# Nash Bargaining Solution for Cooperative Relaying Exploiting Energy Consumption

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Abstract—In this letter, we propose a resource allocation for cooperative relaying in a scenario with a high number of communicating devices. The proposed resource allocation is based on Nash bargaining solution (NBS) and leads to a natural cooperation among devices. The NBS provides an allocation of time intervals maximizing the number of transmitted packets considering energy consumption of devices. The derived NBS is in closed form, thus, it is suitable for wireless communications with time-varying channels as no iterations are needed to find the optimum allocation. Furthermore, linear complexity of the derived NBS allows its application to future mobile networks with a high number of communicating devices.

Index Terms—Cooperative game theory, Nash bargaining, cooperative relaying, resource allocation, energy consumption.

# I. INTRODUCTION

In the upcoming mobile networks, base stations (BSs) are expected to serve a mix of common human traffic and machine type communication (MTC). The number of connected devices, generating both human as well as MTC traffic, is exponentially increasing. This motivates development of efficient strategies handling the traffic generated by these devices. In [1], it is shown that relaying of data using the device-todevice (D2D) communication to the BS is a feasible solution enabling communication of a massive amount of devices. At the same time, the D2D also reduces energy consumption of the devices [2] but the relaying devices should be motivated [3].The benefits of D2D for devices are further described in [2], [3].

The devices acting as relays consume their own energy for delivery of data from other devices. Thus, an appropriate incentive for the relays should be defined so that all the devices benefit from the relaying. Most of existing works focus primarily on maximization of data rates [4]. Nevertheless, an energy consumption of communication and relaying is a crucial factor as it impacts the devices' battery lifetime. The battery lifetime extension motivates cooperation of the devices. The cooperation of devices exploiting D2D can be achieved via Game theory, as shown in [5] for resource allocation, or in [6] for power allocation and channel reuse for D2D communication.

A natural solution based on Game theory for the problem of cooperation among devices is the Nash Bargaining solution (NBS). The NBS has been used to encourage cooperation in other setups of wireless communications, for example, in problems of allocating spectrum over frequency selective channels in OFDMA systems [7], [8], for device association to the BSs [9], or for power and bandwidth allocation to the devices [10]. In [11] the authors propose a NBS to maximize data rates of devices via channel assignment and power allocation. However, the authors do not consider communication energy consumption. Duan et al. [12], consider the NBS for energy efficient resource allocation for D2D relaying. However, the authors in [12] focus on a case of two D2D pairs of devices, where each D2D pair provides relaying for the other D2D pair. Thus, making it applicable only in a case of mutual benefit of D2D pairs and impractical for an arbitrary number of D2D pairs acting as relays. Furthermore, the authors do not provide a closed form bargaining solution, but instead formulate the NBS and then solve numerically.

In this paper, we derive a NBS for allocation of communication resources such that all devices in the network benefit from relaying. Unlike related works, our NBS is based on energy consumption of the communication and is solved for N devices. Moreover, we solve the NBS for a general relaying strategy, and thus, the described solution is independent of actual relaying strategies considered by the devices. In contrast to [10] the proposed solution is in closed form and does not require an iterative approach or auctions to reach the optimal solution. Thus, the NBS is applicable to wireless communications even with rapidly time-varying radio channels and high number of devices due to low complexity and high scalability.

# II. SYSTEM MODEL

Consider a single BS, which serves N devices (mobile phones, sensors, etc.). Each device transmits packets with M bits of data to the BS in the uplink direction. We focus on a case where the devices can act as relays through which other devices transmit their data to the BS as shown in Figure 1.

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Fig. 1. System model with device i relaying its data through device j.

The relays exploit Decode and Forward (DF) relaying scheme. Our system model is based on the system model exploited for example in [13].

In the considered scenario the devices share radio resources by means of time division multiple access (TDMA). The devices compete for a part of a frame with a duration of  $T_F$ . Each device transmits for a portion of  $T_F$  defined as transmission time interval (TTI)  $T_i = \alpha_i T_F$ , where  $\alpha_i \in (0, 1)$  and  $\sum_{i=1}^{N} \alpha_i = 1$ . Note that the proposed solution for TDMA can be extended towards Orthogonal Frequency Division Multiple Access (OFDMA), but we leave this extension for the future due to limited space.

Communication between a source device (i.e., the device, which is willing to transmit the data) and the BS is done either by a direct communication or by a relaying via another device. For the direct communication, the data is transmitted by the *i*-th device (source) to the BS and the BS receives data with Signal to Interference plus Noise Ratio (SINR)  $\gamma_i$ . In case of the relaying, the *i*-th device (source) transmits data to a selected *j*-th device (relay) over a D2D channel. The *j*-th device receives the data with SINR  $\gamma_{i,j}$ . Then, the relay forwards the data of the source device to the BS over its direct channel and SINR at the BS is  $\gamma_j$ .

The data rate of the *i*-th device communicating directly to the BS is  $r_i^d = B\log_2(1 + \gamma_i)$ , where *B* is the bandwidth allocated for the direct communication with the BS. The data rate between the source and the relaying devices is defined as  $r_{i,j}^{D2D} = B^{D2D}\log_2(1 + \gamma_{i,j})$ . The data rate  $r_{i,j}^{D2D}$  of the *i*-th device to the *j*-th device can be higher than the data rate achievable by the *j*-th relay at its direct channel to the BS ( $r_j^d$ ). Thus, we adapt the data rate at the relay channel to match data rate at the direct channel of the relay, i.e., the data rate at relay channel is  $\bar{r}_{i,j}^{D2D} = min(r_j^d, r_{i,j}^{D2D})$ .

The energy consumed by the direct transmission of a packet is expressed as:

$$E_{i}^{d} = \frac{(P_{i}^{\text{tx}} + P_{i}^{c}) M_{i}}{r_{i}^{d}}$$
(1)

where  $P_i^{\text{tx}}$  is the power consumed for the transmission,  $P_i^c$  is the power consumed by the circuitry of the *i*-th device, and  $M_i$  is the amount of bits to be transmitted by the *i*-th device.

The energy consumed by the D2D transmission of the packet from the *i*-th device (source) to the *j*-th device (relay) is then expressed in similar way, i.e.:

$$E_{i,j}^{D2D} = \frac{\left(P_i^{\text{tx},\text{D2D}} + P_i^c\right)M_i}{\bar{r}_{i,j}^{D2D}}$$
(2)

where  $P_i^{\text{tx,D2D}}$  is the power consumed by the D2D transmission of the *i*-th device. The *i*-th device can transmit its data directly or via *j*-th relay by exploiting the relaying strategy  $s_i$  from a set of possible strategies s, given as:

$$s_i = \begin{cases} j & \text{if transmitting via } j\text{-th device} \\ 0 & \text{otherwise (direct transmission to the BS)} \end{cases}$$
(3)

If the device decides not to follow the relaying strategy (i.e.,  $s_i = j$ ), it follows a disagreement strategy d (i.e.,  $s_i = 0$ ). Under the disagreement strategy d, the device does not cooperate with others and transmits data directly to the BS, disregarding strategies of other devices.

Based on the strategy selected by the device, we define the energy consumed for transmission of the *i*-th device following the strategy  $s_i$  as:

$$E_i^{\text{tx}}(s_i) = \begin{cases} E_{i,j}^{D2D} & \text{if } s_i \neq 0\\ E_i^d & \text{otherwise} \end{cases}$$
(4)

Note that the energy consumed by the relaying devices for reception is omitted in the model as it leads to different solution, which is more complex and does not fit to the page limit. We assume that each device has initial energy  $E_i^{\text{init}}$ . Then, the total number of packets transmitted by the device following  $s_i$  before the battery depletion is defined as:

$$N_i(s_i) = \frac{E_i^{\text{init}}}{E_i^{\text{tx}}(s_i)} \tag{5}$$

The coordination of resource allocation is done in a central way by a BS, as described in, e.g., [14]. The only information needed to be collected by the BS is either energy consumption or channel quality, which is anyway reported to control the communication in cellular networks even with D2D relaying.

#### **III. PROBLEM FORMULATION**

Our objective is to allocate TTIs to the devices under cooperation via Nash Bargaining solution. The Nash Bargaining solution is a class of a cooperative games where each player follows strategy, which reaches a mutual agreement among the players and has a higher utility than a non-cooperative strategy.

Let  $\mathbf{N} = \{1, 2, ..., N\}$  be a set of players, in our case represented by the devices willing to transmit data. Let  $\mathbf{Q}$  be a closed and convex subset of  $\mathbb{R}^N$  representing the set of feasible payoff allocations that the players can get by cooperation. Then, let  $\mathbf{A} = \{\alpha_1, \alpha_2, ..., \alpha_N\}$  be a set of feasible allocations of TTIs to the devices. Let  $\overline{N}_i(d) = \frac{N_i(d)}{N}$  be the minimal payoff required by the *i*-th player, otherwise, the *i*-th player does not cooperate. Suppose  $\{\alpha_i N_i(s_i) \in \mathbf{Q} | \alpha_i N_i(s_i) \geq \overline{N}_i(d), \forall i \in \mathbf{N}\}$  is a nonempty bounded set. We define  $\bar{N}(\mathbf{d}) = (\bar{N}_1(d), \dots, \bar{N}_N(d))$ , then the pair  $(\mathbf{Q}, \bar{N}(\mathbf{d}))$  is called the *N*-person bargaining problem.

The objective is to find the NBS of the TTIs allocation  $A^*$ , which maximizes the product (benefit) of the number of transmitted packets gained by the cooperation. This objective is formulated as:

$$\mathbf{A}^{\star} = \underset{\mathbf{A}}{\operatorname{argmax}} \prod_{i=1}^{N} \left( \alpha_{i} N_{i}(s_{i}) - \bar{N}_{i}(d) \right)$$
(6)

subject to 
$$\alpha_i^{\star} N_i(s_i) \ge \alpha_i N_i(s_i), \forall i \in \mathbb{N}$$
 (7)

$$0 < \alpha_i < 1 \tag{8}$$

$$\sum_{i=1}^{N} \alpha_i = 1 \tag{9}$$

The constraint (7) motivates devices to cooperation as it specifies that the number of transmitted packets for each device following  $\alpha_i^*$  must be higher than if the device would follow any other  $\alpha_i$ . The constraint (8) limits  $\alpha_i$  to allocate each device a portion of TTIs while the constraint (9) guarantees that the resources allocated to all devices fit to a single frame.

#### **IV. NASH BARGAINING SOLUTION**

In this section, we first derive the NBS for two devices. Then, we generalize the solution towards N devices. Since the objective function (6) is convex, we explore the Karush-Kuhn-Tucker (KKT) conditions for two as well as for N devices.

#### A. NBS for two devices

In this subsection we consider two devices i.e., N = 2, in line with model in Figure 1. In this case, a device which provides relaying, i.e., Device 2 (*j*), is allocated with a fraction  $\alpha_2$  of the frame and a device exploiting relaying, i.e., Device 1 (*i*) gets  $\alpha_1 = 1 - \alpha_2$  of the frame by Pareto optimality. To derive the NBS, we formulate the Nash product in terms of  $\alpha_2$ (where the constraints are already incorporated by the choice of  $\alpha_1$  and  $\alpha_2$ :

$$L = \left[ (1 - \alpha_2) N_1(s_1) - \frac{N_1(d)}{2} \right] \left( \alpha_2 - \frac{1}{2} \right) N_2(s_2) \quad (10)$$

Then, the derivative of L with respect to  $\alpha_2$  is set equal to zero:

$$\frac{\partial L}{\partial \alpha_2} = \frac{N_2(s_2) \left[ N_1(s_1) \left( 3 - 4\alpha_2 \right) - N_1(d) \right]}{2} = 0 \qquad (11)$$

By solving the linear equation in (11) for  $\alpha_2$  and substituting  $\alpha_1 = 1 - \alpha_2$  we obtain the NBS for TTI allocation for both devices where

$$\alpha_1 = \frac{1}{4} + \frac{E_2^{\text{tx}}(s_2)}{4E_2^{\text{tx}}(d)} \qquad \alpha_2 = \frac{3}{4} - \frac{N_2(d)}{4N_2(s_2)} \tag{12}$$

The numerical analysis of the number of transmitted packets is done in a scenario with parameters from [2], i.e.,  $P_i^{tx} = P_i^{\text{tx},\text{D2D}} = 200 \text{ mW}, P_i^c = 800 \text{ mW}, B = B^{D2D} = 200 \text{ kHz}, M = 100 \text{ B}, E_i^{init} = 100 \text{ J}, \text{ and } T_F = 10 \text{ ms}.$  The derived NBS is compared with Equal TTI allocation when each device is allocated with  $\frac{1}{N}$  of  $T_F$ , MaxMin TTI allocation, where



Fig. 2. Number of transmitted packets  $(N_i(s_i))$  with  $\gamma_1 = 0$  dB and  $\gamma_2 = 30$  dB, (dashed line - Device 1 (source), solid line - Device 2 (relay).

the minimal number of transmitted packets per a device is maximized [4], and to *Direct* transmission scheme without relaying, where each device is allocated with  $\frac{1}{N}$  of  $T_F$ . The derived NBS works independently of relaying strategy. Nevertheless, for comparison with other allocations, we select a commonly exploited opportunistic relay (OR) selection [15]. This strategy considers quality of both the direct channel ( $\gamma_i$ ) and the D2D channel between source and relay devices ( $\gamma_{i,j}$ ) for selection of the relaying device, i.e., the strategy  $s_i$  for the OR is defined as:

$$s_i = \operatorname*{argmax}_{j \in N} \min\left(\gamma_j, \gamma_{i,j}\right) \tag{13}$$

In Figure 2, the number of transmitted packets is shown for the Device 1 ( $N_1$ ) and Device 2 ( $N_2$ ) as a function of  $\gamma_{1,2}$ . The Device 2 acts as the relay for the Device 1. Note that the packets from Device 1 relayed by Device 2 are not included in the number of packets transmitted by the Device 2 (i.e., in  $N_2$ ). For all three relaying algorithms,  $N_1$  increases with  $\gamma_{1,2}$  due to improvement in the relaying channel quality. The MaxMin algorithm results in the highest  $N_1$ , but the lowest  $N_2$  out of all relaying algorithms, because the MaxMin targets to provide fairness among the devices  $(N_1 = N_2 \text{ and lines for the Device})$ 1 and the Device 2 overlap in Figure 2). As a result of fairness, the Device 2 does not cooperate since it looses with respect to the direct transmission. The Equal algorithm improves  $N_i$  for the Device 1 with respect to the direct transmission, however, the performance of the relaying Device 2 is the same as for the direct transmission. This means the Device 2 is still not motivated to cooperate and help the Device 1. In contract to this, the derived NBS results in a gain for both devices with respect to the direct communication. Consequently, the Device 2 is motivated to cooperate with the Device 1, because the Device 2 receives an incentive in terms of additional resources for communication as a reward for its cooperation. Assuming rationality of players (devices), only the derived NBS leads to natural cooperation of devices.

# B. NBS for N devices

In this subsection, we generalize the solution obtained for two devices towards N devices. First, we replace product in (6) by the sum of logarithms:

$$\mathbf{A}^{\star} = \operatorname{argmax} \sum_{i=1}^{N} \log \left( \alpha_i N_i(s_i) - \bar{N}_i(d) \right)$$
(14)

Then the Lagrangian of (14) is derived considering conditions (7), (8), and (9):

$$L = \sum_{i=1}^{N} \log \left( \alpha_i N_i(s_i) - \frac{N_i(d)}{N} \right) + \mu \left( \sum_{i=1}^{N} \alpha_i - 1 \right)$$
(15)

Taking the derivatives of the Lagrangian with respect to  $\alpha_i$ and  $\mu$ , and by setting the derivative equal to zero, we get:

$$\frac{\partial L}{\partial \alpha_i} = \frac{N_i(s_i)}{\alpha_i N_i(s_i) - \frac{N_i(d)}{N}} + \mu N = 0$$
(16)

$$\frac{\partial L}{\partial \mu} = \sum_{i=1}^{N} \alpha_i - 1 = 0 \tag{17}$$

From (16) we obtain

$$\alpha_i = \frac{N_i(d)}{N_i(s_i)N} - \frac{1}{N\mu} \tag{18}$$

where  $\mu$  is determined from (17) and (18) as:

$$\mu = \frac{1}{1 - \sum_{i=1}^{N} \frac{N_i(d)}{N_i(s_i)N}}$$
(19)

Next, by substituting (19) to (18), we obtain the NBS of TTI allocation in a closed form as:

$$\alpha_{i} = \frac{N_{i}(d)}{N_{i}(s_{i})N} + \frac{1 - \sum_{i=1}^{N} \frac{N_{i}(d)}{N_{i}(s_{i})N}}{N}$$
(20)

To obtain the allocations of TTIs to the devices in the terms of the transmission energies, we substitute the number of transmitted packets from (5) into (20):

$$\alpha_{i} = \frac{E_{i}^{tx}(s_{i})}{E_{i}^{tx}(d)N} - \frac{\sum_{i=1}^{N} \frac{E_{i}^{tx}(s_{i})}{E_{i}^{tx}(d)N} - 1}{N}$$
(21)

The derived allocation (21) is in closed form, which makes it suitable for wireless communications with a frequently varying quality radio channel. The complexity of (21) is O(N), thus the solution is suitable even for scenarios with a high number of devices, as envisioned in 5G mobile networks).

## V. SIMULATION RESULTS AND ANALYSIS

In this section, we present numerical results obtained by simulations following parameters defined in Section IV-A in line with [2]. The results for the NBS are compared with all three allocation schemes (MinMax, Equal, Direct) described also in Section IV-A. The devices are uniformly distributed in a simulation area with diameter of 500 m around a single BS. The direct channel is modeled as Urban Macro with Log-normal shadowing with variance of 4 dB and the D2D channel follows Winner II model. Each device has the initial



Fig. 3. Average energy consumed per transmission  $(E_i^{tx}(s_i))$  (a) and total number of transmitted packets  $(\sum N_i)$  (b).

energy generated from exponential distribution with  $\lambda = 1$  and maximal value of 100 J.

The average energy consumed per transmission  $E_i^{tx}(s_i)$  is shown in Figure 3a. This figure, shows that energy efficiency is improved via relaying with respect to the direct transmission, disregarding whether the cooperation is natural (for the NBS) or must be externally enforced (for MaxMin and Equal).

Figure 3b shows the total number of transmitted packets over the number of devices deployed in the area, i.e.,  $\sum N_i(s_i)$ . For the Direct transmission, the number of transmitted packets is almost constant disregarding the number of devices, because each device transmits data directly to the BS. For the MaxMin allocation, the total number of transmitted packets decreases with an increasing number of devices, as the MaxMin targets a fairness in  $N_i$ . For the Equal allocation and for the NBS, the total number of transmitted packets is increasing with the number of devices, because a higher number of possible relays can appear in proximity of the source device due to a higher number of devices in the area.

In the Figure 4a, we show fairness in gained number of



Fig. 4. Jain's fairness index of number of transmitted packets gained by cooperation (a) and CDF of the gain (b).

transmitted packets via Jain's fairness index. The fairness in gain is highest for the NBS, as the NBS motivates devices to cooperate via fair sharing of benefits by all devices. A lower fairness in the distribution of the gain among the devices for the MaxMin is a result of the fact that the algorithm targets fairness in  $N_i$ , but disregards gain in the number of transmissions by individual devices. The Equal allocation splits the time fairly, but disregards channel quality and provides the worst fairness in gain out of all the compared schemes.

Figure 4b shows that the NBS distributes the gain in the number of transmitted packets more fairly among the devices comparing to the Equal allocation for 100 and 200 devices.

## VI. CONCLUSION

In this letter, we have derived energy consumption-based Nash Bargaining solution for allocation of communication resources maximizing the number of transmitted packets. The derived NBS motivates the devices to cooperation. The NBS is in closed form, thus, it is applicable even to wireless communication with frequently varying channel. Due to a very low complexity, the derived NBS is scalable and suitable for scenarios with very high number of devices, as envisioned in 5G mobile networks .

In the future, the NBS should be extended to OFDMA over frequency selective channels and mobility of devices.

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