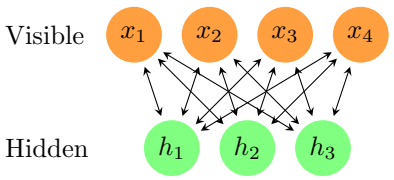
Restricted Boltzmann Machines (RBM)

Jacob Goldberger



$$\begin{aligned} E(x,h) &= -(h^\top w x + a^\top x + b^\top h) \qquad\qquad x \in \{0,1\}^d, \;\; h \in \{0,1\}^m \\ p(x,h) &= \frac{1}{Z}\exp(-E(x,h)) \end{aligned}$$

$$p(h|x) = \prod_i p(h_i|x), \qquad p(h_i = 1|x) = g(\sum_j w_{ij}x_j + b_i) = \frac{1}{1+\exp(-w_i x - b_i)}$$

$$p(x|h) = \prod_j p(x_j|h), \qquad p(x_j = 1|h) = g(\sum_i w_{ij}h_i + a_j)$$

$$\begin{aligned} \frac{\partial \log p(x;w,a,b)}{\partial w_{ij}} &= p(h_i = 1|x)x_j - E_{p(x,h)}h_i x_j = p(h_i = 1|x)x_j - p(h_i x_j = 1) \\ \frac{\partial \log p(x;w,a,b)}{\partial a_j} &= x_j - E_{p(x,h)}x_j = x_j - p(x_j = 1) \\ \frac{\partial \log p(x;w,a,b)}{\partial b_i} &= p(h_i = 1|x) - E_{p(x,h)}h_i = p(h_i = 1|x) - p(h_i = 1) \end{aligned}$$

Training data: $x_1, ..., x_n \in \{0,1\}^d$

Likelihood: $S(w,a,b) = \sum_t \log p(x_t;w,a,b)$

Contrastive Divergence (CD-1):

$$\begin{aligned} h_{t1,i} &\sim p(h_i|x_t) \\ x_{t2,j} &\sim p(x_j|h_{t1}) \end{aligned}$$

$$\begin{aligned} \frac{\partial S(w,a,b)}{\partial w} &\approx \sum_t (p(h = 1|x_t)x_t^\top - p(h = 1|x_{t2})x_{t2}^\top) \\ \frac{\partial S(w,a,b)}{\partial a} &\approx \sum_t (x_t - x_{t2}) \\ \frac{\partial S(w,a,b)}{\partial b} &\approx \sum_t (p(h = 1|x_t) - p(h = 1|x_{t2})) \end{aligned}$$