

Joint Sponsored and Edge Caching Content Service Market: A Game-Theoretic Approach

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Abstract—In a sponsored content scheme, a wireless network operator negotiates with a sponsored content service provider in which the latter can pay the former to lower the cost of the mobile subscribers/users to access certain content. As such, the scheme motivates the entities in the sponsored content ecosystem to be more actively involved. Meanwhile, with the forthcoming 5G cellular networks, edge caching becomes a promising technology for traffic offloading to reduce cost and improve service quality of the content service. The key idea is that an edge caching content service provider caches content on edge networks. The cached content is then delivered to mobile users locally, reducing latency substantially. In this paper, we propose the joint sponsored and edge caching content service market model. We investigate an interplay between the sponsored content service provider and the edge caching content service provider under the non-cooperative game framework. Furthermore, the interactions among the wireless network operator, content service providers and mobile users are modeled as a hierarchical three-stage Stackelberg game. In the game model, we analyze the sub-game perfect equilibrium in each stage through backward induction analytically. Additionally, the existence of the proposed Stackelberg equilibrium is validated by capitalizing on the bilevel optimization programming. Based on the analysis of the game properties, we propose a sub-gradient based iterative algorithm, which guarantees to converge to the Stackelberg equilibrium.

Index Terms—Sponsored content, edge caching, content delivery network, Quality of Service, Stackelberg game, economic analysis, bilevel optimization.

I. INTRODUCTION

The demand of cellular data/content traffic continues to rise sharply, and the high data cost becomes one of the critical concerns for mobile users while consuming the content [2]. Therefore, one of the important challenges for content service providers is how to attract more mobile users to access their content, and thus achieve a higher revenue gain. Recently, the concept of *sponsored content* in wireless networks has been introduced as a new promising scheme. The scheme

utilizes a properly designed incentive mechanism to motivate mobile users to use more services and consume more contents, generating higher revenue to mobile service businesses. For example, as one of wireless network operators, AT&T launched a *sponsored data plan* that allows content service providers to pay for the data bytes that their users consume, thereby not counting into the mobile users' data quota [3]. However, this concept creates complex interactions among the entities in sponsored content ecosystem: wireless network operator, content service providers and mobile users [4].

Figure 1 illustrates the interactions among different entities in the sponsored content scheme. In particular, in a traditional mobile service model, the users are charged by the wireless network operator based on their content traffic consumption, but no charging is applied to the content service provider. With the sponsored content, the content service provider cooperates with the network operator, and the former will pay the latter based on the content traffic requested by mobile users. Note that the content service provider is called “sponsored content service provider” under sponsored content scheme. Correspondingly, the users will be partially charged or they can consume the content for free. The sponsored content scheme generates a positive cycle for the wireless network operators, content service providers and mobile users, i.e., a multi-win situation. In other words, the mobile users want to access more content that will not be counted into their data caps. The sponsored content service providers can encourage more mobile users to have higher demand for content. Accordingly, the higher demand contributes to the more payoff of the content service provider and network operator.

Meanwhile, in the 5th generation (5G) cellular networks, edge caching is emerging as a promising technology to deliver content services with lower cost and higher quality. The key idea that the edge caching content service provider caches the content on edge devices in advance. As such, when users request the content, it can be delivered directly. Furthermore, with edge caching, mobile users can obtain requested content without incurring the cellular data cost, e.g., through a WiFi connection, and thus edge caching can be viewed as a new sponsorship scheme for mobile users. In this sense, the edge caching has the potential to alleviate backbone network burden [5]. In the future 5G cellular network, to meet explosive content traffic demands and support sustainable development, edge caching is one of the most effective solutions. On the one hand, more and more cache-enabled small base stations will be deployed as the infrastructure of edge networks. On the other hand, the storage capacity has been growing rapidly

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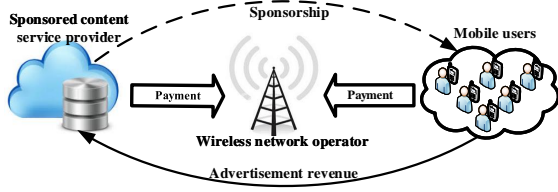


Figure 1: The interactions among the entities in sponsored content ecosystem.

and becomes much cheaper [6]. As a result, equipping caches at small base stations offers a promising way to exploit the potential of edge networks in addition to densifying the existing cellular networks. Therefore, both sponsored and edge caching content are accessible for mobile users simultaneously.

In this work, we introduce the integration of the edge caching and sponsored content schemes. In particular, we investigate the effects of their interaction and their coexistence on the mobile user behavior, the network operator in the data/content traffic market. The main contributions of this paper are summarized as follows:

- 1) We formulate a joint sponsored and edge caching content service market model to analyze the interactions among the wireless network operator, the sponsored content service provider as well as the edge caching content service provider, and mobile users.
- 2) We formulate a novel hierarchical three-stage Stackelberg game to model their interactions to jointly maximize the payoff of the wireless network operator, the profit of each content service providers, and the individual utilities of mobile users.
- 3) Through backward induction, we analyze the sub-game perfect equilibrium in each stage analytically. In particular, we prove that an optimal strategy of the mobile user in Stage III is unique, and accordingly demonstrate the uniqueness of the Nash equilibrium among the content service providers in Stage II.
- 4) Furthermore, the existence of the Stackelberg equilibrium is validated by capitalizing on the bilevel optimization technique. Based on our theoretical discoveries regarding the properties of the equilibrium in the Stackelberg game, we propose a sub-gradient based iterative algorithm that guarantees the convergence to the Stackelberg equilibrium.
- 5) We conduct extensive numerical simulations to evaluate the performance of all the players in the proposed Stackelberg game. The results draw some useful engineering insights, e.g., the monopoly wireless network operator intends to set the maximum possible value as the optimal price for payoff maximization.

The rest of the paper is organized as follows. Section II provides reviews for the related work. Section III presents the system description and Section IV formulates a hierarchical three-stage Stackelberg game to model the interactions among the players in the joint sponsored and edge caching content service market. The equilibrium analysis through backward

induction for optimal strategies of players are shown in Section V. Section VI provides the numerical results for performance evaluation. Section VII concludes the paper with summary and future directions.

II. RELATED WORKS

With remarkable interests from academia and industry, the sponsored content concept has attracted many researchers to investigate and innovate better schemes. The authors in [7] focused on the interaction between the Sponsored Content Service Provider (SCSP) and the Wireless Network Operator (WNO) in a two-sided platform, where the demand of Mobile User (MU) is assumed to be a random variable. In [8], the authors studied the sponsorship competition among multiple SCSPs in a Internet content market and demonstrated that the competitions improve the welfare for the WNO and SCSPs. The interactions among the WNO, SCSP and MUs were modeled as a Stackelberg game in [9]. Then, the authors in [10], [11] studied the similar problem proposed in [12], where the utilities of MUs are coupled due to the underlying social network effects. The authors in [13] studied the service-selection process among MUs as an evolutionary population game and demonstrated that sponsoring can help improve the SCSP's profit and the MU's quality of experience.

Recent studies have shown that edge caching emerges as a promising paradigm to alleviate the increasing data/content traffic burden of traditional cellular networks [5], [6], [14]–[17]. The authors in [14] provided a review on the techniques related to caching in existing wireless networks. The authors introduced the methods to predict the content popularity distributions and MUs' preferences in [5], and further investigated the impact of content delivery and placement on wireless caching. In [6], the authors demonstrated that local caching at base stations can improve the energy efficiency of the cellular networks. The authors in [15] studied the competition among multiple edge caching content service providers (ECCSP) and formulated a resource allocation problem. Each ECCSP decides its optimal reserved cache memory in the edge network for enhancing the QoS of MUs. In [17], the authors considered a simple model, where the single CSP offers both sponsored and edge caching content service to MUs and the requests for the content service from MUs are assumed to be random. A two-stage decision problem is then formulated to model the interaction between the CSP and MUs.

However, to the best of our knowledge, none of existing works explore the great benefit of sponsored content concept and edge caching. They can clearly complement each other, from the users' perspective, which encourages the users to consume more services while improving service quality, and hence this is the objective of this paper.

III. SYSTEM MODEL

As illustrated in Fig. 2(a), we consider a mobile video content delivery network as a market consisting of four entities: WNO, SCSP, ECCSP, and a pool of MUs. Due to the fact that the same video is often available from many content providers, which applies particularly to popular videos [18],

[19]. Therefore, there is a competition among different content providers as they may provide the same video contents to the same pool of MUs. In order to obtain some insights, we consider the scenario where the SCSP and the ECCSP both have the same popular video contents. In particular, the SCSP and the ECCSP compete for the video users, e.g., Netflix [20] and Hulu [21]. In other words, the video contents requested by MUs are available from both the SCSP and the ECCSP. The MUs can choose to access and consume the video contents from the SCSP and ECCSP. If the MU accesses the content from the SCSP, the content is downloaded directly through the network infrastructure of WNO. The SCSP can sponsor partly or fully the data transfer from the WNO to the MU. On the contrary, the MU can access the content stored in an edge caching device from the ECCSP. The MU can download the content from the device locally without involving the WNO. In practice, the MU requests for the content by using an application agent, where the content source, i.e., from the SCSP and ECCSP, can be chosen proactively by the MU. Again, since the MU can choose the services from two sources, the SCSP and ECCSP compete with each other to attract demand from the users.

Let y denote the content (volume) demanded by MUs, and $\sigma_e f(y)$ denote the utility obtained from accessing and consuming the content, where the factor $\sigma_e > 0$ represents the utility coefficient of MUs, e.g., a particular valuation between MUs and content. Similar to that in [9], [12], we adopt the following function:

$$f(y) = \frac{1}{1-\alpha} y^{1-\alpha}, \quad (1)$$

where $0 < \alpha < 1$ is a given coefficient. In particular, $f(\cdot)$ is a non-decreasing and concave function with decreasing marginal satisfaction. This reflects the decreasing marginal preference of MUs. In traditional wireless content access, the WNO charges each MU p for per unit volume of content downloaded. Thus, the general utility of the MU which has the content demand y is given as: $v(y) = \sigma_e f(y) - py$.

A. Content Access from Sponsored Content Service Provider

As discussed in Section I, under the sponsored content scheme launched by the WNO, the payment from MUs to the WNO can be partly sponsored by the SCSP. Let $\theta \in [0, 1]$ represent the sponsorship factor of content decided by the SCSP. Again, the sponsorship fee is paid by the SCSP to the WNO. Thus, the MU pays for $(1-\theta)y$ units of content to the WNO, and thus the cost incurred to the MU is $(1-\theta)py$ [10]. Generally, the MUs are also affected by another variable l_a which is the amount of advertisement imposed by the SCSP and ECCSP per volume of content. We assume that l_a is constant for all contents. For example, Pandora Internet Radio plays advertisement at regular intervals between songs. We assume the normalized $l_a \in [0, 1]$ since both the ECCSP and SCSP cannot have more advertisement than its content. For the ease of derivation later, we introduce an auxiliary variable τ defined as: $\tau = \frac{1}{1+l_a}$, $\tau \in [\frac{1}{2}, 1]$. Thus, the utility of the MU which has the content demand y from the SCSP is expressed by $u_s(y) = \tau \sigma_e f(y) - (1-\theta)py$.

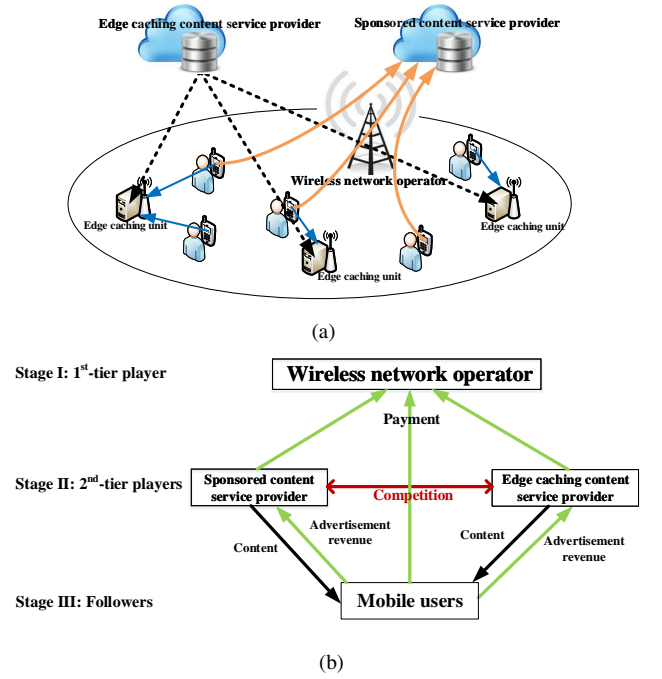


Figure 2: (a) System model for joint sponsored and edge caching content service market; (b) A hierarchical three-stage Stackelberg game to model the interactions among the wireless network operator, the sponsored content service provider as well as the edge caching content service provider, and mobile users.

B. Content Access from Edge Caching Content Service Provider

Considering edge caching, the ECCSP is able to cache the video contents in the edge caching devices, and thus the MUs can access the cached content from the ECCSP through a local network connection. It is worth noting that the ECCSP cannot cache all the content it has in the edge devices due to the limited size, and thus the ECCSP will refresh the video contents in the edge devices after a relatively long time. We denote t ($t \in [0, 1]$) as the caching effort of the ECCSP, which indicates the sponsorship from the ECCSP to MUs. Again, the MU accessing the cached content from the ECCSP, the advertisement from the ECCSP is imposed to the MU which lowers its utility. Nevertheless, accessing the cached content, the MU does not need to pay the WNO [17]. Accordingly, the utility of the MU which has demand z for the cached content from the ECCSP is expressed as $u_e(z) = \tau \sigma_e f(z)g(t) - cz$, where c is the network handover¹ cost [22].

The caching effort t indicates the amount of resources, e.g., storage, transmit power, bandwidth, and computing, allocated

¹According to [17], [22], when the MU switches its access from a cellular link to a local network link such as Wi-Fi, it involves with the discomfort caused by the user rebuffering as well as long join time, and the additional energy loss, which is captured by the handover cost in the model. The evaluation of the synchronized handover scheme or algorithm is a well-studied topic [22], and hence is not the focus of this paper. For example, some multimedia protocol allows applications to specify the current position of playout (pause, resume). The users can easily click the pause and resume to finish the content synchronization. As such, we consider that the content synchronization in the smart phone is auto-complete which is not a manual work for the MU.

for delivering the cached content to the MUs. $g(t)$ is defined as content delivery quality. In particular, $g(t)$ is a monotonically increasing function reflecting the positive influence of caching effort t on MUs' experience. That is, the greater t , the better quality $g(t)$, but the increase in the quality becomes lower when t is larger. Similar to (1), we adopt the a common function to capture the Quality of Service (QoS) experienced by MUs [23], i.e.,

$$g(t) = \frac{1}{1-\beta} t^{1-\beta}, \quad (2)$$

where $0 < \beta < 1$ is a coefficient.

In the next section, based on the system description presented, we provide the detailed analysis for the joint sponsored and edge caching content service model.

IV. GAME FORMULATION FOR JOINT SPONSORED AND EDGE CACHING CONTENT SERVICE MODEL

In this section, we formulate a hierarchical three-stage Stackelberg game to model the interactions among the WNO, SCSP, ECCSP, and MUs as illustrated in Fig. 2(b). We analyze the sub-game problems from each stage in Section IV-A, IV-B and IV-C.

A. Mobile Users in Stage III (Followers)

Given the popular video contents under our consideration, the content demand requested by the MU can be sponsorable and cachable at the same time². Thus, in addition to accessing the sponsored content from the SCSP, the MU has an alternative choice, i.e., to access the cached content from an edge caching device deployed by the ECCSP. Each MU determines a fraction of the content demand to access from the SCSP denoted by $x \in [0, 1]$. Thus, the fraction of cached content demand from the ECCSP through a local network connection is $1 - x$. Note that we normalize the content demand as 1 to facilitate the tractability of the three-stage Stackelberg game analysis and obtain some insights into the problem. In particular, the analytical results will not structurally change even if the content demand is not normalized.

Each myopic MU needs to consider how to balance its content demand from the SCSP and ECCSP to maximize its utility. In particular, the utility of the MU from taking the action x is expressed as follows:

$$\begin{aligned} u(x; \theta, t, p) &= u_s(x) + u_e(1-x) \\ &= \tau \sigma_e f(x) - (1-\theta)xp \\ &\quad + \tau \sigma_e f(1-x)g(t) - (1-x)c, \end{aligned} \quad (3)$$

where $f(x)$ and $g(t)$ are given in (1) and (2), respectively.

²Even if there exist some non-cachable video contents that the MUs can only access through the SCSP, the studied scenario degenerates into the pure sponsored content scenario proposed in [9], i.e., the scenario without caching. This is a special case of the scenario under our consideration, in which the content demand of the MU through the ECCSP is imposed as zero. Likewise, if there exist some non-sponsorable as well as non-cachable video contents that the MU can only fetch with the full payment, the current game model degenerates into the two-stage Stackelberg game model that only includes the WNO as the leader and the MU as the follower. Such traditional pricing problem between the WNO and MUs is a well-studied topic [24], and hence is out of the scope of this paper.

Given the volume unit price set by the WNO, the sponsorship factor θ from the SCSP and the caching effort t from the ECCSP, the MU chooses x to maximize the utility, and thus each MU sub-game problem can be written as follows:

Problem 1. (The MU sub-game):

$$\begin{aligned} &\underset{x}{\text{maximize}} && u(x; \theta, t, p) \\ &\text{subject to} && x \in [0, 1]. \end{aligned} \quad (4)$$

B. SCSP and ECCSP in Stage II (2nd-Tier Players)

Being aware of the pricing strategy of the WNO, both the SCSP and ECCSP determine their individual strategy competitively and simultaneously.

1) *Sponsored Content Service Provider:* The goal of the SCSP is to maximize its profit, i.e., advertisement revenue minus the sponsorship fee provided for the MUs. The profit is expressed as follows:

$$\Pi_s(\theta; p) = \sigma_c h(x) - \theta p x. \quad (5)$$

We denote $\sigma_c h(x)$ as the advertisement revenue [9], [12], where σ_c is the advertisement revenue coefficient and

$$h(x) = \frac{1}{1-\gamma} x^{1-\gamma}, \quad (6)$$

where $0 < \gamma < 1$ which is a coefficient. In fact, we could also adopt a linear dependency between demand and ad revenue. However, most of the related works have proved that a function with diminishing return would model the ad revenue more closely [9], [10], [12], [25].

The SCSP is willing to provide the sponsorship to lower the cost for MUs, which in turn attracts more MUs to access and consume the content. However, the more sponsorship may incur excessive cost. Thus, the SCSP sub-game is defined as follows:

Problem 2-A. (The SCSP sub-game):

$$\begin{aligned} &\underset{\theta}{\text{maximize}} && \Pi_s(\theta; p) \\ &\text{subject to} && \theta \in [0, 1]. \end{aligned} \quad (7)$$

2) *Edge caching content service provider:* Recall that the cost of the ECCSP for caching the content with caching effort t is Ct , as discussed in Section III. Likewise, the ECCSP aims at maximizing its profit, i.e., the advertisement revenue from content traffic minus the cost for content caching, which is expressed as follows:

$$\Pi_e(t; p) = \sigma_c h(1-x) - Ct, \quad (8)$$

where $h(\cdot)$ is given in (6). We assume that the baseline content caching cost is C , and thus the cost of the ECCSP for caching the content with caching effort t is Ct [15], [17], [26]. The ECCSP has an incentive to increase the caching effort which provides better-quality service to the MUs and hence attracts more MUs to access more content from the ECCSP. However, increasing the caching effort increases the cost incurred to the ECCSP. Note that in order to provide cache services, the ECCSP also needs to pay a side payment to the WNO for using the edge cache devices or small base stations deployed by the WNO [27]. In the paper, we consider that the ECCSP

“rents” the cache service in which the payment is fixed for a certain time period. Thus, it does not affect the game strategies of ECCSP or WNO. Similar to [17], [19], we assume such payment to be a constant cost for the ECCSP and thus is omitted for ease of presentation and analysis³. Accordingly, the profit maximization of the ECCSP is expressed as follows:

Problem 2-B. (The ECCSP sub-game):

$$\begin{aligned} & \underset{t}{\text{maximize}} \quad \Pi_e(t; p) \\ & \text{subject to} \quad t \in [0, 1]. \end{aligned} \quad (9)$$

Generally, the ECCSP has the potential of offloading as well as relieving the backbone network burden, and reducing content delivery cost especially when the number of mobile users is increasing considerably.

C. Wireless Network Operator in Stage I (1st-Tier Player)

The WNO sets the volume unit price p for data traffic. In addition to obtaining its revenue from charging the SCSP and MUs, the WNO has the content delivery cost. Accordingly, the objective of the WNO is to maximize its payoff, which is expressed as follows:

$$\mathcal{P}(p) = px - wx^2, \quad (10)$$

where wx^2 denotes the corresponding cost, and w represents the unit cost of content delivery. The quadratic sum form reflects the marginal cost increases as the total demand increases, e.g., due to congestion effects, which is a widely-accepted assumption [28]. We have the strategy space of the WNO be $\{\mathbf{P} : 0 \leq p \leq \bar{p}\}$, where \bar{p} is the maximum price. Therefore, the payoff maximization problem of the WNO is formulated as follows:

Problem 3. (The WNO sub-game):

$$\begin{aligned} & \underset{p}{\text{maximize}} \quad \mathcal{P}(p) \\ & \text{subject to} \quad p \in [0, \bar{p}]. \end{aligned} \quad (11)$$

Note that we assume a monopolistic WNO in this work, while many WNOs, e.g., in the US, are oligopolists. However, due to low churn rates, they are often effective monopolies [9].

The **Problems 1, 2-A, 2-B** and **3** altogether form a hierarchical three-stage Stackelberg game with complete information⁴. The objective of the game is to find the Stackelberg equilibrium. The Stackelberg equilibrium is the point where the

³The optimization of the side payment and the impact of such payment on the interactions among four entities in the current model is worth further investigation in the future studies.

⁴In certain scenarios, the information and strategy of the player cannot be predicted accurately by others, which lead to the incomplete information environments. As such, we can employ the Bayesian game to describe the strategic behaviors among players. Comparing with the complete information game, Bayesian game introduces two new concepts: the types of players, and the probability distribution of the types. The probability distribution is the belief about unknown information of the players, referred to as the type of the player, e.g., the utility coefficient of MUs. Such distribution information can be obtained through, e.g., historical information or long-term learning. In this regard, each MU knows its own expected payoff instead of specific payoff. Thus, each player seeks for a strategy profile that maximizes its expected payoff given the belief on the strategies of other players and type distribution. Thereafter, the game equilibrium analysis for the problem can be similarly applied by following [29].

payoff of the leader is maximized provided that the followers adopt their best responses, i.e., the Nash equilibrium [30].

To investigate the Stackelberg equilibrium of the proposed game, we seek for each sub-game (\mathcal{G}^u , \mathcal{G}^c , and \mathcal{G}^w) a perfect equilibrium using backward induction to determine the strategies of all the players. In Stage III, the MUs are not coupled with each other, and thus we can analyze an optimal fraction of content demand of each MU in the sub-game independently. Since there is only one type of players in this sub-game \mathcal{G}^u , the best response of the MU can be obtained by directly solving **Problem 1**. In Stage II, the sub-game \mathcal{G}^c among the 2nd-tier players, i.e., the SCSP and ECCSP is a non-cooperative game. For a non-cooperative game, the Nash equilibrium is defined as the point at which no player can improve its payoff by changing its strategy unilaterally. The best response of the 2nd-tier players can be obtained by solving **Problems 2-A** and **2-B**. Note that **Problem 1** must be solved first since the 2nd-tier players derive their individual best responses based on those of 1st-tier players, i.e., MUs. Similarly, **Problems 2-A** and **2-B** need to be solved before solving **Problem 3**, since the best response of the 1st-tier player, the WNO, is dependent on those of the 2nd-tier players, i.e., the ECCSP and SCSP. In the next section, we solve the **Problems 1, 2-A, 2-B** and **3** in each stage sequentially.

V. GAME EQUILIBRIUM ANALYSIS

In this section, we consider the sub-game perfect equilibrium in each stage sequentially by employing backward induction. Specifically, we analyze the content demand of MUs in Stage III, the sponsoring strategy of the SCSP as well as the caching strategy of the ECCSP in Stage II, and the pricing strategy of the WNO in Stage I.

A. Stage III: MU's content demand

Given price p decided by the WNO, sponsorship factor θ and caching effort t determined by the SCSP and ECCSP, respectively, the MUs decide on their optimal content demand for utility maximization individually in the sub-game \mathcal{G}^u . Since the utilities of different MUs are not coupled with each other, the sub-game of the MUs can be modeled as an optimization problem. Thus, we analyze the sub-game \mathcal{G}^u by solving **Problem 1**. By substituting $f(\cdot)$ in (1) and $g(\cdot)$ in (2) into (3), we obtain the following expression for the utility of the MU which has the decision variable x ,

$$u(x; \theta, t, p) = \frac{\tau\sigma_e}{1-\alpha} x^{1-\alpha} + \frac{\tau\sigma_e t^{1-\beta}}{(1-\alpha)(1-\beta)} (1-x)^{1-\alpha} - (1-x)c - (1-\theta)xp. \quad (12)$$

Then, we take the first order and second order derivatives of (12) with respect to x to prove its concavity, which can be written as follows:

$$\frac{\partial u}{\partial x} = \tau\sigma_e x^{-\alpha} - \frac{\tau\sigma_e t^{1-\beta}}{1-\beta} (1-x)^{-\alpha} + c - (1-\theta)p, \quad (13)$$

$$\frac{\partial^2 u}{\partial x^2} = -\alpha\tau\sigma_e x^{-\alpha-1} - \frac{\alpha\tau\sigma_e t^{1-\beta}}{1-\beta} (1-x)^{-\alpha-1} < 0. \quad (14)$$

The second order derivative of $u(\cdot)$ is always negative, and thus u is strictly concave with respect to x . We also know that the strategy space of x is a convex and compact subset of the Euclidean space. Accordingly, we can conclude with the following proposition immediately [30].

Proposition 1. *The sub-game perfect equilibrium in the sub-game \mathcal{G}^u is unique.*

Furthermore, based on the first order derivative condition, we have

$$\frac{\partial u}{\partial x} = \tau\sigma_e x^{-\alpha} - \frac{\tau\sigma_e t^{1-\beta}}{1-\beta}(1-x)^{-\alpha} + c - (1-\theta)p = 0, \quad (15)$$

and we can show that

$$x^{-\alpha} - \frac{t^{1-\beta}}{1-\beta}(1-x)^{-\alpha} = \frac{1}{\tau\sigma_e}[(1-\theta)p - c]. \quad (16)$$

We have the following conclusions immediately. If the best response of the MU is larger than 1, then $x^* = 1$, and if the best response of the MU is smaller than 0, then $x^* = 0$. If the best response of the MU with respect to its content demand x^* is within the strategy space $[0, 1]$, the best response of the MU, x^* satisfies the condition in (16), i.e.,

$$x^{*-\alpha} - \frac{t^{1-\beta}}{1-\beta}(1-x^*)^{-\alpha} = \frac{1}{\tau\sigma_e}[(1-\theta)p - c]. \quad (17)$$

B. Stage II: SCSP's sponsoring strategy and ECCSP's caching strategy

Based on the sub-game equilibrium in \mathcal{G}^u obtained from the Stage III, the 2nd-tier players, i.e., the SCSP and ECCSP optimize their sponsoring and caching strategies for profit maximization competitively, respectively. The optimal strategies of both the SCSP and ECCSP are obtained by solving the **Problems 2-A** and **2-B**.

We first analyze the optimal sponsoring strategy of the SCSP. From (5), we have the profit of the SCSP, which is reformulated as follows:

$$\Pi_s(\theta; p) = \sigma_c \frac{1}{1-\gamma} x^{*1-\gamma} - \theta p x^*, \quad (18)$$

where x^* is the best response of the MU given the strategies of other players in the game. The first and second derivatives of profit $\Pi_s(\theta; p)$ with respect to the sponsorship factor θ are given as $\frac{\partial \Pi_s(\theta; p)}{\partial \theta} = \sigma_c x^{*- \gamma} \frac{\partial x^*}{\partial \theta} - p x^* - \theta p \frac{\partial x^*}{\partial \theta}$ and $\frac{\partial^2 \Pi_s(\theta; p)}{\partial \theta^2} = -\gamma \sigma_c x^{*- \gamma - 1} \left(\frac{\partial x^*}{\partial \theta} \right)^2 + \sigma_c x^{*- \gamma} \frac{\partial^2 x^*}{\partial \theta^2} - 2p \frac{\partial x^*}{\partial \theta} - \theta p \frac{\partial^2 x^*}{\partial \theta^2}$.

From the condition in (16), we can obtain $\frac{\partial x^*}{\partial \theta}$ and $\frac{\partial^2 x^*}{\partial \theta^2}$, the steps of which are shown as follows. The first order partial derivatives of (16) with respect to θ is expressed as $\left[x^{*- \alpha - 1} + \frac{t^{1-\beta}}{1-\beta}(1-x^*)^{-\alpha-1} \right] \frac{\partial x^*}{\partial \theta} = \frac{p}{\alpha \tau \sigma_e}$. Consequently, we can obtain

$$\frac{\partial x^*}{\partial \theta} = \frac{p}{\alpha \tau \sigma_e \left[x^{*- \alpha - 1} + \frac{t^{1-\beta}}{1-\beta}(1-x^*)^{-\alpha-1} \right]} > 0. \quad (19)$$

The second order partial derivatives of (16) with respect to θ is expressed as follows:

$$\begin{aligned} & \left[-x^{*- \alpha - 2} + \frac{t^{1-\beta}}{1-\beta}(1-x^*)^{-\alpha-2} \right] (\alpha+1) \left(\frac{\partial x^*}{\partial \theta} \right)^2 \\ & + \left[x^{*- \alpha - 1} + \frac{t^{1-\beta}}{1-\beta}(1-x^*)^{-\alpha-1} \right] \frac{\partial^2 x^*}{\partial \theta^2} = 0. \end{aligned} \quad (20)$$

By substituting (19) into (20), we have

$$\begin{aligned} \frac{\partial^2 x^*}{\partial \theta^2} &= - \left[-x^{*- \alpha - 2} + \frac{t^{1-\beta}}{1-\beta}(1-x^*)^{-\alpha-2} \right] \\ & \times \frac{(\alpha+1)p^2}{\alpha^2 \tau^2 \sigma_e^2 \left[x^{*- \alpha - 1} + \frac{t^{1-\beta}}{1-\beta}(1-x^*)^{-\alpha-1} \right]^3}. \end{aligned} \quad (21)$$

Likewise, we next analyze the optimal caching strategy of the ECCSP. From (8), we have the profit of the ECCSP, which is expressed as follows:

$$\Pi_e(t; p) = \sigma_c \frac{1}{1-\gamma} (1-x^*)^{1-\gamma} - Ct. \quad (22)$$

The first and second derivatives of profit $\Pi_e(t; p)$ with respect to caching effort t are given as follows:

$$\frac{\partial \Pi_e(t; p)}{\partial t} = -\sigma_c (1-x^*)^{-\gamma} \frac{\partial x^*}{\partial t} - C, \quad (23)$$

and

$$\begin{aligned} \frac{\partial^2 \Pi_e(t; p)}{\partial t^2} &= -\gamma \sigma_c (1-x^*)^{-\gamma-1} \left(\frac{\partial x^*}{\partial t} \right)^2 \\ & - \sigma_c (1-x^*)^{-\gamma} \frac{\partial^2 x^*}{\partial t^2}. \end{aligned} \quad (24)$$

From the condition in (16), we derive $\frac{\partial x^*}{\partial t}$ and $\frac{\partial^2 x^*}{\partial t^2}$ with the following steps. The first order partial derivatives of (16) with respect to t is expressed as follows:

$$\left[x^{*- \alpha - 1} + \frac{t^{1-\beta}}{1-\beta}(1-x^*)^{-\alpha-1} \right] \frac{\partial x^*}{\partial t} = -\frac{t^{-\beta}}{\alpha} (1-x^*)^{-\alpha}. \quad (25)$$

Accordingly, we have

$$\frac{\partial x^*}{\partial t} = \frac{-t^{-\beta}(1-x^*)^{-\alpha}}{\alpha \left[x^{*- \alpha - 1} + \frac{t^{1-\beta}}{1-\beta}(1-x^*)^{-\alpha-1} \right]} < 0. \quad (26)$$

Similarly, we obtain the second order partial derivatives of (16) with respect to t as follows:

$$\begin{aligned} & \left[x^{*- \alpha - 2} + \frac{t^{1-\beta}}{1-\beta}(1-x^*)^{-\alpha-2} \right] (\alpha+1) \left(\frac{\partial x^*}{\partial t} \right)^2 \\ & + \left[x^{*- \alpha - 1} + \frac{t^{1-\beta}}{1-\beta}(1-x^*)^{-\alpha-1} \right] \frac{\partial^2 x^*}{\partial t^2} \\ & + 2t^{-\beta}(1-x^*)^{-\alpha-1} \frac{\partial x^*}{\partial t} = \frac{\beta}{\alpha} t^{-\beta-1} (1-x^*)^{-\alpha}. \end{aligned} \quad (27)$$

By substituting (26) into (27), we have

$$\begin{aligned} \frac{\partial^2 x^*}{\partial t^2} = & \frac{t^{-\beta}(1-x^*)^{-\alpha}}{\alpha \left[x^{*- \alpha - 1} + \frac{t^{1-\beta}}{1-\beta} (1-x^*)^{-\alpha-1} \right]} \\ & \times \left\{ \frac{2t^{-\beta}(1-x^*)^{-\alpha-1}}{x^{*- \alpha - 1} + \frac{t^{1-\beta}}{1-\beta} (1-x^*)^{-\alpha-1}} + \frac{\beta}{t} \right. \\ & \left. + \frac{-(\alpha+1)t^{-\beta}(1-x^*)^{-\alpha} \left[-x^{*- \alpha - 2} + \frac{t^{1-\beta}}{1-\beta} (1-x^*)^{-\alpha-2} \right]}{\alpha \left[x^{*- \alpha - 1} + \frac{t^{1-\beta}}{1-\beta} (1-x^*)^{-\alpha-1} \right]^2} \right\}. \end{aligned} \quad (28)$$

By analyzing the profits of the SCSP and ECCSP given in (18) and (22), respectively, we have the following proposition.

Proposition 2. *The existence of the Nash equilibrium in the non-cooperative sub-game \mathcal{G}^c is guaranteed if the following conditions*

$$\sigma_c x^{*- \gamma} - \theta p > 0, \quad (29)$$

$$-x^{*- \alpha - 2} + \frac{t^{1-\beta}}{1-\beta} (1-x^*)^{-\alpha-2} > 0, \quad (30)$$

and

$$\gamma + \alpha - 1 > 0 \quad (31)$$

hold.

Proof. From (21), we can easily know that $\frac{\partial^2 x^*}{\partial \theta^2} < 0$ under the condition in (30). Furthermore, we have $\frac{\partial x^*}{\partial \theta} > 0$ and

$$\begin{aligned} \frac{\partial^2 \Pi_s(\theta; p)}{\partial \theta^2} = & -\gamma \sigma_c x^{*- \gamma - 1} \left(\frac{\partial x^*}{\partial \theta} \right)^2 \\ & - (\sigma_c x^{*- \gamma} - \theta p) \frac{\partial^2 x^*}{\partial \theta^2} - 2p \frac{\partial x^*}{\partial \theta}. \end{aligned} \quad (32)$$

Consequently, we can conclude that $\frac{\partial^2 \Pi_s(\theta; p)}{\partial \theta^2} < 0$ under the condition in (29). Then, we analyze the properties of $\frac{\partial^2 \Pi_e(t; p)}{\partial t^2}$. From (24), in order to prove that $\frac{\partial^2 \Pi_e(t; p)}{\partial t^2}$ is negative, we need to prove that

$$\gamma(1-x^*)^{-1} \left(\frac{\partial x^*}{\partial t} \right)^2 + \frac{\partial^2 x^*}{\partial t^2} > 0. \quad (33)$$

By substituting (26) and (28) into (33), and with simple steps, we have

$$\begin{aligned} & \underbrace{(\gamma + 2\alpha)(1-x^*)^{-1} x^{*- \alpha - 1}}_{>0} + \underbrace{(\alpha + 1)x^{*- \alpha - 2}}_{>0} \\ & + (\gamma + \alpha - 1) \underbrace{\frac{t^{1-\beta}}{1-\beta} (1-x^*)^{-\alpha-2}}_{>0} > 0. \end{aligned} \quad (34)$$

Accordingly, under the condition in (31), the inequality given in (34) is satisfied. Therefore, the negativity of $\frac{\partial^2 \Pi_e(t; p)}{\partial t^2}$ is proved.

The strategy space of the SCSP is defined to be within $[0, 1]$, which is a non-empty, convex, and compact subset of the Euclidean space. As aforementioned, we prove that the second partial derivative of the SCSP's objective function with respect to its decision variable is negative, i.e., $\frac{\partial^2 \Pi_s(\theta; p)}{\partial \theta^2} < 0$. Therefore, the profit function of the SCSP, $\Pi_s(\theta; p)$, is continuous and strictly concave with respect to θ . Likewise, the

profit function of the ECCSP, $\Pi_e(t; p)$, is strictly concave with respect to its decision variable t . This is because we prove that the second order partial derivative of $\Pi_e(t; p)$ with respect to t is negative, i.e., $\frac{\partial^2 \Pi_e(t; p)}{\partial t^2} < 0$. Thus, the strict concavity of the objective function of the ECCSP $\Pi_e(t; p)$ is ensured. Moreover, the strategy space of the ECCSP, $[0, 1]$, is also a non-empty convex and compact subset of the Euclidean space. Therefore, the Nash equilibrium exists in this non-cooperative sub-game \mathcal{G}^c between the SCSP and ECCSP [30]. The proof is now completed. \square

Finally, we prove the uniqueness of the Nash equilibrium in the non-cooperative sub-game between the SCSP and ECCSP, as shown in the following proposition.

Proposition 3. *The Nash equilibrium in the non-cooperative sub-game \mathcal{G}^c is unique provided*

$$2\alpha - 1 > 0. \quad (35)$$

Proof. Please refer to the appendix for details. \square

C. Stage I: WNO's pricing strategy

In Stage I, the monopolist WNO determines the optimal price p^* by solving **Problem 3** with the optimal sponsorship factor θ^* as well as the optimal caching effort t^* obtained from Stage II, and the optimal content demand x^* obtained from Stage I. Thus, **Problem 3** can be reformulated as follows:

$$\text{maximize } \mathcal{P}(p) = px^* - wx^{*2} \quad (36)$$

$$\text{subject to } 0 \leq p \leq \bar{p}, \quad (37)$$

$$\theta^* = \arg \max_{\theta} \Pi_s, \quad (38)$$

$$t^* = \arg \max_t \Pi_e, \quad (39)$$

$$\sigma_c x^{*- \gamma} - \theta^* p > 0, \quad (40)$$

$$-x^{*- \alpha - 2} + \frac{t^{*1-\beta}}{1-\beta} (1-x^*)^{-\alpha-2} > 0, \quad (41)$$

$$x^{*- \alpha} - \frac{t^{*1-\beta}}{1-\beta} (1-x^*)^{-\alpha} = \frac{1}{\tau \sigma_e} [(1-\theta^*)p - c]. \quad (42)$$

The constraint in (37) is used for the strategy space of the WNO, the constraints in (38) and (39) indicate that θ^* and t^* denote the best responses of the SCSP and ECCSP, respectively, provided that the price p is given. The constraint in (42) is derived from the best response of the MU when θ , t and p are given, which represents the implicit function of $x^*(\theta, t, p)$. The constraints in (40) and (41) are given in Proposition 2, which ensure the existence and the uniqueness of the non-cooperative sub-game \mathcal{G}^c .

Let \mathcal{X} denote $x^*(\theta, t, p) : \mathcal{D}_{\theta} \otimes \mathcal{D}_t \otimes \mathcal{D}_p \mapsto \mathcal{D}_x$, where \otimes denotes the Cartesian product, \mathcal{D}_{θ} , \mathcal{D}_t and \mathcal{D}_p represent the domains of θ , t and p , respectively. Note that these domains are all close sets. Furthermore, we define $\Pi(\theta, t, p)$ and $g(\theta, t, p)$ as follows:

$$\begin{aligned} \Pi(\theta, t, p) &= \begin{bmatrix} \sigma_c h(x^*(\theta, t, p)) - \theta p x^*(\theta, t, p) \\ \sigma_c h(1-x^*(\theta, t, p)) - Ct \end{bmatrix} \\ &= \begin{bmatrix} \sigma_c h(\mathcal{X}) - \theta p \mathcal{X} \\ \sigma_c h(1-\mathcal{X}) - Ct \end{bmatrix}, \end{aligned} \quad (43)$$

$$g(\theta, t, p) = \begin{bmatrix} -\sigma_c \mathcal{X}^{-\alpha} + \theta p + v \\ \mathcal{X}^{-\alpha-2} - \frac{t^{1-\beta}}{1-\beta} (1-\mathcal{X})^{-\alpha-2} + v \end{bmatrix}, \quad (44)$$

where v which is a small number. In addition, we define $\rho = \begin{bmatrix} \theta \\ t \end{bmatrix} : \mathcal{D}_\theta \otimes \mathcal{D}_t \mapsto \mathcal{D}_\rho$. As a result, **Problem 3** can be redefined as the bilevel programming problem, which is shown as follows:

$$\begin{aligned} & \underset{0 < p < \bar{p}}{\text{maximize}} && \mathcal{P}(p) = p\mathcal{X} - w\mathcal{X}^2 \\ & \text{subject to} && \rho^* = \arg \max_{\rho} \Pi(\rho, p), \\ & && g(\rho, p) \leq 0, \\ & && \rho \in \mathcal{D}_\rho. \end{aligned} \quad (45)$$

Recall from Section V-B, we prove that for any given p , there exists a unique pair of θ^* and t^* as the optimal solution of **Problem 2**, i.e., the Nash equilibrium in non-cooperative sub-game \mathcal{G}_c . Accordingly, the optimal solution to the lower-

level programming problem $\rho^* = \begin{bmatrix} \theta^* \\ t^* \end{bmatrix}$ is shown to exist

and be unique, for any given p . Therefore, the strong sufficient optimality condition of second order (SSOSC) is satisfied for the bilevel programming problem in (45) because of the existence and the uniqueness of ρ^* (Theorem 3.9 in [31]). This indicates that the optimal solution to the lower-level programming problem of the bilevel programming problem is strongly stable. Thus, our bilevel programming problem can be reduced to a single-level problem, which is expressed as follows:

$$\begin{aligned} & \underset{0 < p < \bar{p}}{\text{maximize}} && \mathcal{P}(p) = p\mathcal{X} - w\mathcal{X}^2, \\ & \text{subject to} && \rho = U(p). \end{aligned} \quad (46)$$

$\rho = U(p)$ in the constraint can be obtained using the KKT condition to the lower-level programming problem of the bilevel programming problem as follows:

$$\begin{cases} \nabla_\rho \Pi(\rho, p) - \lambda^\top \nabla_\rho g(\rho, p) = 0, \\ \lambda \perp g(\rho, p), \lambda \geq 0, g(\rho, p) \leq 0, \end{cases} \quad (47)$$

where λ denotes the Lagrangian multiplier vector. Moreover, the feasible domain of the single-level programming is defined as follows:

$$\Omega(p, \rho) = \{(p, \rho) | \rho = U(p), p \in \mathcal{D}_p\}, \quad (48)$$

which is a non-empty and closed set according to the Weierstrass Theorem [32]. Since we know that the optimal solution to the lower-level programming problem of the bilevel programming problem is unique, Constant Rank Constraint Qualification (CRCQ) and Mangasarian-Fromovitz Constraint Qualification (MFCQ) are satisfied⁵ by all the feasible points in $\Omega(p, \rho)$ (Theorem 3.9 in [31]). Based on Theorem 4.10 in [34], $\rho = U(p)$ is a piecewise continuously differentiable function and $(p, \rho) = (p, U(p))$ is therefore continuous on p . Furthermore, with the closed sets $\Omega(p, \rho)$ and \mathcal{D}_p , according to the well-known Closed Graph Theorem [35], we can conclude that $\rho = U(p)$ being continuous implies that the mapping $\mathcal{D}_p \mapsto \Omega(p, \rho)$ is closed. Therefore, $\Omega(p, \rho)$ is non-empty and closed, and thus the bilevel programming problem admits a

globally optimal solution, namely the Stackelberg equilibrium. Accordingly, we conclude with the following proposition.

Algorithm 1 Sub-gradient based iterative algorithm finding the Stackelberg equilibrium for joint sponsored and edge caching content service market model

- 1: **Initialization:**
Select initial input $p \in [0, \bar{p}]$, $\theta \in [0, 1]$ and $t \in [0, 1]$, $k \leftarrow 1$, step size δ ;
 - 2: **repeat**
 - 3: Each MU decide on its content demanded from the SCSP $x^{[k+1]}$, using a gradient assisted searching algorithm, e.g.,
$$x^{[k+1]} \leftarrow x^{[k]} + \mu \nabla u(x^{[k]}), \quad (49)$$

where $\mu \nabla u(x^{[k]})$ is the gradient with $\frac{\partial u(x^{[k]})}{\partial x^{[k]}}$ and μ is the step size;
 - 4: The SCSP and ECCSP update their sponsorship factor $\theta^{[k+1]}$ and caching effort and $t^{[k+1]}$ using the similar gradient assisted searching algorithm, respectively. The updated sponsorship factor and caching effort are broadcast to all MUs and the WNO;
 - 5: The WNO tries to increase or decrease its price p with a small step size δ , and calculate its corresponding payoff;
 - 6: **if** $\mathcal{P}(p^{[k]}) < \mathcal{P}(p^{[k]} + \delta)$ **then**
 - 7: $p^{\text{new}} \leftarrow \min\{\bar{p}, p^{[k]} + \delta\}$; % Increase the price
 - 8: **else**
 - 9: **if** $\mathcal{P}(p^{[k]}) < \mathcal{P}(p^{[k]} - \delta)$ **then**
 - 10: $p^{\text{new}} \leftarrow \max\{0, p^{[k]} - \delta\}$; % Decrease the price
 - 11: **else**
 - 12: $p^{\text{new}} \leftarrow p^{[k]}$; % Keep the price unchanged
 - 13: **end if**
 - 14: **end if**
 - 15: $p^{[k+1]} \leftarrow p^{\text{new}}$;
 - 16: The updated price information is broadcast to all MUs and both the SCSP and ECCSP;
 - 17: $k \leftarrow k + 1$;
 - 18: **until** $\|p^{[k]} - p^{[k-1]}\|_1 < \delta$
-

Proposition 4. *There exists at least one Stackelberg equilibrium in the proposed hierarchical three-stage Stackelberg game.*

Up to now, each stage of the proposed hierarchical three-stage Stackelberg game has been investigated. Similar to that in [36], [37], we then present the sub-gradient based algorithm to obtain the Stackelberg equilibrium of the proposed game in Algorithm 1.

When Algorithm 1 converges, the WNO cannot increase or decrease the price unilaterally for improving its payoff. The convergence property of the sub-gradient based iterative algorithm has been proved (Lemma 3 in [36]). Specifically, when the initial price value and the step size δ are fixed, the results in the subsequent iterations are fixed. For example, we consider that at the k th iteration, the price of the WNO is given with a fixed value. Then, at the $(k + 1)$ th iteration, the step size is fixed and the search direction of the algorithm from the current iteration to the next iteration is unique because of the property of sub-gradient strategy [36], [37]. Thus, the price of the WNO at the $(k + 1)$ th iteration is also fixed. Based on the above mentioned fact, therefore, the game can converge to a unique outcome, when the initial price and the step size δ are fixed.

⁵Following [33], we can easily check the full rank of the gradient of the constraint for satisfying CRCQ, and the existence of the first derivative of the constraint for satisfying MFCQ.

D. Extension analysis on multi-WNO game

In this part, we briefly discuss a general scenario that incorporates multiple WNOs in the game. In this regard, the individual WNO competes with each other to reach an equilibrium, while constrained by the lower level equilibrium among the lower-layer players, i.e., the SCSP, the ECCSP and MUs. Modeling this competition leads to the Equilibrium Problems with Equilibrium Constraints (EPEC) formulation [38]. Note that our previous analysis on the market with a single WNO can be similarly extended to a multiple WNO scenario with no logical difficulty, but at the price of cumbersome derivation and equilibrium computation. We also adopt the backward induction and first consider the content demand problem of MUs in Stage III with fixed upper strategies, i.e., sponsorship, caching effort and price. Let $\mathbf{x} = \{x_1, \dots, x_m, \dots, x_M\}$ denote the strategy of the MU, where $x_m \in [0, 1]$ represents the fraction of the content demand to access through a cellular link of the WNO m . Let $\boldsymbol{\theta} = \{\theta_1, \dots, \theta_m, \dots, \theta_M\}$ denote the strategy of the SCSP, where θ_m represents the sponsorship offered to the MU using the cellular link provided by the WNO m . Likewise, the strategy of the ECCSP is similarly defined as $\mathbf{t} = \{t_1, \dots, t_m, \dots, t_M\}$. p_m is the price set by the WNO m , $m = 1, 2, \dots, M$.

Then the analysis on the lower equilibrium in the previous section can be readily presented here. The utility of the MU is modified as follows:

$$u(\mathbf{x}) = \sum_{m=1}^M (\tau \sigma_e f(x_m) - (1 - \theta_m) x_m p_m + \tau \sigma_e f(1 - x_m) g(t_m) - (1 - x_m) c). \quad (50)$$

The profit of the SCSP and the ECCSP are respectively reformulated as (51) and (52):

$$\Pi_s(\boldsymbol{\theta}) = \sum_{m=1}^M (\sigma_c h(x_m) - \theta_m p_m x_m), \quad (51)$$

$$\Pi_e(\mathbf{t}) = \sum_{m=1}^M (\sigma_c h(1 - x_m) - C t_m). \quad (52)$$

Following the similar analysis, we can also prove that

$$\frac{\partial^2 u(\mathbf{x})}{\partial x_m^2} = -[\alpha \tau \sigma_e x_m^{-\alpha-1} - \frac{\alpha \tau \sigma_e t_m^{1-\beta}}{1-\beta} (1 - x_m)^{-\alpha-1}] < 0, \quad (53)$$

and we also know that the strategy space of x_m is a convex and compact subset of the Euclidean space. Thus, the sub-game perfect equilibrium of MUs can be validated to be unique. With the optimal content demand solution obtained from Stage III, we can check properties of the second order derivative of the objective function of the SCSP and the ECCSP, i.e., $\frac{\partial^2 \Pi_s(\boldsymbol{\theta})}{\partial \theta_m^2}$ and $\frac{\partial^2 \Pi_e(\mathbf{t})}{\partial t_m^2}$. The existence and uniqueness of the non-cooperative sub-game between the SCSP and the ECCSP can be proved in a way similar to that presented in Section V-B and the details are omitted. Clearly, we can see that the properties of the lower equilibrium remain the same with the case with one WNO.

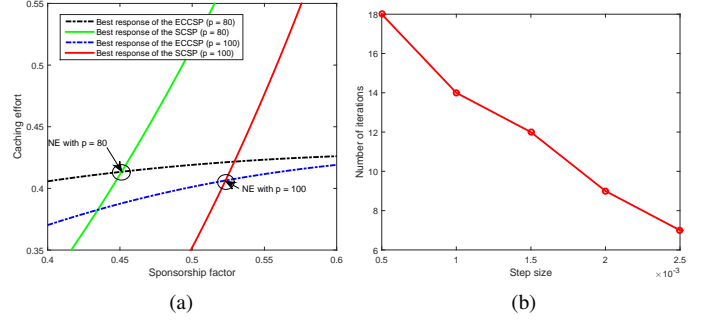


Figure 3: (a) Nash equilibrium for caching effort t and sponsorship factor θ under different price levels; (b) Number of iterations versus the step size.

After solving the problems in the lower layers, we can consider the upper layer multi-objective optimization problems, which are expressed as follows:

$$\begin{aligned} &\text{maximize} \quad \mathcal{P}(p_m) = p_m x_m - w_m x_m^2, \quad m = 1, 2, \dots, M. \\ &0 \leq p_m \leq \bar{p} \end{aligned} \quad (54)$$

Note that such problems are also constrained by the lower level equilibrium among the lower-layer players. The WNOs, as the leaders, are able to predict the lower-layer equilibrium to assist the decision-making at the upper layer. Such EPEC formulation can be solved by following the EPEC decomposition method proposed in [39]. To find the equilibrium solutions from the competition among WNOs, we resort to the sub-gradient based algorithm (Algorithm 1 in [37]), and its convergence is proved in [37]. The intuition is that when WNOs adopt the sub-gradient algorithm, each WNO initially assumes that there is no competition with other WNOs and hence sets its price at a certain value to receive high utility. After several iterations, when the WNO discovers that there exist other WNOs trying to attract MUs with appealing prices, the WNO predicts the responses of the other WNOs and tries to adjust its price competitively. Within the finite number of iterations, all WNOs are able to determine the best pricing decisions to achieve the highest utilities. Consequently, the EPEC problem is solved in an iterative manner.

VI. PERFORMANCE EVALUATION

In this section, we conduct simulations to evaluate performance of the players in the proposed joint sponsored and edge caching content service market through the Stackelberg game. Unless otherwise stated, we set $\alpha = 0.8$, $\beta = 0.5$, $\gamma = 0.8$, $l_a = 1$, $\sigma_e = 40$, $\sigma_c = 120$, $c = 80$, $C = 120$, $w = 1$, and $\bar{p} = 100$.

Figure 3(a) illustrates the Nash equilibrium (NE) of the non-cooperative game between the 2nd-tier players, i.e., the SCSP and ECCSP. The NE is the point at which the best responses of the SCSP and ECCSP intersect. Under different prices, different NE points are observed. Additionally, when the price increases, the optimal sponsorship fee increases and the optimal caching effort decreases. This is because when the price is high, the MUs are reluctant to choose the

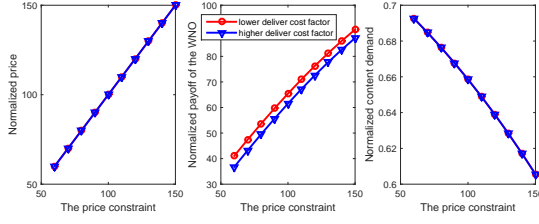


Figure 4: The impact of the price constraint \bar{p} on the WNO from joint sponsored and edge caching content service market. (lower delivery cost factor: $w = 1$, higher delivery cost factor: $w = 2$)

cellular link for content access. In this regard, the SCSP needs to offer higher sponsorship fee to compensate the cost for MUs, i.e., the price paid to the WNO, in order to attract content demand of the MUs. Conversely, the ECCSP has an incentive to decrease its caching effort to save the cost. Furthermore, Fig. 3(b) shows the number of iterations needed for convergence versus different step sizes. As expected, the convergence rate of the proposed algorithm depends on the step size. When the step size is small, the delay is large because of the slow convergence time in the sub-gradient algorithm, but the achieved results are more accurate. The accuracy means that the gap between the achieved results and the optimal values is small because of the small step size. Conversely, when the step size is big, the delay is small but the achieved results are not accurate. We next investigate the impact of the price constraint on the WNO, as illustrated in Fig. 4. It is worth noting that the optimal price offered by the WNO is the same as the maximum price constraint. The intuition is that the lower price can attract the MU to consume more sponsored content from the SCSP, which may incur higher delivery cost maintained by the WNO. Thus, the WNO is reluctant to lower the offered price. In addition, we observe that as the price constraint, i.e., the optimal price increases, the payoff of the WNO increases and the sponsored content demand of the MU decreases. This is because the WNO is able to set a higher price and extract more surplus from the MU, and achieves higher payoff consequently. Moreover, as expected, the payoff of the WNO decreases with the increase of w , i.e., the content delivery cost factor.

Then, we study the impact of the utility coefficient of MUs on the strategies and payoff of players in the game model, as illustrated in Fig. 5. We observe that the sponsored content demand of the MU decreases with the increase of σ_e . This is because when σ_e increases, the sponsorship fee from the SCSP becomes relative lower compared with the improved utility of the MU from consuming the content. Recall that the caching effort positively affects the utility of the MU from consuming the content, therefore, the sponsored content demand of the MU decreases. As a result, the SCSP wants to offer more sponsorship fee to attract the MU to consume the sponsored content, and the ECCSP has an incentive to lower its caching effort to reduce the caching cost. Once the sponsorship fee from the SCSP is large enough, the SCSP is not willing to offer more sponsorship fee to save its cost. Accordingly, the sponsorship fee from the SCSP increases first and then decreases. Therefore, the SCSP's profit decreases

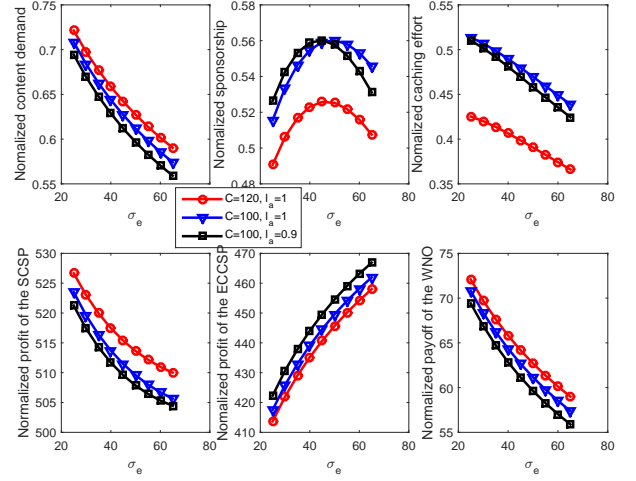


Figure 5: The impact of the utility level of MUs on the strategies and payoff of players (MU, SCSP, ECCSP, and WNO) from joint sponsored and edge caching content service market.

and the ECCSP's profit increases, which is consistent with the results in Fig. 5. Since the sponsored content demand of the MU decreases, and thus the payment from the MU to the WNO is reduced, which leads to the decrease of the WNO's payoff. As expected, we observe that the sponsored content demand of the MU increases and the caching effort of the ECCSP decreases as the content caching cost C increases. This is due to the fact that, with the increase of C , the cost of the ECCSP for increasing caching effort becomes greater. In this case, the ECCSP is willing to reduce its caching effort for saving cost and thus compensate for its decreasing profit. Meanwhile, the SCSP wants to offer higher sponsorship fee to the MU for encouraging the MU's higher sponsored content demand. Consequently, both the SCSP's profit and the WNO's payoff increase. Moreover, comparing curves with different value of the advertisement length l_a in the second row of Fig. 5, we observe that the increase of l_a leads to the increase of the SCSP's profit and the WNO's payoff. This is because when l_a increases, the utility of the MU from consuming the content decreases, and the offered sponsorship fee becomes significant for the MU. This encourages higher sponsored content demand of the MU, which benefits the SCSP and the WNO accordingly. Therefore, the profit of the ECCSP decreases because of its decreasing content traffic.

Furthermore, in Fig. 6, we evaluate the impact of the number of MUs on the strategies and payoff of players in the game model. We consider a set of MUs, in which the utility coefficients of MUs are assumed to follow the normal distribution $\mathcal{N}(\sigma_e, 1)$. Since the MUs are not coupled with each other in the model, the SCSP and the ECCSP can determine their strategies on each MU individually without involvements from other MUs. In this regard, the strategies of both providers only depend on the utility coefficient of the MU, i.e., σ_e . As expected, when σ_e increases, the optimal sponsorship fee decreases and the optimal caching effort decreases. This is consistent with what we have discussed for Fig. 5. We also find that the average sponsorship and average caching effort

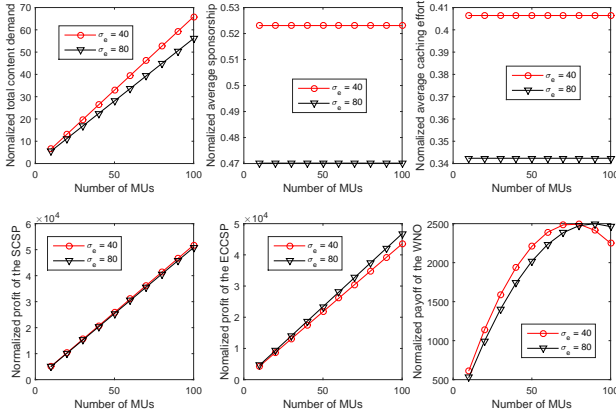


Figure 6: The impact of the number of MUs on the strategies and payoff of players (MUs, SCSP, ECCSP, and WNO) from joint sponsored and edge caching content service market.

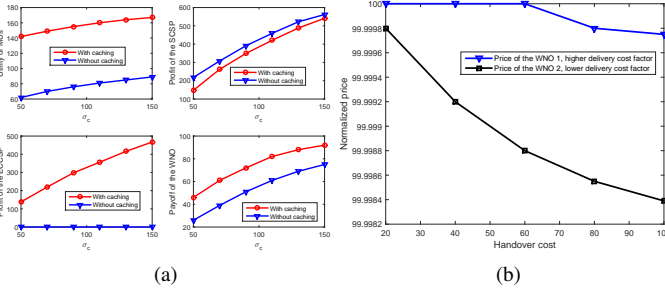


Figure 7: (a) The performance comparison between the joint sponsored and edge caching scenario and the scenario without caching; (b) The normalized price versus the handover cost in the duopolistic WNO scenario.

remain unchanged, when the number of MUs increases. Note that the terms “average sponsorship” or “average caching effort” are the mean value of the offered “sponsorship fees” or devoted “caching resources”, respectively, for each MU. Thus, the total investments of the SCSP and the ECCSP, i.e., the total sponsorship fees and total caching resources, respectively, are increasing, which scales with the number of MUs. Moreover, the increase of the number of MUs leads to the profit improvement of both providers since they can sell more services to the incoming customers. We also find that the marginal increase of the payoff of the WNO decreases when the number of MUs increases. Furthermore, when the number of MUs reaches a certain level, the payoff of the WNO even decreases. When the number of MUs increases, the backhaul traffic increases that causes congestion, which in turn leads to the rapid increase of the maintenance cost because of the limited capacity and bandwidth in the wireless environments.

We also study the performance comparison between the joint sponsored and edge caching scenario and the scenario without caching, as shown in Fig. 7(a). We find that the utilities of MUs are higher under the joint sponsored and edge caching scenario. The intuition is that, with caching, the ECCSP enters the market for selling the content, and extracts the surplus from MUs. Meanwhile, the MUs have the positive payoff

from purchasing and enjoying the content with more efficient content delivery. With the competition of the ECCSP, the profit of the SCSP decreases consequently⁶. The payoff of the WNO is also higher under the joint sponsored and edge caching scenario. The reason is that the ECCSP helps to offload some traffic load from the cellular link. As aforementioned, when the data traffic (number of MUs) reaches a certain level, the payoff of the WNO may decrease. Thus, the edge caching in the model relieves the congestion of the backhaul network maintained by the WNO, especially when the data traffic is high. Under the joint sponsored and edge caching scenario, the profit of the ECCSP is higher than the profit loss of the SCSP when the ECCSP exists in the market. Therefore, the social welfare of all the entities under the joint sponsored and edge caching scenario is also higher than that under the scenario without caching.

Lastly, we investigate the price competitions in the multiple WNO scenario. Figure 7(b) illustrates the normalized price versus the handover cost. For ease of presentation, we consider the duopoly WNO scenario that consists of two WNOs: WNO 1 with a higher delivery cost factor, WNO 2 with a lower delivery cost factor. We find that the WNO 2 is willing to reduce the price when the handover cost increases. The reason is that the MUs are reluctant to access edge caching content as the handover cost is high. In this regard, the WNO 2 has an incentive to reduce its price to attract MUs competing with WNO 1. Thus, when the handover cost increases, the price set by the WNO 2 decreases. However, when the handover cost is relatively small, the WNO 1 is not willing to reduce its price. The reason is that the lower price may attract more MUs and meanwhile cause higher data traffic, therefore, the delivery cost is even higher than the gained revenue. When the handover cost decreases to a certain level, the WNO 1 reduces its price slightly because of the competition since the price of the WNO 2 is relatively low. Note that the prices set by two WNOs approach to their maximum values, i.e., the price constraint. The reason is that the lower price may lead to higher data traffic, which may even decrease the payoff of the WNO. Thus, the price is not reduced significantly.

In summary, we draw the following engineering insights of the proposed scheme:

- In the proposed scheme, the MUs are encouraged to access and consume more services while improving service quality. Furthermore, in the presence of competition between the SCSP and the ECCSP, they are tempted to put more investment, i.e., sponsorship fee or caching resources. This further benefits MUs at the expense of the content providers, and thus the utility of user is improved compared with that under the scenario without caching. Consequently, the profit of the SCSP decreases because of the competition. Nevertheless, we find that the total

⁶Note that the SCSP must participate in the joint sponsored and edge caching content model which is applicable to the real video content market. If the SCSP chooses not to participate, it will obtain zero profit. Therefore, the SCSP chooses to compete with the ECCSP for gaining the loyalty of users. Furthermore, although the SCSP cannot benefit from the joint sponsored and edge caching content model, the other parties such as the WNO, the ECCSP and MUs will all benefit. Moreover, the social welfare of all parties in the model is improved, which is more valuable from the market's perspective.

profit of the SCSP and the ECCSP in the proposed model is higher than the profit of the SCSP in the model without caching. This provides a motivation for cooperation between the SCSP and the ECCSP. Indeed, this may even encourage them to collaborate to extract more surplus from MUs. The monopoly WNO is willing to set the price as high as possible to reduce the congestion cost. In the scenario with multiple WNOs, the prices set by the WNOs also approach their maximum values. However, the WNO with a lower delivery cost factor inclines to set a lower price than those of the WNOs.

- With the demand of cellular data/content traffic increasing sharply, the resource scarcity in wireless networks becomes more serious which deteriorates the service quality of users. In the presence of limited bandwidth and capacity of the WNO, we find that when the number of users is increasing which is often the case in practice, the backhaul traffic significantly increases that causes severe congestion and in turn leads to rapid increase of the maintenance cost, i.e., the content delivery cost. Nevertheless, the edge caching can alleviate such backbone network burden, and thus improve the payoff of the WNO in a large extent. This is also confirmed in numerical results. Therefore, the proposed scheme is shown to be of high importance to well accommodate the growing content traffic demand of users. On the one hand, the proposed scheme relieves the congestion of the backhaul network. On the other hand, the proposed scheme also improves the service quality. This benefit is especially valuable in consideration of radio resource scarcity in wireless networks. Finally, the social welfare of the proposed model is higher than that of the model without caching.

VII. CONCLUSION AND FUTURE STUDIES

In this work, we have formulated a joint sponsored and edge caching content service market model, where the interactions among the wireless network operator, the sponsored content service provider as well as the edge caching content service provider have been modeled as a hierarchical three-stage Stackelberg game. Then, we have analytically analyzed the sub-game perfect equilibrium in each stage using backward induction. Furthermore, we have validated the existence of the Stackelberg equilibrium by capitalizing on the bilevel optimization technique. Additionally, we have proposed a sub-gradient based iterative algorithm, which is ensured to converge to the Stackelberg equilibrium. At last, we have presented the performance evaluation.

For the future work, we mainly have the following directions:

- Radio transmission with multiple access: The impacts of such wireless characteristic on the proposed scheme can be incorporated straightforwardly in the model. Specifically, on view of such wireless characteristic, the service quality experienced by MUs is also negatively affected by other MUs while using a local network connection because of the multiple access technique. Similar to our previous work [10], the congestion component in the

utility function of MU which accesses the edge caching content can capture the impacts of congestion, i.e., the negative impacts from other MUs. In addition, different MUs may have different congestion sensitivity factors due to the inherent heterogeneity. In this regard, the caching effort t cannot benefit different MUs in the same way. For example, the same caching effort may benefit the congestion-tolerant MUs (with lower congestion sensitivity factor) more.

- Bounded Rationality: Note that the decision making of users in our model is considered to be rational and selfish. Nonetheless, the model can be extended to capture human behaviors, i.e., internal bounded rationality. In this regard, the prediction of user strategies can be intractable that may change the actual decision-making process. Therefore, similar to [40], we can seek for more general theoretical model called prospect theory coming from behavioral economics [41] to analyze the human behaviors with more psychologically accurate description.

APPENDIX

Proof of Proposition 3. The Jacobian matrix of point-to-set mapping with respect to the profit profile of the SCSP and ECCSP, i.e., $\nabla \mathbf{F}$ is defined as follows:

$$\begin{aligned} \nabla \mathbf{F} &= \nabla \mathbf{F}(\Pi_s(\theta; p), \Pi_e(t; p)) \\ &= \begin{bmatrix} \frac{\partial^2 \Pi_s(\theta; p)}{\partial \theta^2} & \frac{\partial^2 \Pi_s(\theta; p)}{\partial \theta \partial t} \\ \left(\frac{\partial^2 \Pi_e(t; p)}{\partial \theta \partial t} \right)^\top & \frac{\partial^2 \Pi_e(t; p)}{\partial t^2} \end{bmatrix}. \end{aligned} \quad (55)$$

Then, we have

$$\nabla \mathbf{F} + \nabla \mathbf{F}^\top = \begin{bmatrix} 2 \frac{\partial^2 \Pi_s(\theta; p)}{\partial \theta^2} & \frac{\partial^2 \Pi_s(\theta; p)}{\partial \theta \partial t} + \frac{\partial^2 \Pi_e(t; p)}{\partial \theta \partial t} \\ \frac{\partial^2 \Pi_s(\theta; p)}{\partial \theta \partial t} + \frac{\partial^2 \Pi_e(t; p)}{\partial \theta \partial t} & 2 \frac{\partial^2 \Pi_e(t; p)}{\partial t^2} \end{bmatrix}. \quad (56)$$

In order to guarantee the uniqueness of the Nash equilibrium in the non-cooperative sub-game \mathcal{G}^c , we need to prove that the matrix in (56) is negative definite [42]. Recall from Proposition 2 that $\frac{\partial^2 \Pi_s(\theta; p)}{\partial \theta^2} < 0$ and $\frac{\partial^2 \Pi_e(t; p)}{\partial t^2} < 0$. Then, we only need to ensure the negativity of the determinant for the matrix given in (56).

Since we have (57) and (58), in order to guarantee the negativity of the determinant for the matrix in (56), we only need

to ensure the negativity of $\begin{bmatrix} \frac{\partial^2 \Pi_s(\theta; p)}{\partial \theta^2} & \frac{\partial^2 \Pi_s(\theta; p)}{\partial \theta \partial t} \\ \frac{\partial^2 \Pi_e(t; p)}{\partial \theta \partial t} & \frac{\partial^2 \Pi_e(t; p)}{\partial t^2} \end{bmatrix}$. Accord-

ingly, we prove the positivity of $\begin{bmatrix} -\frac{\partial^2 \Pi_s(\theta; p)}{\partial \theta^2} & -\frac{\partial^2 \Pi_s(\theta; p)}{\partial \theta \partial t} \\ -\frac{\partial^2 \Pi_e(t; p)}{\partial \theta \partial t} & -\frac{\partial^2 \Pi_e(t; p)}{\partial t^2} \end{bmatrix}$ with the steps shown in (59). With simple transformations, we have (60) in the final step.

Consequently, to prove the positivity of above equality, one necessary condition is that the following constraints are satisfied:

$$\begin{aligned} \frac{\partial x^*}{\partial \theta} \frac{\partial^2 x^*}{\partial t^2} - \frac{\partial x^*}{\partial t} \frac{\partial^2 x^*}{\partial \theta \partial t} &> 0, \quad \frac{\partial^2 x^*}{\partial \theta^2} \frac{\partial x^*}{\partial t} - \frac{\partial^2 x^*}{\partial \theta \partial t} \frac{\partial x^*}{\partial \theta} > 0, \\ \frac{\partial^2 x^*}{\partial \theta^2} \frac{\partial^2 x^*}{\partial t^2} - \left(\frac{\partial^2 x^*}{\partial \theta \partial t} \right)^2 &< 0, \quad 2 \frac{\partial x^*}{\partial \theta} \frac{\partial^2 x^*}{\partial t^2} - \frac{\partial x^*}{\partial t} \frac{\partial^2 x^*}{\partial \theta \partial t} > 0. \end{aligned} \quad (61)$$

$$\begin{aligned}
& \det \begin{bmatrix} 2 \frac{\partial^2 \Pi_s(\theta;p)}{\partial \theta^2} & \frac{\partial^2 \Pi_s(\theta;p)}{\partial \theta \partial t} + \frac{\partial^2 \Pi_e(t;p)}{\partial \theta \partial t} \\ \frac{\partial^2 \Pi_s(\theta;p)}{\partial \theta \partial t} + \frac{\partial^2 \Pi_e(t;p)}{\partial \theta \partial t} & 2 \frac{\partial^2 \Pi_e(t;p)}{\partial t^2} \end{bmatrix} \\
&= \det \begin{bmatrix} \frac{\partial^2 \Pi_s(\theta;p)}{\partial \theta^2} & \frac{\partial^2 \Pi_e(t;p)}{\partial \theta \partial t} \\ \frac{\partial^2 \Pi_s(\theta;p)}{\partial \theta \partial t} & \frac{\partial^2 \Pi_e(t;p)}{\partial t^2} \end{bmatrix} + \det \begin{bmatrix} \frac{\partial^2 \Pi_s(\theta;p)}{\partial \theta^2} & \frac{\partial^2 \Pi_s(\theta;p)}{\partial \theta \partial t} \\ \frac{\partial^2 \Pi_s(\theta;p)}{\partial \theta \partial t} & \frac{\partial^2 \Pi_e(t;p)}{\partial t^2} \end{bmatrix} \\
&+ \det \begin{bmatrix} \frac{\partial^2 \Pi_s(\theta;p)}{\partial \theta^2} & \frac{\partial^2 \Pi_s(\theta;p)}{\partial \theta \partial t} \\ \frac{\partial^2 \Pi_e(t;p)}{\partial \theta \partial t} & \frac{\partial^2 \Pi_e(t;p)}{\partial t^2} \end{bmatrix} + \det \begin{bmatrix} \frac{\partial^2 \Pi_s(\theta;p)}{\partial \theta^2} & \frac{\partial^2 \Pi_e(t;p)}{\partial \theta \partial t} \\ \frac{\partial^2 \Pi_e(t;p)}{\partial \theta \partial t} & \frac{\partial^2 \Pi_e(t;p)}{\partial t^2} \end{bmatrix}, \tag{57}
\end{aligned}$$

$$\begin{aligned}
& \det \begin{bmatrix} \frac{\partial^2 \Pi_s(\theta;p)}{\partial \theta^2} & \frac{\partial^2 \Pi_s(\theta;p)}{\partial \theta \partial t} \\ \frac{\partial^2 \Pi_s(\theta;p)}{\partial \theta \partial t} & \frac{\partial^2 \Pi_e(t;p)}{\partial t^2} \end{bmatrix} + \det \begin{bmatrix} \frac{\partial^2 \Pi_s(\theta;p)}{\partial \theta^2} & \frac{\partial^2 \Pi_e(t;p)}{\partial \theta \partial t} \\ \frac{\partial^2 \Pi_e(t;p)}{\partial \theta \partial t} & \frac{\partial^2 \Pi_e(t;p)}{\partial t^2} \end{bmatrix} \\
&= 2 \frac{\partial^2 \Pi_s(\theta;p)}{\partial \theta^2} \frac{\partial^2 \Pi_e(t;p)}{\partial t^2} - \left(\frac{\partial^2 \Pi_s(\theta;p)}{\partial \theta \partial t} \right)^2 - \left(\frac{\partial^2 \Pi_e(t;p)}{\partial \theta \partial t} \right)^2 \\
&< 2 \frac{\partial^2 \Pi_s(\theta;p)}{\partial \theta^2} \frac{\partial^2 \Pi_e(t;p)}{\partial t^2} - 2 \frac{\partial^2 \Pi_s(\theta;p)}{\partial \theta \partial t} \frac{\partial^2 \Pi_e(t;p)}{\partial \theta \partial t} \\
&= \det \begin{bmatrix} \frac{\partial^2 \Pi_s(\theta;p)}{\partial \theta^2} & \frac{\partial^2 \Pi_e(t;p)}{\partial \theta \partial t} \\ \frac{\partial^2 \Pi_s(\theta;p)}{\partial \theta \partial t} & \frac{\partial^2 \Pi_e(t;p)}{\partial t^2} \end{bmatrix} + \det \begin{bmatrix} \frac{\partial^2 \Pi_s(\theta;p)}{\partial \theta^2} & \frac{\partial^2 \Pi_e(t;p)}{\partial \theta \partial t} \\ \frac{\partial^2 \Pi_e(t;p)}{\partial \theta \partial t} & \frac{\partial^2 \Pi_e(t;p)}{\partial t^2} \end{bmatrix}. \tag{58}
\end{aligned}$$

$$\begin{aligned}
& \det \begin{bmatrix} -\frac{\partial^2 \Pi_s(\theta;p)}{\partial \theta^2} & -\frac{\partial^2 \Pi_s(\theta;p)}{\partial \theta \partial t} \\ -\frac{\partial^2 \Pi_e(t;p)}{\partial \theta \partial t} & -\frac{\partial^2 \Pi_e(t;p)}{\partial t^2} \end{bmatrix} \\
&= \gamma^2 \sigma_c^2 (1-x^*)^{-\gamma-1} \left(\frac{\partial x^*}{\partial \theta} \right)^2 \left(\frac{\partial x^*}{\partial t} \right)^2 + \gamma \sigma_c^2 x^{*- \gamma-1} (1-x^*)^{-\gamma} \left(\frac{\partial x^*}{\partial \theta} \right)^2 \frac{\partial^2 x^*}{\partial t^2} \\
&- \gamma \sigma_c (\sigma_c x^{*- \gamma} - \theta p) (1-x^*)^{-\gamma} \frac{\partial^2 x^*}{\partial \theta^2} \left(\frac{\partial x^*}{\partial t} \right)^2 - \sigma_c (\sigma_c x^{*- \gamma} - \theta p) (1-x^*)^{-\gamma} \frac{\partial^2 x^*}{\partial \theta^2} \frac{\partial^2 x^*}{\partial t^2} \\
&+ 2p\gamma \sigma_c (1-x^*)^{-\gamma-1} \left(\frac{\partial x^*}{\partial t} \right)^2 \frac{\partial x^*}{\partial \theta} + 2p\sigma_c (1-x^*)^{-\gamma} \frac{\partial x^*}{\partial \theta} \frac{\partial^2 x^*}{\partial t^2} - \gamma^2 \sigma_c^2 x^{*- \gamma-1} (1-x^*)^{-\gamma-1} \\
&\times \left(\frac{\partial x^*}{\partial \theta} \right)^2 \left(\frac{\partial x^*}{\partial t} \right)^2 - \gamma \sigma_c^2 x^{*- \gamma-1} (1-x^*)^{-\gamma} \frac{\partial^2 x^*}{\partial \theta \partial t} \frac{\partial x^*}{\partial \theta} \frac{\partial x^*}{\partial t} + \gamma \sigma_c (\sigma_c x^{*- \gamma} - \theta p) (1-x^*)^{-\gamma} \\
&\times \frac{\partial^2 x^*}{\partial \theta \partial t} \frac{\partial x^*}{\partial \theta} \frac{\partial x^*}{\partial t} + \sigma_c (\sigma_c x^{*- \gamma} - \theta p) (1-x^*)^{-\gamma} \left(\frac{\partial^2 x^*}{\partial \theta \partial t} \right)^2 - p\gamma \sigma_c (1-x^*)^{-\gamma-1} \frac{\partial x^*}{\partial \theta} \left(\frac{\partial x^*}{\partial t} \right)^2 \\
&- p\sigma_c (1-x^*)^{-\gamma} \frac{\partial x^*}{\partial t} \frac{\partial^2 x^*}{\partial \theta \partial t}. \tag{59}
\end{aligned}$$

Since we obtain the expression of $\frac{\partial x^*}{\partial \theta}$, $\frac{\partial^2 x^*}{\partial \theta^2}$, $\frac{\partial x^*}{\partial t}$ and $\frac{\partial^2 x^*}{\partial t^2}$ aforementioned, in the following, we derive the expression for $\frac{\partial^2 x^*}{\partial \theta \partial t}$. From (16), we have

$$\begin{aligned}
& \left[-x^{*- \alpha-2} + \frac{t^{1-\beta}}{1-\beta} (1-x^*)^{-\alpha-2} \right] (\alpha+1) \frac{\partial x^*}{\partial \theta} \frac{\partial x^*}{\partial t} \\
&+ \left[x^{*- \alpha-1} + \frac{t^{1-\beta}}{1-\beta} (1-x^*)^{-\alpha-1} \right] \frac{\partial^2 x^*}{\partial \theta \partial t} \\
&+ t^{-\beta} (1-x^*)^{-\alpha-1} \frac{\partial x^*}{\partial \theta} = 0. \tag{62}
\end{aligned}$$

With simple manipulations, we obtain the final expression as follows:

$$\begin{aligned}
\frac{\partial^2 x^*}{\partial \theta \partial t} &= \frac{pt^{-\beta} (1-x^*)^{-\alpha}}{\alpha \tau \sigma_e \left[x^{*- \alpha-2} + \frac{t^{1-\beta}}{1-\beta} (1-x^*)^{-\alpha-1} \right]^2} \\
&\times \left\{ \frac{(\alpha+1) \left[-x^{*- \alpha-2} + \frac{t^{1-\beta}}{1-\beta} (1-x^*)^{-\alpha-2} \right]}{\alpha \left[x^{*- \alpha-1} + \frac{t^{1-\beta}}{1-\beta} (1-x^*)^{-\alpha-1} \right]} - \right. \\
&\quad \left. (1-x^*)^{-1} \right\}. \tag{63}
\end{aligned}$$

Then, to check whether the constraints in (61) are satisfied, we substitute the specific expressions of $\frac{\partial x^*}{\partial \theta}$, $\frac{\partial^2 x^*}{\partial \theta^2}$, $\frac{\partial x^*}{\partial t}$, $\frac{\partial^2 x^*}{\partial t^2}$ and $\frac{\partial^2 x^*}{\partial \theta \partial t}$ into (61). Accordingly, for the first constraint in (61), we have $\left(\frac{\partial x^*}{\partial \theta} \frac{\partial^2 x^*}{\partial t^2} - \frac{\partial x^*}{\partial t} \frac{\partial^2 x^*}{\partial \theta \partial t} \right) =$

$$\begin{aligned}
& \frac{pt^{-\beta} (1-x)^{-\alpha}}{\alpha^2 \tau \sigma_e \left[x^{-\alpha-1} + \frac{t^{1-\beta}}{1-\beta} (1-x)^{-\alpha-1} \right]^2} \\
&\times \left\{ \frac{t^{-\beta} (1-x)^{-\alpha-1}}{\left[x^{-\alpha-1} + \frac{t^{1-\beta}}{1-\beta} (1-x)^{-\alpha-1} \right]} + \frac{\beta}{t} \right\}, \tag{64}
\end{aligned}$$

from which the first constraint in (61) is satisfied. Furthermore, the second constraint in (61) is also satisfied without any conditions, since we have $\left(\frac{\partial^2 x^*}{\partial \theta^2} \frac{\partial x^*}{\partial t} - \frac{\partial^2 x^*}{\partial \theta \partial t} \frac{\partial x^*}{\partial \theta} \right) =$

$$\frac{p^2 t^{-\beta} (1-x)^{-\alpha-1}}{\alpha^2 \tau^2 \sigma_e^2 \left[x^{-\alpha-1} + \frac{t^{1-\beta}}{1-\beta} (1-x)^{-\alpha-1} \right]^3}, \tag{65}$$

which is positive without any conditions. Additionally, we

$$\begin{aligned}
& \det \begin{bmatrix} -\frac{\partial^2 \Pi_s(\theta;p)}{\partial \theta^2} & -\frac{\partial^2 \Pi_s(\theta;p)}{\partial \theta \partial t} \\ -\frac{\partial^2 \Pi_e(t;p)}{\partial \theta \partial t} & -\frac{\partial^2 \Pi_e(t;p)}{\partial t^2} \end{bmatrix} \\
&= \underbrace{\gamma \sigma_c^2 x^{*- \gamma - 1} (1 - x^*)^{- \gamma - 1} \frac{\partial x^*}{\partial \theta}}_{>0} \left(\frac{\partial x^*}{\partial \theta} \frac{\partial^2 x^*}{\partial t^2} - \frac{\partial x^*}{\partial t} \frac{\partial^2 x^*}{\partial \theta \partial t} \right) \\
&\quad - \underbrace{\gamma \sigma_c (\sigma_c x^{*- \gamma} - \theta p)}_{>0} (1 - x^*)^{- \gamma - 1} \underbrace{\frac{\partial x^*}{\partial t} \left(\frac{\partial^2 x^*}{\partial \theta^2} \frac{\partial x^*}{\partial t} - \frac{\partial^2 x^*}{\partial \theta \partial t} \frac{\partial x^*}{\partial \theta} \right)}_{<0} - \underbrace{\sigma_c (\sigma_c x^{*- \gamma} - \theta p)}_{>0} (1 - x^*)^{- \gamma} \left[\frac{\partial^2 x^*}{\partial \theta^2} \frac{\partial^2 x^*}{\partial t^2} - \left(\frac{\partial^2 x^*}{\partial \theta \partial t} \right)^2 \right] \\
&\quad + \underbrace{p \sigma_c (1 - x^*)^{- \gamma} \left(2 \frac{\partial x^*}{\partial \theta} \frac{\partial^2 x^*}{\partial t^2} - \frac{\partial x^*}{\partial t} \frac{\partial^2 x^*}{\partial \theta \partial t} \right)}_{>0} + \underbrace{p \gamma \sigma_c (1 - x^*)^{- \gamma - 1} \frac{\partial x^*}{\partial \theta} \left(\frac{\partial x^*}{\partial t} \right)^2}_{>0}. \tag{60}
\end{aligned}$$

$$\begin{aligned}
& \text{have } \left[\frac{\partial^2 x^*}{\partial \theta^2} \frac{\partial^2 x^*}{\partial t^2} - \left(\frac{\partial^2 x^*}{\partial \theta \partial t} \right)^2 \right] = \\
& \frac{p^2 t^{-2\beta} (1 - x)^{-2\alpha}}{\alpha^2 \tau^2 \sigma_e^2 \left[x^{-\alpha-1} + \frac{t^{1-\beta}}{1-\beta} (1 - x)^{-\alpha-1} \right]^4} \\
& \times \left\{ \frac{-(\alpha+1)\beta \left[-x^{-\alpha-2} + \frac{t^{1-\beta}}{1-\beta} (1 - x)^{-\alpha-2} \right]}{\alpha t} \right. \\
& \quad \left. - t^{-\beta} (1 - x)^{-\alpha-2} \right\}. \tag{66}
\end{aligned}$$

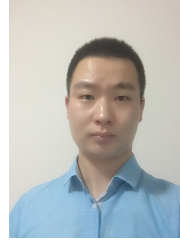
Thus, under the condition in (30), the negativity of $\left[\frac{\partial^2 x^*}{\partial \theta^2} \frac{\partial^2 x^*}{\partial t^2} - \left(\frac{\partial^2 x^*}{\partial \theta \partial t} \right)^2 \right]$ is ensured. Lastly, with some manipulations, we have (67). If the condition in (35), i.e., $2\alpha - 1 > 0$ holds, the last constraint in (61) is satisfied. Therefore, the Jacobian matrix of point-to-set mapping with respect to the profit profile of both the SCSP and ECCSP is negative definite. Consequently, ∇F is diagonally strictly concave, from which the uniqueness of the Nash equilibrium in the non-cooperative sub-game \mathcal{G}^c is guaranteed [42]. Thus, the proof is now completed. \square

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$$\left(2 \frac{\partial x^*}{\partial \theta} \frac{\partial^2 x^*}{\partial t^2} - \frac{\partial x^*}{\partial t} \frac{\partial^2 x^*}{\partial \theta \partial t}\right) = \left\{ \frac{(\alpha+1)t^{-\beta}(1-x)^{-\alpha-2} + 3\alpha t^{-\beta}(1-x)^{-\alpha-1} + (2\alpha-1)t^{-\beta}(1-x)^{-2\alpha-2} \frac{t^{1-\beta}}{1-\beta} + \frac{\beta}{t}}{\alpha \left[x^{-\alpha-1} + \frac{t^{1-\beta}}{1-\beta} (1-x)^{-\alpha-1} \right]^2} + \frac{\beta}{t} \right\} \times \frac{pt^{-\beta}(1-x)^{-\alpha}}{\alpha^2 \tau \sigma_e \left[x^{-\alpha-1} + \frac{t^{1-\beta}}{1-\beta} (1-x)^{-\alpha-1} \right]^2}. \quad (67)$$

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