Markov Random Field (MRF)

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MRF model:

$$p(x_1, .., x_n) = \frac{1}{Z} \prod_i \psi_i(x_i) \prod_{ij \in E} \psi_{ij}(x_i, x_j)$$

Belief Propagation (Sum Product):

$$\begin{aligned} m_{j \to i}(x_i) &= \sum_{x_j} (\psi_j(x_j) \psi_{ij}(x_i, x_j) \prod_{k \in N(j) \setminus i} m_{k \to j}(x_j)) \\ b_i(x_i) &= \psi_i(x_i) \prod_{k \in N(i)} m_{k \to i}(x_i) \end{aligned}$$

Belief Revision (Max Product):

$$\begin{aligned} m_{j \to i}(x_i) &= \max_{x_j} (\psi_j(x_j) \psi_{ij}(x_i, x_j) \prod_{k \in N(j) \setminus i} m_{k \to j}(x_j)) \\ \hat{x}_i &= \arg\max_{x_i} (\psi_i(x_i) \prod_{k \in N(i)} m_{k \to i}(x_i)) \end{aligned}$$

Iterated conditional mode (ICM) algorithm:

$$\hat{x}_i = \arg\max_{x_i} (\psi_i(x_i) \prod_{j \in N(i)} \psi_{ij}(x_i, x_j))$$

Gibbs Sampling:

$$p(x_i | x \setminus x_i) \propto \psi_i(x_i) \prod_{j \in N(i)} \psi_{ij}(x_i, x_j)$$

$$\hat{p}(x_i = a) = \frac{1}{n} |\{1 \le t \le n | x_i(t) = a\}|$$

Mean Field Algorithm:

$$q_i(x_i) \propto \exp \left(\log \psi_i(x_i) + \sum_{j \in N(i)} \sum_{x_j} q_j(x_j) \log \psi_{ij}(x_i, x_j) \right)$$