triangleq \sup_{x,y \in A}|x - y|$. Denote by $\theta_\rho(\lambda)$ the probability that the origin is an element of an unbounded occupied component of the Gilbert disk (Boolean) model $(X\,,\rho\,,\lambda)$, that is

$$\theta_\rho(\lambda) \triangleq \Pr(d(W) = \infty).$$

The critical density $\lambda_c(2\rho)$ is defined by

$$\lambda_c(2\rho) \triangleq \inf\{\lambda \geq 0 : \theta_\rho(\lambda) > 0\}. \tag{3}$$

As we next state, the critical probability has a strong tie to the crossing probabilities which we define next.

E. Unit Transformations

We now define unit transformations of the heterogeneous model. We later use this definition in the discussion of the ergodicity of the heterogenous model.

Definition 2.8: Let $\mathcal{B}^2$ denote the Borel sets of $\mathbb{R}^2$ and let Ω be a set of simple counting measures on $\mathcal{B}^2$. Let $t \in \mathbb{R}^2$ and $T_t : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by the translation $T_t x = x + t$. T_t then induces the transformation $S_t : \Omega \to \Omega$ for each $A \in \mathcal{B}^2$ through the equation (see [32, p. 22])

$$(S_t\omega)(A) = \omega(T_t^{-1}A)\,,\ \forall \omega \in \Omega. \tag{4}$$

Let $\tilde{\Omega} = \Omega_p \times \Omega_s$, $\tilde{\mathcal{F}} = \mathcal{F}_p \times \mathcal{F}_s$ and $\tilde{P} = P_p \times P_s$. Denote the unit vectors of $\mathbb{R}^2$ by $e_1\,,e_2$. It follows that T_t induces the transformation $\tilde{T}_t$ on $\tilde{\Omega}$ where

$$\tilde{T}_t = (S_t\omega_p\,,S_t\omega_s). \tag{5}$$

More specifically, T_{e_i} induces the transformation $\tilde{T}_{e_i}$ on $\tilde{\Omega}$ where

$$\tilde{T}_{e_i} = (S_{e_i}\omega_p\,,S_{e_i}\omega_s). \tag{6}$$

III. Simultaneously Connected Components

In this section we establish the connectedness of the simultaneous connectivity region, and several results regarding the number of unbounded connected components in the primary and secondary networks and their existence.

Proposition 3.1: The simultaneous connectivity region $\mathcal{C}$ is connected.

The proof of this proposition can be found in [33, Proposition 3.1].

Proposition 3.2: The heterogeneous model is ergodic (with respect to the unit transformations).

The proof of this proposition can be found in [33, Proposition 4.1].

From the ergodicity of the model we deduce that the number of unbounded connected components in each of the networks is constant a.s.

Proposition 3.3: The number of unbounded connected components in the primary network and the number of unbounded connected components in the secondary network are constant a.s.

Due to space limitations the proof of this proposition is omitted and can be found in [33, Proposition 4.2].

Theorem 3.4: There is at most one unbounded connected component in the primary network and at most one unbounded connected component in the secondary network.

Theorem 3.5: Let $D_t\,,d_t > 0$ be given. For every $\lambda_p > \lambda_c(D_t)$ and $\lambda_s > \lambda_c(d_t)$ there exists $D_f > 0$ such that there is an unbounded connected component in the primary network and also an unbounded connected component in the secondary network.

The proofs of Theorems 3.4 and 3.5 are presented in [33, Theorems 4.3, 4.4], respectively.

IV. Necessary Conditions for Simultaneous Connectivity

In this section we state necessary conditions for simultaneous percolation in both the primary and secondary networks. These two conditions are found by implementing two different methods. The first condition, stated in Theorem 4.1, is found by considering the fact that there cannot exist both an unbounded vacant component and an occupied component in a Gilbert disk (Boolean) model a.s.

Theorem 4.1: Suppose that $2D_f > d_t$, then

$$\begin{aligned} &\lambda_s > d_t^{-2}\lambda_c(1), \\ &D_t^{-2}\lambda_c(1) < \lambda_p < \left(4D_f^2 - d_t^2\right)^{-1}\lambda_c(1) \end{aligned} \tag{7}$$

are necessary conditions for simultaneous percolation in both networks.

The proof of this theorem is presented in [33, Theorem 5.1].

Another set of conditions for simultaneous connectivity is obtained by discretization onto a site percolation in which each site has eight neighbors. This set of conditions explores the relationship between the densities of the primary secondary networks under the simultaneous percolation regime.

Theorem 4.2: Let $n_p \triangleq \left\lceil \frac{\sqrt{2}d_t}{D_f} \right\rceil^2$. Denote by p_8 the critical probability of site percolation with eight neighbors. Then

$$\begin{aligned} &\lambda_s > d_t^{-2}\lambda_c(1), \\ &\lambda_p > D_t^{-2}\lambda_c(1), \\ &\lambda_p < -\left(\frac{n_p}{d_t}\right)^2 \ln\left(1 - \left(1 - e^{-\lambda_s d_t^2} - p_8\right)^{1/n_p}\right) \end{aligned} \tag{8}$$

are necessary conditions for simultaneous percolation in both networks.

Due to space limitations the proof of this theorem is omitted and can be found in [33, Theorem 5.2].

By applying the lower bound $\frac{1}{3} \leq p_8$ (see [34, Chapter 2.2]) to Theorem 4.2 we obtain the following corollary.

Corollary 4.3: Let $n_p \triangleq \left\lceil \frac{\sqrt{2}d_t}{D_f} \right\rceil^2$. Then

$$\begin{aligned} &\lambda_s > d_t^{-2}\lambda_c(1), \\ &\lambda_p > D_t^{-2}\lambda_c(1), \\ &\lambda_p < -\left(\frac{n_p}{d_t}\right)^2 \ln\left(1 - \left(\frac{2}{3} - e^{-\lambda_s d_t^2}\right)^{1/n_p}\right). \end{aligned} \tag{9}$$

are necessary conditions for simultaneous percolation in both networks.

The proof of this corollary is presented in [33, Corollary 5.3].

V. Sufficient Conditions for Simultaneous Connectivity

In this section we present the sufficient conditions for the existence of both primary and secondary connected unbounded components. We find these conditions by discretizing the continuous model onto a dependent site percolation model [34], [35]. We discretize the heterogeneous model in a way that ensures that if there exists an unbounded connected occupied component in the discrete site percolation, an unbounded connected component exists in the continuous model as well.

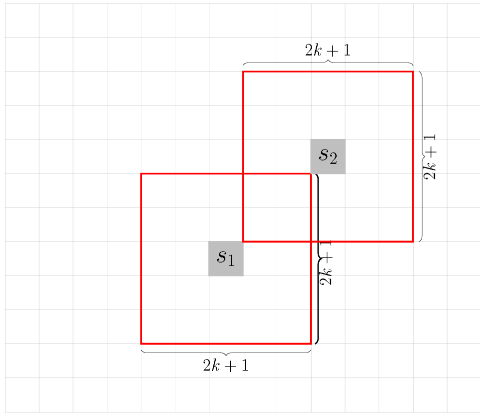

Fig. 4. A k-dependent site percolation. A red box around a site encloses its dependent sites. The sites s_1 and s_2 are independent.

A dependent site percolation is a site percolation in which the state of a site may depend on the states of other sites. If the state of a site only depends on the states of the sites that are separated by a path of minimum length $k < \infty$ we say that the model is k-dependent (see Fig. 4).

Let p be the marginal probability for a site to be open in a stationary dependent site model. By [34, Theorem 2.3.1], there exists $p(k)$ such that for $p > p(k)$ there exists an unbounded occupied component a.s. It follows that there exists $p_8(k)$ such that for all $p > p_8(k)$ there exists an unbounded occupied component in the k-dependent site percolation models with eight neighbors.

Let $S(t, D_f)$ stand for the (random) intersected area between two disks of radius D_f with centers at distance[1] t. Further, let

$$\begin{aligned} f_{T,\ell}(t) =& \mathbb{1}_{\{t\in[0,\ell]\}} \frac{4t}{\ell^2}\left(\frac{\pi\ell^2}{2} - 2\ell t + \frac{t^2}{2}\right) \\ &+ \mathbb{1}_{\{t\in[\ell,\sqrt{2}\ell]\}} \frac{4t}{\ell^2}\left(\ell^2 \arcsin\left(\frac{2\ell^2 - t^2}{t^2}\right)\right. \\ &\left. + 2\ell\sqrt{t^2-\ell^2} - \ell^2 + \frac{t^2}{2}\right) \end{aligned} \tag{10}$$

be the probability density function of the random distance t between the two centers, each generated independently from the box of side length ℓ. Denote, $\ell_8 = \frac{D_t}{2\sqrt{2}}$,

$$\begin{aligned} &\tilde{p}_8(D_f, d_t, \lambda_p, \lambda_s) \triangleq \lambda_s e^{-\lambda_s \frac{d_t^2}{8}} e^{-\lambda_p \pi D_f^2} \\ &+ \left(1 - \lambda_s e^{-\lambda_s \frac{d_t^2}{8}} - e^{-\lambda_s \frac{d_t^2}{8}}\right) \\ &\cdot e^{-\lambda_p \pi D_f^2}\left[2 - \int_{-\infty}^{\infty} f_{T,\ell_8}(t) e^{\lambda_p [S(t,D_f) - \pi D_f^2]} dt\right]. \end{aligned} \tag{11}$$

[1] $S(d, r) = -\frac{1}{2}d\sqrt{4r^2 - d^2} - 2r^2 \arctan\left(\frac{d}{\sqrt{4r^2-d^2}}\right) + \pi r^2$

Theorem 5.1: Let $k_8 = \left\lceil \frac{4\sqrt{2}D_f}{d_t} \right\rceil$. There exist unbounded connected components simultaneously in both the primary and secondary networks if (both) the following conditions hold

$$\lambda_p > D_t^{-2}\lambda_c(1)\,, \quad \tilde{p}_8(D_f\,, d_t\,, \lambda_p\,, \lambda_s) > p_8(k_8). \quad (12)$$

The proof of this theorem is presented in [33, Theorem 6.1].

A simpler but looser bound can be derived.

Corollary 5.2: There exists an unbounded connected component in both the primary and secondary networks if the following condition holds

$$D_t^{-2}\lambda_c(1) < \lambda_p < (\pi D_f^2)^{-1} \ln \left(\frac{1 - e^{-\lambda_s d_t^2/8}}{1 - \left(\frac{1}{3}\right)^{(2k_8+1)^2}} \right). \quad (13)$$

The proof of this corollary is presented in [33, Corollary 6.2].

VI. Conclusion

In this paper we presented several results regarding the simultaneous connectivity of cognitive radio ad-hoc networks. We discussed the uniqueness of the unbounded connected components in each of the networks. We also claimed that for each pair of densities greater than the critical density without inter-network interference, there exists a small enough guard zone such that there exist unbounded connected components in both networks. Furthermore, we presented sufficient as well as necessary conditions for the simultaneous connectivity of the heterogeneous model.

References

[1] O. Dousse, F. Baccelli, and P. Thiran, "Impact of interference on connectivity in ad hoc networks," *IEEE/ACM Trans. Netw*, vol. 13, no. 2, pp. 425–436, April 2005.

[2] O. Dousse, M. Franceschetti, N. Macris, R. Meester, and P. Thiran, "Percolation in the signal to interference ratio graph," *J. Appl. Probabil.*, vol. 43, no. 2, pp. 552–562, June 2006.

[3] W. Ren, Q. Zhao, and A. Swami, "On the connectivity and multihop delay of ad hoc cognitive radio networks," *IEEE Journal on Selected Areas in Communications*, vol. 29, no. 4, pp. 805–818, April 2011.

[4] ——, "Connectivity of heterogeneous wireless networks," *IEEE Trans. Inf. Theory*, vol. 57, no. 7, pp. 4315–4332, July 2011.

[5] D. Lu, X. Huang, P. Li, and J. Fan, "Connectivity of large-scale cognitive radio ad hoc networks," in *Proceedings of the IEEE International Conference on Computer Communications (INFOCOM12)*, March 2012, pp. 1260–1268.

[6] D. Liu, E. Liu, Y. Ren, Z. Zhang, R. Wang, and F. Liu, "Bounds on secondary user connectivity in cognitive radio networks," *IEEE Communications Letters*, vol. 19, no. 4, pp. 617–620, April 2015.

[7] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Transactions on Information Theory*, vol. 42, no. 2, pp. 388–404, March 2000.

[8] T. K. Philips, S. S. Panwar, and A. N. Tantawi, "Connectivity properties of a packet radio network model," *IEEE Transactions on Information Theroy*, vol. 35, no. 5, pp. 1044–1047, September 1989.

[9] C. Bettstetter, "On the minimum node degree and connectivity of a wireless multihop network," in *Proceedings of the ACM International Symposium on Mobile Ad Hoc Networking and Computing (MobiHoc)*, June 2002, pp. 80–91.

[10] J. Ni and S. A. G. Chandler, "Connectivity properties of a random radio network," *IEEE Proceedings Communications*, vol. 141, no. 4, pp. 289–296, August 1994.

[11] W. C. Ao, S.-M. Cheng, and K.-C. Chen, "Connectivity of multiple cooperative cognitive radio ad hoc networks," *IEEE Journal on Selected Areas in Communications*, vol. 30, no. 2, pp. 263–270, February 2012.

[12] O. A. Al-Tameemi and M. Chatterjee, "Percolation in multi-channel secondary cognitive radio networks under the SINR model," in *Proceedings of the IEEE International Symposium on Dynamic Spectrum Access Networks (DYSPAN'14)*, April 2014, pp. 170–181.

[13] L. T. Dung and B. An, "Connectivity analysis of cognitive radio adhoc networks with shadow fading," *KSII Transactions on Internet and Information Systems*, vol. 9, no. 9, pp. 3335–3356, September 2015.

[14] M. Haenggi, "On routing in random rayleigh fading networks," *IEEE Transactions on Wireless Communications*, vol. 4, no. 4, pp. 1553–1562, July 2005.

[15] S. Ping, A. Aijaz, O. Holland, and A. H. Aghvami, "SACRP: A spectrum aggregation-based cooperative routing protocol for cognitive radio adhoc networks," *IEEE Transactions on Communications*, vol. 63, no. 6, pp. 2015–2030, June 2015.

[16] M. Haenggi and D. Puccinelli, "Routing in ad hoc networks: a case for long hops," *IEEE Communications Magazine*, vol. 43, no. 10, pp. 93–101, October 2005.

[17] A. Zanella, A. Bazzi, G. Pasolini, and B. M. Masini, "On the impact of routing strategies on the interference of ad hoc wireless networks," *IEEE Transactions on Communications*, vol. 61, no. 10, pp. 4322–4333, October 2013.

[18] F. Baccelli, B. Blaszczyszyn, and P. Muhlethaler, "An Aloha protocol for multihop mobile wireless networks," *IEEE Transactions on Information Theory*, vol. 52, no. 2, pp. 421–436, February 2006.

[19] ——, "Stochastic analysis of spatial and opportunistic aloha," *Selected Areas in Communications, IEEE Journal on*, vol. 27, no. 7, pp. 1105–1119, September 2009.

[20] T. V. Nguyen and F. Baccelli, "A probabilistic model of carrier sensing based cognitive radio," in *2010 IEEE Symposium on New Frontiers in Dynamic Spectrum*, April 2010, pp. 1–12.

[21] M. K. Hanawal, E. Altman, and F. Baccelli, "Stochastic geometry based medium access games in wireless ad hoc networks," *IEEE Journal on Selected Areas in Communications*, vol. 30, no. 11, pp. 2146–2157, December 2012.

[22] T. Kwon and J. M. Cioffi, "Spatial spectrum sharing for heterogeneous SIMO networks," *IEEE Transactions on Vehicular Technology*, vol. 63, no. 2, pp. 688–702, February 2014.

[23] O. Dousse, M. Franceschetti, N. Macris, R. Meester, and P. Thiran, "Percolation in the signal to interference ratio graph," *Journal of Applied Probability*, vol. 43, no. 2, pp. 552–562, 2006.

[24] A. Ghasemi and E. Sousa, "Interference aggregation in spectrum-sensing cognitive wireless networks," *IEEE Journal of Selected Topics in Signal Processing*, vol. 2, no. 1, pp. 41–56, February 2008.

[25] M. Haenggi and R. K. Ganti, "Interference in large wireless networks," *Foundations and Trends in Networking*, vol. 3, no. 2, pp. 127–248, 2008.

[26] C.-H. Lee and M. Haenggi, "Interference and outage in poisson cognitive networks," *IEEE Transactions on Wireless Communications*, vol. 11, no. 4, pp. 1392–1401, April 2012.

[27] R. W. Heath Jr., M. Kountouris, and T. Bai, "Modeling heterogeneous network interference using Poisson point processes," *IEEE Transactions on Signal Processing*, vol. 61, no. 16, pp. 4114–4126, August 2013.

[28] F. Baccelli and B. Blaszczyszyn, "Stochastic geometry and wireless networks: Volume I theory," *Foundations and Trends in Networking*, vol. 3, no. 3-4, pp. 249–449, 2008.

[29] ——, "Stochastic geometry and wireless networks: Volume II applications," *Foundations and Trends in Networking*, vol. 4, no. 1-2, pp. 1–312, 2009.

[30] M. Haenggi, J. G. Andrews, F. Baccelli, O. Dousse, and M. Franceschetti, "Stochastic geometry and random graphs for the analysis and design of wireless networks," *IEEE Journal on Selected Areas in Communications*, vol. 27, no. 7, pp. 1029–1046, September 2009.

[31] M. Haenggi, *Stochastic Geometry for Wireless Networks*. Cambridge University Press, 2013.

[32] R. Meester and R. Roy, *Continuum Percolation*. Cambridge University Press, 1996.

[33] M. Yemini, A. Somekh-Baruch, R. Cohen, and A. Leshem, "On simultaneous percolation with two disk types," *arXiv:1601.04471*, 2016.

[34] M. Franceschetti and R. Meester, *Random networks for communication: From statistical physics to information systems*. Cambridge University Press, 2007.

[35] G. Grimmett, *Percolation*, 2nd ed. New York: Springer, 1999.