Efficient and Asymptotically Optimal Resource Block Allocation

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***Abstract*—Consider a channel allocation problem over a frequency-selective channel. There are K channels (frequency-bands) and N users such that $K = bN$ for some positive integer b. We want to allocate b channels (or resource blocks) for each user. Due to the nature of the frequency-selective channel, each user considers some channels to be better than others. Allocating each user only good channels will result in better performance than an allocation that ignores the selectivity of the channel. The optimal solution for this resource allocation problem can be computed using the Hungarian algorithm. However, this requires knowledge of the numerical value of all the channel gains, which makes this approach impractical for large networks. We suggest a suboptimal approach, that only requires knowing what the M-best channels of each user are. We find the minimal value of M such that there exists an allocation where all the b channels each user gets are among his M-best. This leads to a feedback of significantly less than one bit per user per channel. For a large class of fading distributions, including Rayleigh, Rician, m-Nakagami and more, this suboptimal approach leads to both an asymptotically (in K) optimal sum-rate and asymptotically optimal minimal rate. Our non-opportunistic approach achieves asymptotically full multiuser diversity and optimal fairness, in contrast to all existing limited feedback algorithms.**

***Index Terms*—Resource Allocation, Multiuser Diversity, Channel State Information, Random Bipartite Graphs.**

I. Introduction

The problem of multiple users accessing a common medium is a central issue in every communication network [1]–[3]. The most common access schemes in practice are orthogonal transmission techniques such as TDMA, FDMA , CDMA and OFDMA [4], [5].

When using a frequency division technique in a frequency-selective channel, not all channels have the same quality. Furthermore, since different users have different positions, the channel gains of users in a given frequency band are independent. This introduces a resource allocation problem between users and frequency bands (channels). Theoretically, if the exact knowledge of all these channel gains were available, an optimal solution for this resource allocation could be computed. The performance gain that can be achieved by solving this problem over a frequency allocation that ignores the channel gains is known as multiuser diversity [6].

As networks grow and bandwidth expands the number of channels increases significantly. This is typical for OFDMA systems that tend to have a large number of subcarriers. For example, in LTE [7] there are 128 up to 2048 subcarriers in the downlink for each user. This implies that reporting the channel state information (CSI) to the base station (BS) inflicts a large communication overhead that makes such a scheme infeasible.

In this paper, we propose a novel limited feedback scheme which significantly reduces feedback overhead compared to state of the art techniques [8]–[11]. The proposed technique provides a provably asymptotically (in K) both optimal sum-rate and max-min fairness. The proof applies to a wide range of fading distributions that includes Rayleigh, Rician and m-Nakagami. Our technique uses ~0.85 bit per user per channel already for $K = N = 20$, and ~0.52 bit per user per channel for $K = 512$ and $N = 128$.

In our approach, users only need to feed back the indices (and not the values) of their M-best channels. If M is large enough, there is a high probability of an allocation of users to channels where each user gets all of his b channels from his M-best channels. We prove that for $M \geq (b + 1)(1 + \varepsilon)\ln K$, where $\varepsilon > 0$, this probability approaches one as $K \to \infty$. While our approach is asymptotic, the probability that such an allocation exists is already more than 99% for $K = 20$ channels and $N = 20$ users.

Figure 1 presents a toy example of our system with $N = K = 4$. Each user transmits the indices of his M best channels to the base station. The base station constructs a preferences matrix, computes a perfect matching in the resulting bipartite graph and transmits back the allocated channel to each user.

The existence of a perfect matching (PM) allocation relies on the theory of random bipartite graphs. We define a novel random bipartite graph and prove that it has a PM with high probability. Due to its dependent edges, our graph is not an Erdős–Rényi graph. Consequently, proving the existence of a PM requires revisiting the original proof. Our proof is in the spirit of the original result of Erdős–Rényi on random bipartite graphs [12]. A generalization of our results to the case of unequal number of channels between users can be achieved using the same proof strategy. It is omitted due to the complicated formalism. However, we present simulations that indicate that our results are also valid in this case.

Throughout this paper, we use the term “channel”, which may represent several united subcarriers in case the coherence bandwidth is large and they are highly correlated (e.g., a resource block).

A. Previous Work

There is extensive literature on OFDMA resource allocation (see the surveys in [13], [14] and the references therein). The state of the art algorithms achieve a close to optimal sum-rate with reduced computational complexity compared to the optimal solution. All these approaches assume that users estimate their CSI perfectly and feed all of it back to the BS. While this assumption may be reasonable for small networks, it is highly infeasible for large networks with a large number of subcarriers, such as LTE [7].

In order to overcome the need for the whole CSI to exploit the selectivity of the channel, many works have suggested suboptimal schemes with limited feedback. For an excellent overview of limited feedback systems, see [15].

State of the art algorithms consider an opportunistic scheduling approach, where each available channel is assigned to a user with a high instantaneous channel gain [8]–[11]. These approaches suffer from an inherent unfairness.

The rationale behind the opportunistic approach is to exploit the multiuser diversity. By contrast, here we argue that this widespread approach is too conservative and that achieving multiuser diversity should not come at the expense of fairness. We argue that fairness is undermined because all these approaches schedule a user for each channel separately. Somewhat surprisingly, when analyzing all the channels together, each user is very likely to be "an opportunistic choice" for some channel. This calls for a more sophisticated mathematical analysis of the random matchings between users and channels as we employ in this paper, in contrast to the simplistic arguments dealing with one user at a time.

In contrast to [9], [10], [16], [17] our work is not limited to the case of Rayleigh-fading channels, but applies to a much broader class of exponentially-dominated tail distributions. Besides Rayleigh fading, this class includes Rician, m-Nakagami and more.

Another disadvantage of the existing techniques is that they rely on a thresholding approach for the channel gains [8]–[10]. The thresholding approach requires the BS or the users to know the fading distribution in order to compute the optimal threshold. Hence, the M-best channels approach is more robust and less informational demanding.

In [18], [19], a distributed channel allocation for Ad-Hoc networks was introduced, based on a carrier-sensing multiple access (CSMA) scheme and a thresholding approach for the channel gains. Although considering a different scenario, the analysis in [18], [19] also used a random bipartite graph, but a very simplified one. The random bipartite graph of [18], [19] was an Erdős–Rényi graph (i.e., edges existence is independent), for which PM existence results are well known [12]. Since not all edges in our random bipartite graph are independent, there are no PM existence results that can be employed and a novel analysis is required.

Moreover, the analysis in [18], [19] was limited to bounded channel gains and proved asymptotic optimality of only the expected sum-rate. In contrast, we prove asymptotic optimality for the random sum-rate in probability. More importantly, we prove asymptotic optimality for the random minimal rate in probability.

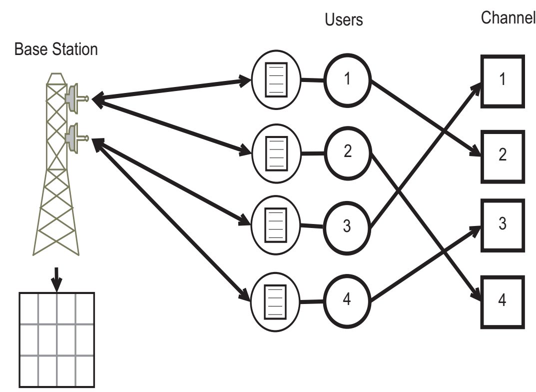

Fig. 1. The System

Nevertheless, [18] introduced an efficient algorithm to compute a PM for every given bipartite graph. We employ this algorithm in our scheme. It requires a time complexity of $O\left(K\log_2^2 K\right)$ [20] instead of $O\left(K^3\right)$ for the Hungarian algorithm [21]. Since this computation has to be done repeatedly by the BS, this major saving in computation time can make our scheme practical in cases where the Hungarian algorithm is not.

The existence of PM allocations is closely related to the pure Nash equilibrium (NE) of interference games. In our previous work [22], we designed a game where all pure NE are an almost (or an exact) PM between users and channels. We assumed that each user was allocated a single channel. The results of this paper can be exploited to generalize our previous results for the case of $b > 1$ channels per user.

B. Outline

The rest of this paper is organized as follows. In Section II we formulate our channel allocation problem and define the user-channel bipartite graph. In Section III we prove that as $K \to \infty$, the probability that a PM exists in the user-channel graph approaches one for $M \geq (b+1)(1+\varepsilon)\ln K$ (Theorem 2) and that it cannot approach one for smaller M (Lemma 3). In Section IV we prove that a PM allocation asymptotically attains both an optimal sum-rate and max-min fairness (Theorem 5). In Section V we suggest an efficient algorithm for the BS to compute this PM. Section VI provides simulation results and Section VII concludes the paper.

II. Problem Formulation

There are K channels and N users such that $K = bN$ for some positive integer b. We want to allocate b channels for each user. For each user n, the channel gains $g_{n,1}, ..., g_{n,K}$ are assumed to be K i.i.d random variables, which are known to the user. We assume that the channel gains of different users are independent. We also assume quasi-static fading, where the coherence time is larger than the time it takes to feed back the CSI. Our assumptions are standard in the channel allocation literature [9], [10], [16], [17], [23], [24]. However, in contrast to most of those works, we do not assume Rayleigh fading or that channel gains of different users are identically distributed.

An allocation a is a mapping from a user index to his set of allocated channels. Denote by $\boldsymbol{a}_n$ the set of allocated channels of user n in allocation a. We want to solve the following combinatorial optimization problem

$$\begin{gathered}\max_{\mathbf{a}} \sum_{n=1}^{N} R_n(\boldsymbol{a}_n) \\ s.t. \\ \boldsymbol{a}_n \bigcap \boldsymbol{a}_m = \emptyset \; \forall n, m, n \neq m \quad, |\boldsymbol{a}_n| = b, \forall n\end{gathered} \tag{1}$$

where the first constraint means that each channel is allocated to a single user (orthogonal transmission) and the second constraint states that each user is allocated exactly b channels.

The achievable rates of the users in bits per channel use, $\{R_n\}$, are defined as

$$R_n(\boldsymbol{a}_n) = \sum_{k \in \boldsymbol{a}_n} \log_2 \left(1 + \frac{g_{n,k}^2 P_n}{N_0}\right) \tag{2}$$

where P_n is the transmission power of user n and N_0 is the variance of the Gaussian noise in his receiver. Note that by assuming that the transmission power is the same in each channel, we assume no water-filling technique is used. This assumption is later justified both analytically and in simulations, showing that the water-filling gain is negligible.

For the rest of this paper, we think of each user as having b agents, each of whom is allocated a single channel. This allows us to introduce the following reduction for the problem

$$\max_{\pi} \sum_{i=1}^{bN} R_i(\{\pi(i)\}) \tag{3}$$

where π is a permutation of the agent indices. This is a combinatorial optimization problem with $(bN)!$ possible solutions. Somewhat surprisingly, this problem can be solved with complexity $O(b^3 N^3)$ using a method that is known as the Hungarian algorithm [21]. The optimal solution of (3) requires the BS to know all NK channel gains. In our limited feedback scheme, the information available at the BS is summarized in a bipartite graph, defined as follows.

Definition 1. A user-channel graph Γ is a balanced bipartite graph consisting of a user nodes set $\mathfrak{N}$ and a channel nodes set $\mathcal{K}$. Every user $n \in \mathcal{N}$ has b user nodes ("agents") and every channel has a single channel node. An edge is connected between $n_i \in \mathfrak{N}$ and $k \in \mathcal{K}$ if and only if channel k is one of the M-best channels for user n. A perfect matching allocation corresponds to a perfect matching in Γ.

Our graph is not a typical random bipartite graph. Each left side node has exactly M edges whereas each right side node may have any other degree. Additionally, groups of b consecutive left side edges are connected to the exact same right side edges. This is why a novel proof for the existence of a PM is needed.

The bipartite graph can also be represented using a $K \times K$ preferences matrix U, as used in the Hungarian Algorithm. Our matrix U is a simplified binary matrix where $u_{i,k} = 0$ if channel k is one of the M-best channels of agent i and $u_{i,k} = 1$ otherwise. Examples of user-channel graphs can be seen in Figure 2. In the left graph $N = 3$ and $b = 2$. A PM exists that consists of the grey edges. In the right graph $N = 2$ and $b = 2$. No user is connected to channel 4 so no PM can exist.

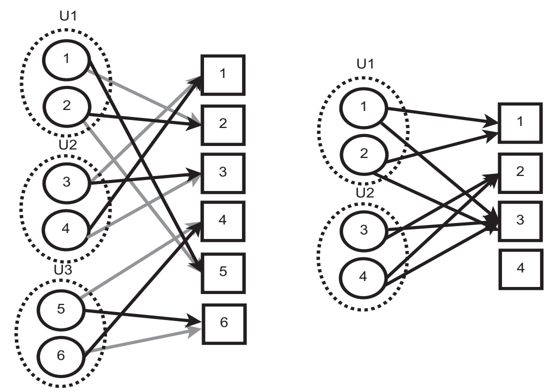

Fig. 2. User-channel graphs

III. Asymptotic Existence of a PM Allocation

We want to show that a PM allocation, where each user only gets M-best channels, exists. There is a natural trade-off for the parameter M. A large value of M increases the probability for the existence of a PM allocation at the cost of reduced performance, since the M-best channel gets worse as M increases. Hence, it is important to identify the smallest value of M for which a PM exists.

The main theorem of this paper is stated as follows.

Theorem 2 (Main Theorem)**.** *Let $K = bN$ for a positive integer b. If $M \geq (b+1)(1+\varepsilon)\ln K$ for some $\varepsilon > 0$ then the probability of the existence of an allocation where each user gets b channels that are among his M-best approaches 1 as $K \to \infty$.*

The above theorem sets the feedback requirement of our algorithm to be $\frac{1}{K}\log_2 \begin{pmatrix} K \\ M \end{pmatrix}$ bits per user per channel. For $M = (b+1)(1+\varepsilon)\ln K$, this is $O\left(\frac{\log_2^2 K}{K}\right)$, instead of a quantization of q bits per user per channel that an approach that requires the whole NK channel gains needs. Additionally, already for $N = K = 20$ and $M = 9$ $(\varepsilon = 0.5)$, we require ~0.86 bit per user per channel, less than one bit per user per channel required by all thresholding approaches (such as in [8]–[10]). For these values, more than 99% of channel realizations have a PM. For $N = 64$, $b = 4$, $K = 256$ and $M = 56$ $(\varepsilon = 1)$ our method requires ~0.74 bit per user per channel and more than 99.9% of channel realizations have a PM. For $N = 128$, $b = 4$ and $K = 512$ and $M = 63$ $(\varepsilon = 1)$ we require ~0.53 bit per user per channel and more than 99.99% of channel realizations have a PM.

For a PM to exist, every channel should be one of the M-best channels for at least one user. The following lemma shows that for that to occur, M must grow with respect to K (faster than $b \ln K$) and that $M = b(1+\varepsilon)\ln K$ is enough for that purpose.

Lemma 3. *Let $K = bN$ for a positive integer b. Let $M > 0$. Denote by E the event in which there is a channel which is not one of the M-best channels for any user. Let $\varepsilon > 0$.*

1) *If $M \leq b \ln K$ then $\lim_{K\to\infty} \Pr(E) \geq \frac{1}{2}$.*
2) *If $M \geq b(1+\varepsilon)\ln K$ then $\lim_{K\to\infty} \Pr(E) = 0$.*

Proof: Denote by E_k the event in which channel k is not one of the M-best channels for any of the users. We want to bound from below and above the probability that there exists a channel that is not good for any of the users, i.e., the probability of $E = \bigcup_k E_k$. Due to the i.i.d. assumption on the channel gains and their independency between users we have $\Pr(E_k) = \left(1 - \frac{M}{K}\right)^N$. From the union bound we obtain that

$$\Pr(E) = \Pr\left(\bigcup_k E_k\right) \leq K\left(1 - \frac{M}{K}\right)^N. \quad (4)$$

Since $\left(1 - \frac{M}{K}\right)^N$ is decreasing with M, using $M \geq b(1+\varepsilon)\ln K$ leads to

$$\lim_{K\to\infty} K\left(1 - \frac{M}{K}\right)^N \leq \lim_{K\to\infty} K\left(1 - \frac{b(1+\varepsilon)\ln K}{K}\right)^{\frac{K}{b}} = \lim_{K\to\infty} K e^{-(1+\varepsilon)\ln K} = 0. \quad (5)$$

The lower bound for the probability for a PM is obtained by using the inequality

$$\Pr\left(\bigcup_k E_k\right) \geq \sum_k \Pr(E_k) - \sum_{k_1,k_2} \Pr\left(E_{k_1} \bigcap E_{k_2}\right). \quad (6)$$

which is a consequence of the inclusion-exclusion principle. By direct counting of the number of user-channel graph with two undesired channels we obtain (details appear in [25])

$$\sum_{k_1,k_2} \Pr\left(E_{k_1} \bigcap E_{k_2}\right) \leq \frac{K^2}{2}\left(1 - \frac{2}{K}\right)^{M\frac{K}{b}}. \quad (7)$$

So we conclude that

$$\Pr(E) \geq K\left(1 - \frac{M}{K}\right)^N - \frac{K^2}{2}\left(1 - \frac{2}{K}\right)^{M\frac{K}{b}}. \quad (8)$$

Obviously, $\Pr(E)$ decreases with M. By using $M = b\ln K$ we conclude that

$$\lim_{K\to\infty} \Pr(E) \geq \lim_{K\to\infty}\left(K\left(1 - \frac{M}{K}\right)^N - \frac{K^2}{2}\left(1 - \frac{2}{K}\right)^{M\frac{K}{b}}\right) = \frac{1}{2}. \quad (9)$$

■

Now we can prove our main Theorem.

Proof Strategy of Theorem 2: The proof uses the Hungarian algorithm on our $K \times K$ binary matrix of preferences U, which consists of $\frac{K}{b}$ submatrices of b identical rows each, and exactly M zeros at each row. We want to show that the probability that the Hungarian algorithm converges after one iteration approaches one as $K \to \infty$; hence the resulting cost is zero, which represents a PM in the equivalent bipartite graph. Denote by $Q_l(K, M)$ the probability that U can be covered by $K - l - 1$ rows and l columns such that all its zeros are covered, and that there is no smaller l with this property. We want to upper bound $Q_l(K, M)$ for each l. By direct counting of the number of combinations for each such matrix, we can show that $\sum_{l=1}^{K-1} Q_l(K, M) \to 0$ as $K \to \infty$. Hence, the probability that a PM does not exist in U vanishes to zero as $K \to \infty$. The details are omitted due to page constraints, and appear in [25]. ■

IV. Asymptotic Optimality of the PM Allocation

In the previous section we showed that $M = (b+1)(1+\varepsilon)\ln K$, for some $\varepsilon > 0$, is enough to guarantee that a PM allocation exists. In this section we show how good a PM allocation actually is. This has to do with how much the M-best channel of each user is worse than his best channel. This definitely depends on the distribution of the channel gains. Luckily, all of the fading distributions used in practice tend to belong to the following class. Among them are Rayleigh, Rician and m-Nakagami fadings.

Definition 4 (Exponentially-Dominated Tail Distribution)**.** Let X be a random variable with a CDF F_X. We say that X has an exponentially-dominated tail distribution if there exist $\alpha > 0, \beta \in \mathbb{R}, \lambda > 0, \gamma > 0$ such that

$$\lim_{x\to\infty} \frac{1 - F_X(x)}{\alpha x^\beta e^{-\lambda x^\gamma}} = 1 \quad (10)$$

The following theorem establishes the asymptotic optimality of a PM allocation for this family of distributions.

Theorem 5. *Let the channel gains be the i.i.d. variables $g_{n,1}, \ldots, g_{n,K}$ and $K = bN$ for a positive integer b. If, for each k and n, $g_{n,k}$ has an exponentially-dominated tail and $M = \lceil (b+1)(1+\varepsilon)\ln K \rceil$ for some $\varepsilon > 0$, then*

$$\operatorname*{plim}_{K\to\infty} \frac{\min_{\mathbf{a}\in\mathcal{P}} \sum_{n=1}^N R_n(\boldsymbol{a}_n)}{\max_{\mathbf{a}\in\mathcal{A}} \sum_{n=1}^N R_n(\boldsymbol{a}_n)} = 1 \quad (11)$$

where $\mathcal{P}$ is the set of PMs in the user-channel graph, $\mathcal{A}$ is the set of all possible allocations of channels to users, and plim *denotes convergence in probability. Furthermore,*

$$\operatorname*{plim}_{K\to\infty} \frac{\min_{\mathbf{a}\in\mathcal{P}} \min_n R_n(\boldsymbol{a}_n)}{\max_{\mathbf{a}\in\mathcal{A}} \max_n R_n(\boldsymbol{a}_n)} = 1. \quad (12)$$

Proof: Denote by $g_{n,(i)}$ the i-th smallest channel gain among $g_{n,1}, \ldots, g_{n,K}$. We have

$$\frac{\min_{\mathbf{a}\in\mathcal{P}} \sum_n R_n(\boldsymbol{a}_n)}{\max_{\mathbf{a}\in\mathcal{A}} \sum_n R_n(\boldsymbol{a}_n)} \geq \frac{b\sum_n \log_2\left(1 + \frac{g^2_{n,(K-M+1)}P_n}{N_0}\right)}{b\sum_n \log_2\left(1 + \frac{g^2_{n,(K)}P_n}{N_0}\right)} \geq \frac{\min_n \log_2\left(1 + \frac{g^2_{n,(K-M+1)}P_n}{N_0}\right)}{\max_n \log_2\left(1 + \frac{g^2_{n,(K)}P_n}{N_0}\right)} \quad (13)$$

The proof for why the last term approaches one as $K \to \infty$ appears in [26, Theorem V.11]. ∎

In fact, the max-min fairness of (12) also holds for agents as well as users. This means that different M-best channels allocated to the same user cannot have a significant quality difference in a large network. Hence, the asymptotic gain of optimally allocating the transmission power (by water-filling) between these channels vanishes to zero.

V. Computation of a PM Allocation

Given the feedbacked information, the base station can construct the user-channel bipartite graph. Then, a PM allocation can be computed by finding a PM in this graph. This can be done efficiently by using the simplified auction algorithm [18], [20]. The expected time complexity of this algorithm is $O\left(\frac{K\log^2(K)}{\log(pK)}\right)$ for an Erdős–Rényi bipartite random graph with edge probability p, and can be improved to $O\left(\frac{K\log^2(K)}{\log(\log(K))}\right)$ by reducing the density of the graph. A factor of $O\left(\log(K)\right)$ comes from the computations done in each iteration of the algorithm.

VI. Simulation Results

For each value from $N = 10, 25, 50, 75, 100$, we simulated 100 random realizations of the channel gains. We used a Rayleigh fading network; i.e., $\{g_{n,k}\}$ are independent Rayleigh random variables. Hence $\left\{g_{n,k}^2\right\}$ are independent exponential random variables, and we chose the parameter $\lambda = 1$ for all of them. The transmission powers were chosen such that the mean SNR for each link is 20[dB]. All the rates are measured in bits per second, assuming that the bandwidth of a subchannel is 15kHz (as in LTE [7]). We used $M = \lceil 1.5\,(b+1)\ln(K)\rceil$ and got that a PM existed in every realization. This implies that our asymptotic results are already valid for $N = 10$.

In Figure 3 the mean (solid lines) and minimal (dashed lines) rate are presented as a function of N, averaged over 100 realizations. We used $b = 4$. The optimal solution was computed using the Hungarian algorithm. We also compare our results to those of [10], as a state of the art thresholding algorithm. It is evident that the performance of a PM allocation is close to the performance of the optimal allocation. As anticipated by our results, it improved with N. The random allocation rates, which can be thought of as the result of an allocation that ignores the CSI, were far behind (especially the minimal rate). Our mean rate is slightly better than that of [10] for all $N \geq 20$. The minimal rate of [10] is much lower (~78% of our minimal rate), and is not increasing with N while our minimal rate does. Last but not least, we require $\frac{1}{K}\log_2\begin{pmatrix} K \\ M \end{pmatrix}$ bit per user per channel (0.81 for $N = 10$ and 0.34 for $N = 200$), while [10] always require one bit per user per channel.

We ran the same experiment but with $b = 2$ and $M = \lceil b\ln(K)\rceil$. This time a PM did not always exist. This is in accordance with Lemma 3. The frequencies of existence of a PM are listed in Table I.

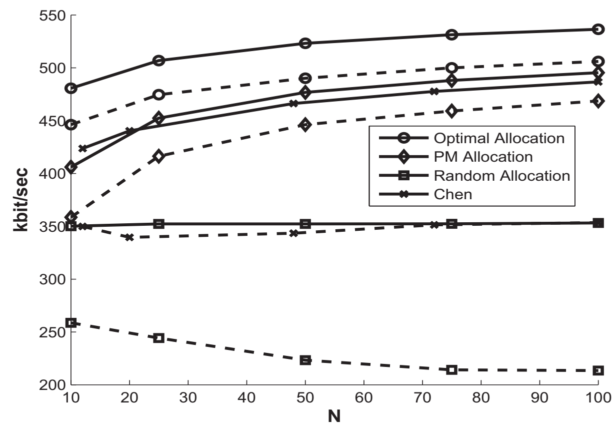

Fig. 3. Achievable rates as a function of N. Solid lines represent the mean rates and dashed lines the minimal rates.

TABLE I
Existence of PM as a function of N for $M = \lceil b\ln(K)\rceil$.

N	10	25	50	75	100
$\Pr(E)$	0.56	0.45	0.64	0.59	0.52

In Figure 4 we present the rates as a function of b for $N = 30$. All the rates increased almost linearly with b, and the ratio PM allocation rates to the optimal allocation rates remained almost constant for all b. This suggests that the results of Figure 3 are similar for each b value.

Next we allocated users a different number of channels. Specifically, $\frac{N}{4}$ of the users were allocated a single channel, $\frac{N}{4}$ two channels, $\frac{N}{4}$ three channels and $\frac{N}{4}$ four channels. We used $M = \lceil 3.75\ln(K)\rceil$ for all users. We simulated 100 realizations for each value from $N = 12, 20, 48, 72, 100$. PMs still existed in all realizations. Figure 5 shows the empirical CDFs of all the mean rates of $N = 100$ users in 100 different channel realizations, for the four classes (each in a different sub-figure). We note that the PM allocation rates are quite concentrated and most of the users get similar rates. These rates are always better than those of a random allocation, and close to the optimal allocation rates. These results, together with the proof itself, support our claim that our results also hold for an unequal allocation.

Since each user is allocated multiple channels, he can optimize his power allocation over these channels instead of using an equal power for all of them. We ignored this possibility in this paper, for a good reason. While increasing the complexity of the transmission, in all the above results water-filling had a negligible gain. To be exact, it never improved any of the rates by more than 0.1 kbps.

VII. Conclusions

We suggested an approach aimed at achieving the multiuser diversity gain for resource block allocation under the frequency-selective channel. Our approach requires knowledge of the indices of the M-best channels for each of the N users, instead of the whole NK channel gains.

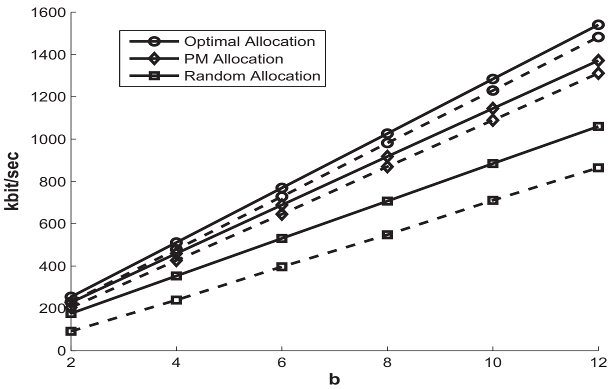

Fig. 4. Achievable rates as a function of b for $N = 30$. Solid lines represent the mean rates and dashed lines the minimal rates.

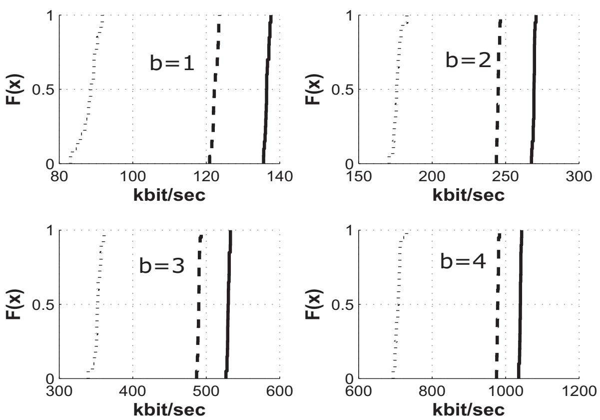

Fig. 5. Empirical CDFs of the mean rates for different classes of users with $N = 100$. Solid lines represent the optimal allocation rates, dashed lines the PM allocation rates and dotted lines the random allocation rates.

We showed that $M = (b+1)(1+\varepsilon)\ln(K)$ for $\varepsilon > 0$ is sufficient to guarantee that a PM allocation, where each user gets all of his b channels from his M-best channels, asymptotically exists. For this M, our suboptimal approach was shown to be asymptotically optimal for a broad class of fading distributions, both in the sum-rate and min-rate senses.

Our algorithm is the first limited feedback algorithm that has provably asymptotic optimality both of the sum-rate and the minimal rate. It requires significantly less feedback than the state of the art algorithms that achieve a good sum-rate.

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