Ergodic Spatial Nulling for Achieving Interference Free Rates

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***Abstract*—It is shown that a receiver equipped with two antennas may null an arbitrary large number of spatial directions to any desired level, while maintaining the interference-free signal-to-noise ratio, by judiciously adjusting the distance between the antenna elements. The main theoretical result builds on ergodic theory. The practicality of the scheme for systems operating at a moderate signal-to-noise ratio is demonstrated for a scenario where each transmitter is equipped with a single antenna and each receiver has two antenna elements, the separation of which can be arbitrarily set. As an example, for a five-user planar line-of-sight interference channel, with the directions of users being uniformly distributed, at a signal-to-noise ratio of 10 dB, a near interference-free average transmission rate is achievable. This amounts to roughly doubling the average rate attained by non-naive time-division multiple access.**

I. Introduction

The information-theoretic model of an interference channel is an abstraction that is motivated by the physical channel model of transmitter-receiver pairs that communicate over a shared wireless medium. While abstraction often leads to insights that may then be translated to more complicated real-life models, it is now recognized that the interference channel is an example that generalization also carries with it the risk of over-abstraction, i.e., losing some key features of the true problem. It is therefore worthwhile to re-examine the problem formulation from time to time as has been demonstrated, e.g., in the case of magnetic recording channels; see e.g., [1] for an overview of the evolution of the physical models and its impact on the relevant information-theoretic and coding techniques. Another example is the evolution that led to the V.90 voice-band modem [2], [3].

We revisit the wireless interference channel where each of the receivers is equipped with multiple antennas. It is well known that given an adaptive array with N_r receive antennas, one can null out N_r-1 (single-antenna) interferers and enjoy a full degree-of-freedom (DoF) for one (single-antenna) desired source. This leads to low utilization of the receive antennas, since only $1/N_r$ of the degrees of freedom convey useful information. The main advantages of receive beamforming are its ease of implementation and its robustness, since channel state information (CSI) feedback is not required.

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Nevertheless, works on interference alignment [4]–[6] demonstrate that half of the DoFs can be achieved, independent of the number of interferers, employing a single antenna at each node. While appealing from a theoretical point of view, interference alignment techniques (in all forms) face some major challenges in real-life applications. Beyond knowledge of full CSI of the complete interference network being required, the results are highly asymptotic. Specifically, the signal-to-noise ratio (SNR) at which a tangible improvement over naive schemes is achieved is extremely high.

We consider the particular class of line-of-sight (as well as specular multipath) interference channels; such channels are prevalent in recent applications of wireless communications. The main result is as follows: given $N_r = 2$ receive antennas and the possibility of *setting their separation*, one can approximately null out *any* number of sources in the plane, affording (with probability one) a full DoF to a single desired source. This corresponds to a utilization of half of the overall system degrees of freedom, so that for a system with K users, we achieve K DoFs out of the $2K$ total number of DoFs. This utilization is similar to the best achievable DoFs of interference alignment schemes, while requiring only receive-side CSI. Furthermore, the proposed technique achieves the desired DoFs even at practical values of SNR. We also show that the scheme can be implemented using a simple linear array and an antenna selection mechanism.

A. Related Work

Handling interference efficiently is a major challenge in multi-user wireless communication. Recently, it has become clear that this challenge can sometimes be overcome via *interference alignment* [5], [7]. Specifically, for the K-user Gaussian interference channel, it is now known that each user can achieve asymptotically roughly half the rate that would be available if there were no interference whatsoever; i.e., $K/2$ DoF are available. However, many schemes, such as the Cadambe-Jafar framework [7], require a large number of independent channel realizations to achieve near-perfect alignment and suffer from a significant SNR penalty due to channel inversion. For the static Gaussian K-user interference channel, Motahari *et al.* showed that $K/2$ DoFs are achievable for almost all channel realizations [8] but thus far this result has not been translated into real gains outside of the very high SNR regime.

Apart from the obvious connection to works on the interference channel, the idea of altering the physical propagation channel, in the present paper the alteration being the distance between antenna elements bears some similarity to "media-based modulation", "spatial modulation" and "index modulation" schemes; see, e.g., [9]. In these works, however, the physical medium is *modulated* based on the information-bearing signal. In contrast, the present work only requires sub-sampling of the spatial channel at the receiver whereas any standard coding technique for the additive white Gaussian noise channel can be used for the the transmitted signals.

II. The interference channel

Consider an interference channel with K transmitters and K corresponding receivers. We assume for simplicity that all transmitters are equipped with the same number of antennas N_t and all receivers are equipped with N_r antennas. Denoting by H_{ij}, the channel matrix from transmitter j to receiver i, the received signal is given by

$$\mathbf{y}_i = \sum_{j=1}^{K} \mathbf{H}_{ij}\mathbf{x}_j + \mathbf{z}_j, \quad i = 1 \ldots K, \tag{1}$$

where $\mathbf{z}_j$ is i.i.d. (between users and over time) circularly-symmetric complex Gaussian noise.

Several variants of this problem have been addressed. For instance, the case of $N_t = N_r = 1$ and real time-varying (which can be thought of as a diagonal matrix) coefficients has been studied in [4] where it was shown that for almost all channel coefficients, interference alignment attains half a DoF per user. A similar result was shown for scalar but time-invariant channels in [5] through alignment on the signal scale using lattice codes. Both of these approaches are very asymptotic in nature and require high resolution transmit-side CSI as well as very high SNR conditions to start to play a beneficial role.

A. The Line-of-Sight Interference Channel

The use of high-frequency communication has prompted recent interest in line-of-sight (LOS) communication as well as specular multipath channels. In this paper, we analyze LOS and finite specular multipath interference channels. Specifically, we make the following assumptions:

- A1 We assume a single transmit antenna per user, and two receive antennas per user, i.e. the matrices $\mathbf{H}_{i,j}$ in (1) are reduced to 2×1 vectors $\mathbf{h}_{i,j} \in \mathbb{C}^2$.
- A2 The vectors $\mathbf{h}_{i,j}$ consist of array manifold vectors.
- A3 We allow the spacing between the two receive antennas to be altered as needed to optimize performance.
- A4 We assume that each receiver has perfect CSI w.r.t. all channel gains corresponding to impinging signals. Transmitters on the other hand need not have access to any CSI beyond the rate at which they should communicate with their respective receiver.
- A5 For simplicity, we use a linear array and planar geometry where all sources are far field point sources.
- A6 Without loss of generality, we use the array manifold as the channel, since the signal attenuation can be absorbed in the power of x_j.
- A7 We assume that the locations of all transmitters and receivers are independently uniformly distributed in angle with respect to the origin.
- A8 We assume that the transmit power of all transmitters is bounded by P.

Note that by A7, the incidence angle of each received signal is uniformly distributed as well. Therefore, it suffices to consider the achievable rate of a single receiver.

III. Ergodic nulling

We propose a novel approach to the interference channel. The classical signal processing literature deals primarily with Nyquist-resolution beamformers, where at least some antennas are separated by at most $\lambda/2$. In this case, the array has a single main lobe in the desired direction, and the resolution of the array is determined by the farthermost elements. This is so since when all distances between antennas are larger than $\lambda/2$, an ambiguous beam pattern occurs. An example of this phenomenon is depicted in Figure 1. Interestingly, an ambiguous beam pattern can prove extremely advantageous when dealing with interference, since such patterns have multiple nulls. Indeed, we will show that by judiciously designing the beam pattern, we can point multiple nulls at the interferers simultaneously. More specifically, with a highly under-sampled array, any (finite) number of interferers at almost any set of directions can be suppressed. This follows from an ergodic theory argument.

To develop the general framework for receiver antenna array design, we first introduce some notations for LOS channels. Let $\mathbf{h}(\theta)$ be the array response towards direction θ. Assuming a spacing of d (in units of λ), the array response is given by

$$\mathbf{h}(\theta) = \frac{1}{\sqrt{2}} \left[1, e^{j2\pi d \cos\theta}\right]^T. \tag{2}$$

so that (1) becomes:

$$\mathbf{y}_i = \sum_{k=1}^{K} \mathbf{h}_{i,k}(\theta)\mathbf{x}_k + \mathbf{z}_i, \quad i = 1 \ldots K, \tag{3}$$

Therefore, using a received beamforming vector

$$\mathbf{w}_i = \frac{1}{\sqrt{2}}[1, e^{j\phi_i}],$$

the received signal of the i'th user becomes:

$$y_i = \sum_{k=1}^{K} \mathbf{w}_i^T \mathbf{h}_{i,k}(\theta)\mathbf{x}_k + z_i, \tag{4}$$

In the next section we show that by properly selecting d_i, ϕ_i, we can obtain the following:

$$\mathbf{w}_i^T \mathbf{h}(\theta_k) \approx \delta_{i,k}, \quad k = 1, .., K, \tag{5}$$

where $\delta_{i,k}$ is Kroneckers's delta function. Henceforth, we will omit the index i, as $\mathbf{w}_i$ is chosen independently for each user.

IV. Proof of Main Result

We now prove that by judiciously adjusting the distance between the receive antennas we can (with probability 1) suppress all interferers to any desired level. First, we leverage the theory of Diophantine approximation to establish that we may do so while treating the angle and gain of the desired signal as a random variable. We then significantly strengthen the results by proving that for almost all angles of arrival, one can approach the interference-free rate of any desired user arbitrarily closely. This is proved using the uniform distribution property of sequences modulo 1.

A. Diophantine Nulling

Consider a single receiver with a desired signal impinging from direction θ_i. Let $\theta_j : j \neq i$ be the directions of the interfering signals. The gain towards direction θ with a beamforming vector $\mathbf{w}$ is given by

$$g(\theta) = |\mathbf{w}^T \mathbf{h}(\theta)|^2 = \frac{1}{2} \left| 1 + e^{j(2\pi d \cos\theta + \phi)} \right|^2. \tag{6}$$

Straightforward algebraic simplification yields:

$$g(\theta) = 1 + \cos(2\pi d \cos(\theta) + \phi) \tag{7}$$

Theorem 1. *For any $\varepsilon > 0$, there exists a spacing d and a beamforming vector $\mathbf{w}$ towards direction θ such that $I(\theta) < \varepsilon$ where*

$$I(\theta) = \sum_{k \neq i} P_k g(\theta_k - \theta) \tag{8}$$

is the total received interference. Furthermore, the expected power of the desired signal satisfies

$$E_{\theta_i} \left[g(\theta_i - \theta) \right] > 0. \tag{9}$$

Proof. Note that $g(\psi) = 0$ whenever

$$\cos(2\pi d \cos(\psi) + \phi) = -1. \tag{10}$$

Therefore, setting $\phi = \pi$, we obtain that $g(\psi) = 0$ whenever

$$2\pi d \cos(\psi) = 2\pi p \tag{11}$$

and p is an integer. Let $q = d$. Then, attaining a null amounts to choosing a pair (p, q) such that

$$\cos\psi = \frac{p}{q}. \tag{12}$$

and p is an integer. Note that this in itself is not a Diophantine approximation problem since q is not required to be integer and therefore the equation can be satisfied exactly for any choice of n. Nonetheless, *simultaneous* Diophantine approximation will be useful when we consider a number of angles (directions) we wish to null. To that end, we assume that q is an integer as well.

That is, given $\theta_j : j \neq i$, our goal is to find $p_j : j \neq i$ and q (where the latter is preferably "small") such that the following holds simultaneously for $j \neq i$:

$$\cos\psi_j \approx \frac{p_j}{q}. \tag{13}$$

where $\psi_j = \theta_j - \theta$.

Let us now recall some basic results from the theory of metrical Diophantine approximation. We refer the reader to Section 4.3 of [10] for an overview, from which we cite the following (special case of the) high-dimensional Khinchine result.

Let $\mathbb{I}_n := [0,1)^n$ denote the unit cube in $\mathbb{R}^n$. We say that a point $\mathbf{x} = (x_1, \ldots, x_n) \in \mathbb{I}_n$ is well approximable if there exist infinitely many rational points

$$\left(\frac{p_1}{q}, \ldots, \frac{p_n}{q} \right)$$

with $q > 0$ such that the inequalities

$$\left| x_i - \frac{p_i}{q} \right| < \frac{1}{q^{1+1/n}} \tag{14}$$

are simultaneously satisfied for $1 \leq i \leq n$. Then Khinchine's theorem states that the measure of well approximable points in $\mathbb{I}_n$ is one.

To complete the proof we note that for all ψ, we choose $d = q$ and $\phi = \pi$. Hence, we obtain that for all $j \neq i$

$$|q \cdot \cos(2\pi\psi_j) - p| < \frac{1}{q^{\frac{1}{K-1}}}. \tag{15}$$

since $g(2\pi p) = 0$ and

$$\left| \frac{dg(\psi)}{d\psi} \right| < 2\pi d \tag{16}$$

by the mean value theorem we obtain that for all j and for all q

$$g(\psi_j) < \frac{2\pi}{q^{\frac{1}{K-1}}}. \tag{17}$$

By Khintchine's theorem, we have infinitely many such q's. Taking q large enough, the claim follows. ☐

Since the desired signal is uniformly distributed in angle, it follows that almost surely, it is possible to achieve one degree of freedom per user. Therefore, the overall utilization is one half of the total degrees-of-freedom.

B. Ergodic Nulling

While Theorem 1 provides a proof that for any set of interfering signals there is a two element array and a beamformer which nulls the interferers, the gain of the desired user is random. We now strengthen the result by proving that for almost all sets of directions, one can suppress the interference to any desired level without sacrificing the desired signal power. Moreover, we show that without loss of generality, the vector $\mathbf{w}$ can be chosen as $\mathbf{w} = \frac{1}{\sqrt{2}}[1, 1]^T$.

Theorem 2 (Main Theorem)**.** *Assume that the directions $\theta_1, .., \theta_K$ are such that $\cos(\theta_1), ..., \cos(\theta_K)$ are irrational and independent over $\mathbb{Q}$. Then for every i and every $\delta > 0$, one can find itegers, $d = q \in \mathbb{N}$ such that beamforming with the vector $\mathbf{w} = \frac{1}{\sqrt{2}}[1, 1]^T$ yields:*

$$\begin{array}{lcll} g(\theta_k) & < & \delta, & k \neq i \\ g(\theta_i) & > & 1 - \delta. & \end{array} \tag{18}$$

Note that this provides the interference-free capacity with half the degrees of freedom. This is the case since the gain in the desired direction can be made arbitrarily close to 1 while the total interference is suppressed to any desired level. We also note that for example whenever $\theta = \frac{2\pi}{N}$ and $N > 6$, then $\cos\left(\frac{2\pi}{N}\right)$ is irrational.

The proof of the main theorem will appear in the full version of this paper [11]. Nonetheless, the following example of a four-user interference channel demonstrates the idea of ergodic nulling. The optimal array dimension was 5λ. Figure 1 depicts the beam pattern resulting from the optimization of the rate of user 1 for a single channel realization for which the directions of the signals are: $[175^o, 59^o, 151^o, 133^o]$. The desired user's gain is close to 2 which is the interference-free gain, while the gains corresponding to the signals of all other users are suppressed to almost 0. Theorem 2 proves that such a beam pattern is almost always achievable provided that the array is sufficiently large.

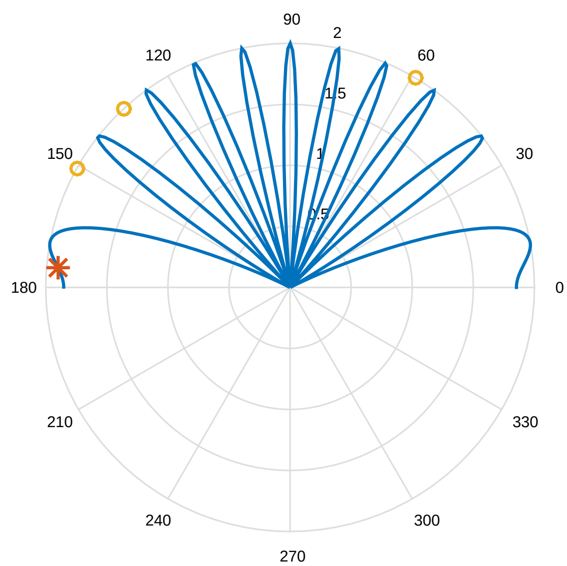

Fig. 1. Optimal beam pattern of user 1. Four-user interference channel. $d_{\max} = 25\lambda$. Directions: $[175^o, 59^o, 151^o, 133^o]$. Optimal $d = 5\lambda$. The powers of all users are $P = 1$.

V. Extension to Multipath Channels

We now show that our approach generalizes to the case of multipath (due to a finite number of reflections). Adhering to discrete time and allowing a different path loss for each reflection, the channel, as given in (3), now generalizes to

$$\mathbf{y}_i(e^{j\omega}) = \sum_{k=1}^{K}\sum_{\ell=1}^{L_{i,j}} \gamma_{i,k,\ell}\mathbf{h}(\theta_{i,k,\ell})e^{j\omega\tau_{i,k,\ell}}\mathbf{x}_k(e^{j\omega}) + \mathbf{z}_i(e^{j\omega}), \tag{19}$$

for $i = 1 \dots K$, where L_{ij} is the number of reflections of the j'th signal received by user i, and $\gamma_{i,k,\ell}$ is the complex path loss of the signal arriving from direction $\theta_{i,k,\ell}$. Using similar ideas to those in the main theorem by nulling all undesired signals and their reflections, while maintaining the desired signal and its reflections arbitrarily close to 1, the resulting received signal is given by:

$$\mathbf{y}_i(e^{j\omega}) = \sum_{\ell=1}^{L_{i,i}} g(\theta_{i,i,\ell})\gamma_{i,i,\ell}\mathbf{h}_{i,i}(\theta_{i,i,\ell})e^{j\omega\tau_{i,i,\ell}}\mathbf{x}_i(e^{j\omega}) + \zeta_i(e^{j\omega}) \tag{20}$$

where $\zeta_i(e^{j\omega}) = \mathbf{z}_i(e^{j\omega}) + \mathbf{z}'_i(e^{j\omega})$ is composed of the receiver noise as well as the residual interference at receiver i, $\mathbf{z}'_i(e^{j\omega})$. Note that the power $\mathbf{z}'_i(e^{j\omega})$ can be made arbitrarily small and $g(\theta_{i,i,\ell})$ are (simultaneously) arbitrarily close to 1 by a proper choice of δ. It follows that (20) amounts to a standard ISI channel in the frequency domain.

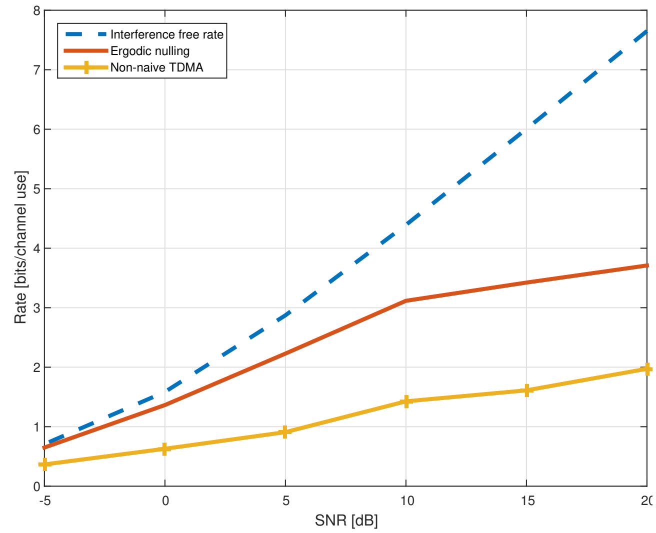

Fig. 2. Four-user interference channel where 100 random channel realizations are drawn. SIR=-5 dB. $d_{\max} = 500\lambda$.

VI. Optimizing the Beamformer

We now discuss the practical implementation of the proposed method. While Theorems 1 and 2 guarantee that interference can be suppressed to any desired level, they do not exploit the full optimization parameter space. Ultimately, our goal is to maximize the signal-to-interference-plus-noise ratio by properly choosing d and ϕ. Explicitly, the desired solution is given by

$$(\hat{d}, \hat{\phi}) = \arg\max_{d,\phi} \frac{P_d g(\theta_d)}{\sum_{k\neq i} P_k g(\theta_k) + \sigma^2} \tag{21}$$

where P_d, P_k are the receive power of the desired and interfering signals, and θ_d, θ_k are the directions of the desired and interfering signals, respectively. While this equation is highly non-linear, given the received signal and interference CSI, we can optimize the receive array by a full grid search over d, ϕ. This is a two dimensional search with a moderate complexity.

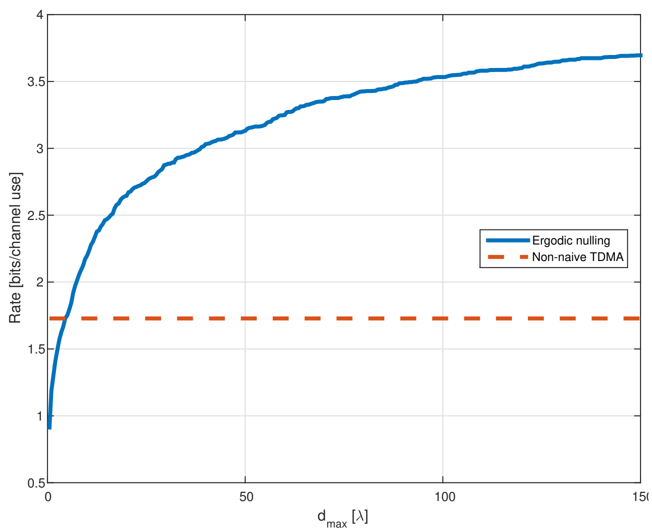

Fig. 3. Four-user interference channel where 100 random channel realizations are drawn. 100 random channels. SNR=10 dB, SIR=-5 dB.

VII. Simulations

To test the proposed ergodic interference nulling scheme, we generated 100 LOS interference channels, with five users. The array was limited to $d = 500\lambda$, which is a reasonable number for practical mm-wave scenarios. We tested the capacity of user 1 with all transmitters randomly located at directions randomly chosen between 0 and 180 degrees. The signal-to-interference ratio was -5dB, since all interferers were assumed to be received with the same power. As naive nulling of a single degree of freedom achieves nearly zero rate, we took as a baseline for comparison non-naive time-division multiple access (TDMA), with two users transmitting per time slot. We calculated the average achievable rate over all the randomly drawn channel realizations, where for each realization we optimized over d, ϕ using a full search with 1^o resolution in ϕ. We also compared the performance to the interference-free capacity of a two-element receive array. The results are depicted in Figure 2. The interference-free rate is clearly nearly attained up to an SNR of roughly 10 dB, and even at 20 dB we attain over 4 bits per channel use, which is roughly double the rate achieved by non-naive TDMA. The slowing of the growth of the rate (as a functio of the SNR) achieved by the ergodic nulling scheme is due to the limited size of the array. To test the dependence on $d_{\max}$ we chose SNR$=10$ dB, and computed the achievable rate as a function of $d_{\max}$. The results are depicted in Figure 3. While attaining the interference-free rate requires about 100λ separation, there is a very significant performance gain, compared to non-naive TDMA, even at $d_{\max} = 15\lambda$.

VIII. Discussion

In practice, moving the antennas to set the desired separation may be difficult to implement. One possibility to overcome this difficulty is to use the standard approach taken in massive MIMO systems, choosing two antennas out of a large array and switching them into two receive chains; see, e.g., [12]. We note that it is possible to reduce the number of total antenna elements when implementing the proposed nulling scheme via antenna selection by the use of a non-redundant array [13] as is commonly used in radio telescopes [14]. In a practical implementation, it is preferable to limit the dimensions of the array. To that end, a receiver could divide the interferers into two groups, a small group of strong interferers to which approximate nulling is applied, and a residual that is treated as noise. Moreover, from a system perspective, it may be advantageous to partition the users into disjoint sets in which the number of strong interferers is limited. Finally, we would like to mention that quantifying the robustness of the proposed scheme to CSI accuracy is an important research direction.

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