

Belief Propagation

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Markov random Field (MRF):

$$p(x_1, \dots, x_n) = \frac{1}{Z} \prod_i \psi_i(x_i) \prod_{ij \in E} \psi_{ij}(x_i, x_j)$$

BP:

$$m_{j \rightarrow i}(x_i) = \sum_{x_j} (\psi_j(x_j) \psi_{ij}(x_i, x_j) \prod_{k \in N(j) \setminus i} m_{k \rightarrow j}(x_j))$$

$$b_i(x_i) = \psi_i(x_i) \prod_{k \in N(i)} m_{k \rightarrow i}(x_i)$$

MP:

$$m_{j \rightarrow i}(x_i) = \max_{x_j} (\psi_j(x_j) \psi_{ij}(x_i, x_j) \prod_{k \in N(j) \setminus i} m_{k \rightarrow j}(x_j))$$

$$\hat{x}_i = \arg \max_{x_i} (\psi_i(x_i) \prod_{k \in N(i)} m_{k \rightarrow i}(x_i))$$

Factor Graphs:

$$p(x_1, \dots, x_n) = \frac{1}{Z} \prod_a f_a(x_a)$$

BP:

$$m_{a \rightarrow i}(x_i) = \sum_{x_a | x_i = x_i} f_a(x_a) \prod_{j \in N(a) \setminus i} m_{j \rightarrow a}(x_j)$$

$$m_{i \rightarrow a}(x_i) = \prod_{b \in N(i) \setminus a} m_{b \rightarrow i}(x_i)$$

$$b(x_i) = \prod_{a \in N(i)} m_{a \rightarrow i}(x_i)$$

MP:

$$m_{a \rightarrow i}(x_i) = \max_{x_a | x_i = x_i} f_a(x_a) \prod_{j \in N(a) \setminus i} m_{j \rightarrow a}(x_j)$$

$$m_{i \rightarrow a}(x_i) = \prod_{b \in N(i) \setminus a} m_{b \rightarrow i}(x_i)$$

$$\hat{x}_i = \arg \max_{x_i} (\prod_{a \in N(i)} m_{a \rightarrow i}(x_i))$$

Bayesian Network:

$$p(x_1, \dots, x_n) = \prod_i p(x_i | x_{pa(i)})$$

BP:

$$\pi(x) = \sum_u p(x|u)\pi_{u \rightarrow x}(x)$$

$$\lambda(x) = \prod_i \lambda_{y_i \rightarrow x}(x)$$

$$b(x) = \pi(x)\lambda(x)$$

$$\pi_{x \rightarrow y}(x) = b(x)/\lambda_{y \rightarrow x}(x)$$

$$\lambda_{x \rightarrow u}(u) = \sum_x p(x|u)\lambda_x(x)$$

MP:

$$\pi(x) = \max_u p(x|u)\pi_{u \rightarrow x}(x)$$

$$\lambda(x) = \prod_i \lambda_{y_i \rightarrow x}(x)$$

$$b(x) = \pi(x)\lambda(x)$$

$$\pi_{x \rightarrow y}(x) = b(x)/\lambda_{y \rightarrow x}(x)$$

$$\lambda_{x \rightarrow u}(u) = \max_x p(x|u)\lambda_x(x)$$

$$\hat{x} = \arg \max_x b(x)$$