Distributed Game Theoretic Optimization Of Frequency Selective Interference Channels: A Cross Layer Approach

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Abstract—In this paper we propose a smart multichannel extension of CSMA (SXMA) with collision avoidance as a mean to perform distributed spectrum allocation using the Nash Bargaining solution. The smart carrier sensing replaces explicit CSI signalling required by standard approaches to cooperative game theoretic spectrum allocation in cognitive radio networks. It allows fast convergence of the network to a cooperative Pareto optimal point.

Keywords: Cognitive radio, distributed spectrum management, game theory, Nash bargaining solution, interference channel, multiple access channel, carrier sensing.

I. INTRODUCTION

In recent years cooperative approaches derived from game theory have been used for efficient radio resource allocation. The most popular approach is the Nash Bargaining Solution (NBS) [1],[2]. The NBS is based on four axioms that lead to a unique solution for the bargaining problem [3]. The NBS was applied for a flat fading channel (SISO [4], MISO [5] and MIMO [6]), as well as for a frequency selective channel under two types of constraints: an average power constraint [1], [7] and a power mask constraint [2].

Most published results are based on a central controller that provides the allocation for NBS. The central controller collects gets from each player the rate vector and the feasible rate with competition. Base on this information the central controller allocates the subchannels to the player. Recently, Gao et al. [8] proposed a semi distributed algorithm with a coordinator using the Lagrange dual decomposition technique. In this approach each player solves the allocation problem independently based on the thresholds that are provided by the coordinator, and transmits his allocation to the coordinator. The coordinator updates the threshold and broadcasts them.

It is highly desirable to achieve the game theoretic solution using a purely distributed approach. In recent years a new family of opportunistic medium access control protocols has emerged in the context of wireless sensor networks, where energy and bandwidth constraints are the dominant limitations [9], [10], [11]. These protocols use the channel response of each sensor in order to determine the back-off function. Recently, more advanced techniques have been proposed, where the back-off function depends on a network utility function [12]. However these techniques were single channel techniques.

In this paper we establish a novel approach for distributed computation of the NBS with no central coordinator and with no explicit transmission of the achievable rates. The proposed approach is based on a novel multichannel smart carrier sensing and can be applied to other cooperative solutions like stable matching [13] and to generalized NBS [7]. The paper is organized as follows. In section II we define the Nash axioms and the Nash Bargaining solution. In section III we describe the cooperative bargaining over frequency selective channel under mask constraint. In section IV we presents a novel multichannel smart carrier sensing approach that solves the NBS without a central controller. Simulation results are shown in section V followed by conclusions.

II. THE NASH BARGAINING SOLUTION

In this section we describe the Nash bargaining solution (NBS) and describe the axioms used in order to derive this cooperative solution. While, Nash equilibria [14] are inevitable whenever a non-cooperative zero sum game is played, they can lead to a substantial loss to all players, compared to a cooperative strategy in the non-zero sum case, where players can cooperate. An example of this situation is the well known prisoner’s dilemma. The main issue in this case is how to achieve the cooperation in a stable manner and what rates can be achieved through cooperation. One of the earliest solutions to cooperative $N$ player games is the Nash bargaining solution [15]. [3]. The underlying structure for a Nash bargaining problem in an $N$ player game is a set of outcomes of the bargaining process $S \subseteq \mathbb{R}^N$ which is compact and convex and a designated disagreement outcome $d$ (which represents the agreement to disagree and solve the problem competitively). $S$ can be considered as a set of outcomes of the possible joint strategies or states, Alternatively, some authors consider $S$ as a set of states, $d$ a disagreement state and a multiuser utility function $U : S \cup \{d\} \rightarrow \mathbb{R}^N$ such that $U(S \cup \{d\})$ is compact and convex. The two approaches are identical and the first is obtained from the second by defining the game by the set of utilities of the possible outcomes. We will use the first formulation since it simplifies notation. However, in some cases we will define the outcomes of the game in terms of strategies. The set $S$ in the first definition is then obtained by identifying it with $U(S \cup \{d\})$ of the second definition.
The Nash bargaining solution is a function $F$ which assigns to each bargaining problem $S \cup \{d\}$ as above an element of $S \cup \{d\}$, satisfying the following four axioms:

*Linearity.* Assume that we consider the bargaining problem $S' \cup \{d'\}$ obtained from the problem $S \cup \{d\}$ by transformations: $s'_i = \alpha_i s_i + \beta_i$, $i = 1, \ldots, N$. $d'_i = \alpha_i d_i + \beta_i$. Then the solution satisfies $F_i (S' \cup \{d'\}) = \alpha_i F_i (S \cup \{d\}) + \beta_i$, for all $i = 1, \ldots, N$.  

*Independence of irrelevant alternatives.* This axiom states that if the bargaining solution of a large game $T \cup \{d\}$ is obtained in a small set $S$. Then the bargaining solution assigns the same solution to the smaller game, i.e., The irrelevant alternatives in $T \setminus S$ do not affect the outcome of the bargaining.

*Symmetry.* If two players $i < j$ are identical in the sense that $S$ is symmetric with respect to changing the $i$'th and the $j$'th coordinates, then $F_i (S \cup \{d\}) = F_j (S \cup \{d\})$. Equivalently, players which have identical bargaining preferences, get the same outcome at the end of the bargaining process.

*Pareto optimality.* If $s$ is the outcome of the bargaining then no other state $t$ exists such that $s \prec t$ (coordinate wise).

A good discussion of these axioms can be found in [16]. Nash proved that there exists a unique solution to the bargaining problem satisfying these four axioms. The solution is obtained by solving the following problem:

$$s_{\text{nash}} = \arg \max_{s \in S \cup \{d\}} \prod_{n=1}^{N} (s_n - d_n).$$

(1)

Typically, one assumes that there exist at least one feasible $s \in S$ such that $d < s$ coordinatewise, but otherwise we can assume that the bargaining solution is $d$. We also define the Nash function $F(s): S \cup \{d\} \rightarrow R$

$$F(s) = \prod_{n=1}^{N} (s_n - d_n),$$

(2)

where $s = [s_1, \ldots, s_N]^T$. The Nash bargaining solution is obtained by maximizing the Nash function over all possible states. Since the set of possible outcomes $S \cup \{d\}$ is compact and convex $F(s)$ has a unique maximum on the boundary of $S \cup \{d\}$.

Whenever the disagreement situation can be decided by a competitive game, it is reasonable to assume that the disagreement state is given by a Nash equilibrium of the relevant competitive game. In some cases there are other possibilities for the disagreement point. When the utility for user $n$ is given by the rate $R_n$, and $d$ is the competitive Nash equilibrium, it is obtained by iterative waterfilling for general ISI channels. For the case of mask constraints the competitive solution is simply given by all users using the maximal PSD at all tones.

**III. BARGAINING OVER FREQUENCY SELECTIVE CHANNELS UNDER MASK CONSTRAINT**

In this section we define the cooperative game corresponding to the joint FDM/TDM achievable rate region for the frequency selective $N$ user interference channel. We limit ourselves to the PSD mask constrained case since this case is actually the more practical one. In real applications, the regulator limits the PSD mask and not only the total power constraint. Let the $K$ channel matrices at frequencies $k = 1, \ldots, K$ be given by $\mathbf{H}_k: k = 1, \ldots, K$. Each player is allowed to transmit at maximum power $p(k)$ in the $k$'th frequency bin. In non-cooperative scenario, under mask constraint, all players transmit at the maximal power they can use. Thus, all players choose the PSD, $\mathbf{p}_i = [p_i(k) : 1 \leq k \leq K]$. The payoff for user $i$ in the non-cooperative game is therefore given by:

$$R_i^C (\mathbf{p}_i) = \sum_{k=1}^{K} \log_2 (1 + \text{SINR}_i(k)).$$

(3)

Here, $R_i^C$ is the capacity available to player $i$ given a PSD mask constraint distributions $\mathbf{p}$, and $\text{SINR}_i(k)$ is the received signal to interference-plus-noise ratio. Note that without loss of generality, and in order to simplify notation, we assume that the width of each bin is normalized to 1. We now define the cooperative game $G_{TF} (N, K, \mathbf{p})$.

**Definition 3.1:** The FDM/TDM game $G_{TF} (N, K, \mathbf{p})$ is a game between $N$ players transmitting over $K$ frequency bins under common PSD mask constraint. Each user has full knowledge of the channel matrices $\mathbf{H}_k$. The following conditions hold:

1. Player $i$ transmits using a PSD limited by $\mathbf{H}_k: k = 1, \ldots, K$.

2. Strategies for player $i$ are vectors $\alpha = [\alpha_i(1), \ldots, \alpha_i(K)]^T$ where $\alpha_i(k)$ is the proportion of time player $i$ uses the $k$'th frequency channel. This is the TDM part of the strategy.

3. The utility of the $i$'th player is given by

$$R_i = \sum_{k=1}^{K} R_i(k) = \sum_{k=1}^{K} \alpha_i(k) \log_2 \left(1 + \frac{|h_i(k)|^2 p_i(k)}{\sigma_i^2(k)} \right).$$

(4)

Note that interference is avoided by time sharing at each frequency band, i.e only one player transmits at a given frequency bin at any time. Furthermore, since at each time instance each frequency is used by a single user, each user can transmit using maximal power.

The Nash bargaining can be posed as an optimization problem

$$\max \prod_{i=1}^{N} (R_i(\alpha_i) - R_i^C)$$

subject to:

$$\forall i, \sum_{k=1}^{K} \alpha_i(k) = 1,$$

$$\forall i, \alpha_i(k) \geq 0,$$

$$\forall i R_i^C \leq R_i(\alpha_i),$$

(5)

where,

$$R_i(\alpha_i) = \sum_{k=1}^{K} \alpha_i(k) \log_2 \left(1 + \frac{|h_i(k)|^2 p_{\text{max}}(k)}{\sigma_i^2(k)} \right) = \sum_{k=1}^{K} \alpha_i(k) R_i(k).$$

(6)

This problem is convex and therefore can be solved efficiently using convex optimization techniques. Furthermore, the following theorem holds [2]:

**Theorem 3.1:** Assume that for all $i \neq j$ the values $\{L_{ij}(k) : k = 1, \ldots, K\}$ are all distinct. Then in the optimal
This theorem suggests, that when \( N/2 \ll K \) the optimal FDM NBS is very close to the joint FDM/TDM solution. It is obtained by allocating the common frequencies to one of the users. Therefore, we will turn now to a pure FDM game, where each frequency is allocated to a single user.

IV. SMART CARRIER SENSING WITH COLLISION AVOIDANCE

In this section we present a multichannel carrier sensing scheme with a time varying back-off function. The back-off function at each channel is derived based on the utility function of each user, the achievable rate at that channel and the total rate currently available for this user.

Similarly to standard CSMA protocols, each user performs sensing on each of its intended channels up to a time defined by a back-off function. If the back-off period expired and he did not sense any activity on the channel he is allowed to transmit its own packet. However, in contrast to standard CSMA, the smart carrier sensing has the following properties:

1) The purpose of the smart carrier sensing is not to provide a random access MAC, but to allow a distributed convergence of the different users to the Nash Bargaining solution for the resource allocation problem. As such convergence time should be short compared to the time scale of channel statistics variations.

2) The sensing is performed simultaneously on all the channels. This is reasonable in view of recent advances in cognitive radio technologies.

3) The back-off function depends on the achievable rate of the user and the total rate it has currently allocated on all channels. This allows to optimize complicated network utility functions.

4) The back-off function is both frequency and time dependent, since after a channel has been allocated, the back-off function might allocate it to a different user at the next packet.

5) In contrast to our work on stable matching [13], users are allowed to transmit on multiple frequencies simultaneously.

The specific choice of the back-off function at each time and each frequency is the important part in the smart carrier sensing. It should be derived from the total utility optimization. The smart carrier sensing algorithm is performed as described in Table I.

Proper choice of the back-off function \( b_n(k) \) for each user and each carrier can lead to optimization of various network utility functions. In the next section we will discuss the solution of the Nash Bargaining problem using smart carrier sensing.

V. DISTRIBUTED COMPUTATION OF THE NASH BARGAINING SOLUTION

We would like to optimize a total utility function of the form

\[
U(\alpha_1, \ldots, \alpha_N) = \sum_{n=1}^{N} \log \left( \sum_{k=1}^{K} \alpha_n(k) R_n(k) - R_{nc} \right) .
\]  

(7)

Note that \( \log(R) \) is a monotonic function of \( R \) and by Pareto optimality

\[
\sum_{n=1}^{N} \alpha_n(k) = 1 \quad \text{for all } k .
\]  

(8)

and

\[
0 \leq \alpha_n(k) \leq 1
\]

Computing \( R_{nc} \) is easy to do without cooperation, since each user can estimate its own channel, and the interference it experiences during the channel acquisition and sensing phase. However, in random access environment we can use \( R_{nc} = 0 \) since a collision results in a packet loss. To simplify notation we assume \( R_{nc} = 0 \). We propose to use the carrier sensing function given by:

\[
b_n(k) = \frac{R_{\text{tot}}^{(\ell+1)}(n)}{R_n(k)^2}
\]  

(9)

Theorem 5.1: Smart carrier sensing with the function \( b_n(k) \) converges to the Nash Bargaining Solution.

The proof of the theorem is postponed to the full version of this paper. However, we hint that the function \( b_n(k) \) was chosen, such that at each step the user with largest gradient of the Nash function at the given channel, receives the access to the channel. The iteration ensures, that we optimize a first order approximation of the Nash Bargaining at each step.

VI. SIMULATIONS

We now present several examples for the performance of the smart carrier sensing for optimizing the Nash Bargaining function. In the first example we assume that two users share a channel composed of 32 i.i.d Rayleigh sub-channels. The
transmit power to receiver noise ratio at each sub channel was 10 dB and 13 dB for users 1 and 2 respectively.

Figure 1 describes the utility as a function of iteration number, where each iteration includes serial update of all frequency channels. Figure 2 demonstrates the rate of each user. We can clearly see the convergence within few iterations.

We repeated the experiment with 16 users and 512 frequency channels. The SNR of the users was set from 0 dB to 45 dB in steps of 3 dB. The results are depicted in Figures 3 and 4. We can clearly see the rapid convergence as well as the final rate of each user. Figure 5 demonstrates the rates achieved after convergence.

VII. CONCLUSIONS

In this paper we propose a new smart multichannel CSMA (SXMA) protocol that can optimize the spectrum allocation under a total PSD mask constraint. The proposed protocol
achieves rapid convergence to the Nash Bargaining Solution (up to second order terms). The technique is easy to implement and fully distributed.

REFERENCES