Other-Cell Interference in Cellular Power-Controlled CDMA

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Abstract—An improved series of bounds is presented for the other-cell interference in cellular power-controlled CDMA. The bounds are based on allowing control by one of a limited set of base stations. In particular, it is shown that the choice of cellular base station with least interference among the set of \( N_c > 1 \) nearest base stations yields much lower total mean interference from the mobile subscribers than the choice of only the single nearest base station.

I. INTRODUCTION

We consider an arbitrary cluster of hexagonal cells, each with a base station which communicates with a subset of all the mobiles in the system and also controls the power of this subset of users so that they are all received with the same power level at that base station. Each mobile user should be claimed and controlled by the base station which receives its signal with the least attenuation. Then, if the given user is power controlled by the base station with least attenuation, its transmitted power will be proportional to this minimum attenuation and hence will produce the least interference to all other cell base stations.

On the other hand, the mobile may not be aware or be able to find all base stations so as to minimize over the entire set. Suppose then that the set of base stations among which to select is limited to the \( N_c \) nearest (i.e. for which the distance \( r \) is minimum). Clearly the interference to other cells decreases monotonically with \( N_c \). We shall show by numerical methods that a large improvement, through reduction of interference to other cells, occurs with an increase of \( N_c \) from 1 to 2 with much smaller relative improvements with increases of \( N_c \) to 3 and 4.

II. PROPAGATION MODEL

The propagation attenuation is generally modeled as the product of the \( \mu \)th power of distance and a log-normal component representing shadowing losses [1]. These represent slowly varying variations even for users in motion and apply to both reverse and forward links. Thus for a user at a distance \( r \) from a base station, attenuation is proportional to

\[
\alpha(r, \xi) = r^\mu \cdot 10^{\xi/10}\]

(1)

where \( \xi \) is the dB attenuation due to shadowing, with zero mean and standard deviation \( \sigma \). Experimental data suggest the choices of \( \mu = 4 \) for power law and \( \sigma = 8 \) dB for standard deviation of \( \xi \), the log-normal shadowing.

Since any analysis of other-cell interference involves comparison of propagation losses among two or more base stations, the model must take into consideration the dependence of the propagation losses to two different base stations from a mobile user. Since the propagation losses in dB are Gaussian, we assume a joint Gaussian probability density for losses to two or more base stations. Equivalently, we may express the random component of the dB loss as the sum of a component in the near field of the user, which is common to all base stations, and a component which pertains solely to the receiving base station and is independent from one base station to another. Thus we may express the random component of the dB loss for the \( i \)th base station \((i = 0, 1, 2, \ldots)\) as

\[
\zeta_i = a_i + b_i \xi_i \quad \text{where } a_i^2 + b_i^2 = 1
\]

(2)

with \( \mathbb{E}(\zeta_i) = \mathbb{E}(\xi) = \mathbb{E}(\xi_i) = 0 \),

\[
\text{Var}(\zeta_i) = \text{Var}(\xi) = \text{Var}(\xi_i) = \sigma^2 \quad \text{for all } i,
\]

\[
\mathbb{E}(\xi_i \xi_j) = 0 \quad \text{for all } i \text{ and } j, \quad i \neq j.
\]

Thus the normalized covariance (correlation coefficient) of the losses to two base stations, \( i \) and \( j \), is

\[
\mathbb{E}(\zeta_i \zeta_j) / \sigma^2 = a_i^2 = 1 - b_i^2.
\]

(3)

A reasonable assumption is that the near field and base station specific propagation uncertainties have equal standard deviations, in which case \( a_i^2 = b_i^2 = 1/2 \) and the normalized covariance is 1/2 for all pairs of base stations. We shall use these values throughout all numerical results, although all expressions will be derived for arbitrary covariance value, (3).

A. Case I: Region where \( N_c \) Nearest Cells do not Include Base Station at Origin

As illustrated in Fig. 1 for the example of \( N_c = 3 \), any point in the shaded triangle will not include the base station at the origin in its minimum distance set. This will be the case for any point in the region \( S_0 \), which again for \( N_c = 3 \) is the region outside the hatched hexagon in the center, which region is therefore denoted \( S_0 \). The expected interference to

\[\footnote{The classical propagation model represented by (1) does not consider the dependence of propagation attenuations from one point to two or more others. The dependence assumed here is a logical extension of the classical model. The choice \( a = b = 1/\sqrt{2} \) in (2) also follows from assuming equal variances for near-field and for far-field attenuations.} \]


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the base station at the origin by the totality of uniformly
distributed users in Region $S_0$ is

$$I_{S_0} = \int_{S_0} E \min_{\mu=1}^{N_\mu} \frac{r_0^\mu (x, y)}{10^{5\mu/10}} 10^{5\mu/10} dA \rho (x, y)$$  (4)

Fig. 1: Region $S_0$ and Distances for $N_C = 3$

where the positive index $i$ refers to any of the $N_C$ base
stations in the set and the index 0 refers to the base station at
the origin, with $r_i$ and $\zeta_i$ being respectively the
corresponding distance and the normally distributed random
component (in dB) of the attenuation to the $i$th base station.
The density of users is assumed to be uniform, with
magnitude $\rho = 2 N_u / (3\sqrt{3})$ users/unit area, such that a
hexagonal cell, with radius normalized to unity, has an
expected number of users equal to $N_u$. Further we define

$$M_j = 10 \mu \log_{10} r_j (dB), R_j = r_j / r_0, \beta = (4\pi) / 10$$  (5)

With these definitions and the assumptions of (2) and
(3), (4) becomes

$$I_{S_0} = \int_{S_0} \sum_{j=1}^{N_j} R_j^\mu E \left[ 10^{5\mu/10} / \right. 10^{5\mu/10} < r_j^\mu / 10^{5\mu/10} \right.$$  

for all $i \neq j, i > 0 \rho dA$

$$= \int_{S_0} \sum_{j=1}^{N_j} R_j^\mu E \left[ e^{\beta r_j (\zeta_j - \zeta_0)} ; \zeta_j > \zeta_j + (M_j - M_i) / b \right.$$  

for all $i \neq j, i > 0 \rho dA$

$$= e^{b^2 \beta (\sigma)^2} \int_{S_0} \sum_{j=1}^{N_j} R_j^\mu \left[ \int_{-\infty}^{\infty} \frac{e^{-t^2/2}}{\sqrt{2\pi}} dt \right.$$  

for all $i \neq j, i > 0 \rho dA$

$$= e^{b^2 \beta (\sigma)^2} \int_{S_0} \sum_{j=1}^{N_j} R_j^\mu \left[ \int_{-\infty}^{\infty} \frac{e^{-t^2/2}}{\sqrt{2\pi}} dt \right.$$  

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for all $i \neq j, j > 0 \rho dA$

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for all $i \neq j, j > 0 \rho dA$

$$= \int_{S_0} \sum_{j=1}^{N_j} R_j^\mu E \left[ e^{\beta r_j (\zeta_j - \zeta_0)} ; \zeta_i > \zeta_i + (M_j - M_i) / b \right.$$  

for all $i \neq j, j > 0 \rho dA$

$$B. Case II: Region where $N_C$ Nearest Cells Do Include Base
Station at Origin

As already noted, the region $S_0$, illustrated for $N_C = 3$
by the hatched hexagon of Fig. 1, is the region where the $N_C$
nearest base stations include the one at the origin. Hence we
proceed nearly as before, but recognize that to be included,
the minimum attenuation to the $N_C - 1$ base stations not
including the origin must be less than that to the origin, for
otherwise it does not represent other-cell interference. Thus
in place of (6) we have,

$$I_{S_0} = \int_{S_0} \sum_{j=1}^{N_j-1} R_j^\mu E \left[ 10^{5\mu/10} / \right. 10^{5\mu/10} < r_j^\mu / 10^{5\mu/10} \right.$$  

for all $i \neq j, i > 0 \rho dA$

$$= \int_{S_0} \sum_{j=1}^{N_j-1} R_j^\mu E \left[ e^{\beta r_j (\zeta_j - \zeta_0)} ; \zeta_i + M_0 > \zeta_j + M_j \right.$$  

for all $i \neq j, j > 0 \rho dA$
\[ f = \left( \left( \frac{\beta \sigma}{\sqrt{2}} \right)^{2/\mu} \right) \int_{s_0} \int_{s_0} R_j^\mu e^{-\left( \frac{\beta \sigma}{\sqrt{2}} \right)^2} \, \text{d}A \]

where the effect of the log-normal variation in attenuation appears as a scale factor to the distance-dependent interference. This is, of course, a consequence of the fact that for \( N_C = 1 \), the nearest base station is always selected.

III. RESULTS AND CONCLUSIONS

Table I shows the relative interference factor, \( f \), as a function of total standard deviation \( \sigma_T = \sqrt{2} \sigma \) for control by the best of \( N_C = 1, 2, 3 \) and 4 base stations when the deterministic attenuation power law is \( \mu = 4 \). Table II shows \( f \) as a function of \( \sigma \) for \( N_C = 2 \) with power laws 3, 4 and 5. All results for \( N_C = 3 \) and 4 are computed numerically using (6), (7) and (8), for \( N_C = 2 \) using (9) and for \( N_C = 1 \) using (10). Note in Table I that the results are all the same for \( \sigma = 0 \) because the best base station is always the nearest if the attenuation is purely deterministic. Note also that a small amount of randomness (e.g. \( \sigma_T = 2 \text{ dB} \)) can actually improve performance over the strictly deterministic case when \( N_C > 1 \).

### Table I

<table>
<thead>
<tr>
<th>Total Standard Deviation</th>
<th>( N_C = 1 )</th>
<th>( N_C = 2 )</th>
<th>( N_C = 3 )</th>
<th>( N_C = 4 )</th>
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<td>0.44</td>
<td>0.44</td>
<td>0.44</td>
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<td>20.0</td>
<td>2.62</td>
<td>1.17</td>
<td>0.91</td>
</tr>
</tbody>
</table>

Fig. 2: Integration Regions \( S_0 \) for \( N_C = 1, 2 \) and 3

Here the region \( S_0 \) shrinks from the hexagon of Fig. 1 for \( N_C = 3 \) to the six-pointed star for \( N_C = 2 \) to the single-cell hexagon for \( N_C = 1 \) (see Fig. 2).
TABLE II
RELATIVE OTHER-CELL INTERFERENCE FACTOR, F FOR \( \mu = 3, 4 \) AND 5 (\( N_c = 2 \))

<table>
<thead>
<tr>
<th>Total Standard Deviation</th>
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<th>( \mu = 4 )</th>
<th>( \mu = 5 )</th>
</tr>
</thead>
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<td>0.47</td>
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<td>5.93</td>
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<td>1.42</td>
</tr>
</tbody>
</table>

A major conclusion is that, particularly for larger \( \sigma \), the reduction of other-cell interference in going from using just the nearest cell (\( N_c = 1 \)) to using the better of the nearest pair of cells (\( N_c = 2 \)) is quite dramatic. The relative improvement in using the best of the three nearest cells is much less, though noticeable, because it reduces the interference from users in the corners of the cell, which are equidistant from three base stations.

ACKNOWLEDGMENT

The authors acknowledge the insight provided by T. Chebaro and P. Godlewski in [2], who pointed out that a previous interference analysis presented in Appendix I of [3] was an approximation rather than a bound, which led to our reformulation of the problem presented here. In addition, the same authors also proposed in [4] the series of approximations involving an ever increasing number of nearest cells, as used herein.

REFERENCES