Changing of the Guards: Strip Cover with Duty Cycling*

Amotz Bar-Noy† amotz@sci.brooklyn.cuny.edu

Ben Baumer‡ bbaumer@smith.edu

Dror Rawitz§ rawitz@eng.tau.ac.il

September 29, 2012

Abstract

The notion of *duty cycling* is common in problems which seek to maximize the lifetime of a wireless sensor network. In the duty cycling model, sensors are grouped into *shifts* that take turns covering the region in question, and each sensor can belong to at most one shift. We consider the imposition of the duty cycling model upon the STRIP COVER problem, where we are given n sensors on a one-dimensional region, and each shift can contain at most $k \leq n$ sensors. We call the problem of finding the optimal set of shifts so as to maximize the length of time that the entire region can be covered by a wireless sensor network, k-DUTY CYCLE STRIP COVER (k-DUTYSC). In this paper, we present a polynomial-time algorithm for 2-DUTYSC. Furthermore, we show that this algorithm is a $\frac{35}{24}$-approximation algorithm for k-DUTYSC. We also give two lower bounds: $\frac{15}{11}$, for $k \geq 4$, and $\frac{6}{5}$, for $k = 3$, and provide experimental evidence suggesting that these lower bounds are tight. Finally, we propose a fault tolerance model and find thresholds on the sensor failure rate, over which our algorithm has the highest expected performance.

Keywords: approximation algorithms, duty cycling, wireless sensor networks, adjustable ranges, strip cover.

1 Introduction

We consider the following problem: Suppose we have a one-dimensional region (or interval) that we wish to cover with a wireless sensor network. We are given the locations of n sensors located on that interval, and each sensor is equipped with an identical battery of finite charge. We have the ability to set the sensing radius of each sensor, but its battery charge drains in inverse proportion to the radius that we set. Our goal is to organize the sensors into disjoint coverage groups (or *shifts*), who will take turns covering the entire region for as long as possible. We call this length of time the *lifetime* of the network.

*A preliminary version of this paper appeared in the proceedings of the 19th International Colloquium on Structural Information and Communication Complexity (SIROCCO), 2012.

†The Graduate Center of the City University of New York, New York, NY 10016, USA.

‡Smith College, Northampton, MA 01063, USA.

§School of Electrical Engineering, Tel Aviv University, Tel-Aviv 69978, Israel.

More specifically, we consider the STRIP COVER problem with identical batteries under a *duty cycling* restriction. An instance consists of a set $X \subset [0, 1]$ of n sensor locations, and a rational number B representing the initial battery charge of each sensor. Each battery discharges in inverse linear proportion to its radius, so that a sensor i whose radius is set to r_i survives for B/r_i units of time. In the *duty cycling* model, the sensors are partitioned into disjoint coverage groups, called *shifts*, which take turns covering the entire interval for as long as their batteries allow. The sum of these lengths of time is called the *lifetime* of the network and is denoted by T. For any fixed $k \leq n$, the k-DUTY CYCLE STRIP COVER (k-DUTYSC) problem seeks an optimal partitioning of the sensors such that the network lifetime T is maximized, yet no coverage group contains more than k sensors. In the fault tolerant variant, each sensor may fail to activate with some fixed probability $p \in [0, 1]$, and we seek to maximize the *expected* lifetime of the network (i.e., the expected sum of lifetimes of surviving shifts).

Motivation. Applications of scheduling problems similar to STRIP COVER are increasingly common. One such application involves monitoring a fence, or supply line, that exists in inhospitable territory. In this case, it may be feasible (even cost-effective) to deploy a set of sensors along the fence, but unfeasible to position them at pre-determined locations. For example, it might be easy to drop the senors from an airplane, but impossible to dispatch human beings to place them. While the scheduler may have access to the location of each sensor via GPS, technical limitations may require that a single assignment be given. In such a scenario, we might be incentivized to organize the sensors into disjoint shifts, providing motivation for our duty cycling model. Finally, any physical device will have some nonzero failure rate, and thus a fault-tolerant solution will be more robust.

Solutions to the general STRIP COVER problem contain both the radial assignments and activation and de-activation times for each sensors. As a result, these solutions can be complicated to implement and understand. Moreover, interdependence among multiple sensors can make such solutions susceptible to catastrophic decline in network lifetime if there is a non-zero probability of sensor failure. Conversely, since in the duty cycling model each sensor can participate in at most one cover, the scheduling of the covers is of little importance. Furthermore, by minimizing the number of sensors participating in each cover, duty cycling solutions can be insulated from the risk imposed by sensor failure.

Related work. This line of research began with Buchsbaum, et al.'s [5] study of the RESTRICTED STRIP COVER (RSC) problem. In RSC, the locations and sensing radii of n sensors placed on an interval are given, and the problem is to compute an optimal set of activation times, so as to maximize the network lifetime. They showed that RSC is NP-hard, and presented an $O(\log \log n)$-approximation algorithm. Gibson and Varadarajan [11] later improved on this result by discovering a constant factor approximation algorithm.

The problem of finding the optimal set of radial assignments for sensors deployed on an interval, rather than the activation times, is more tractable. Peleg and Lev-Tov [12] considered the problem of covering a finite set of m target points while minimizing the sum of the radii assigned, and found an optimal polynomial-time solution via dynamic programming. The situation wherein the whole interval must be covered corresponds to a "one shift" version of n-DUTYSC, wherein the restriction is not upon the size of each shift, but upon the number of shifts. Bar-Noy, et al. [4] provided an optimal polynomial-time algorithm for this problem.

The interest in duty cycling developed in part from the introduction of the SET k-COVER problem by Slijepcevic and Potkonjak [16]. This problem, which they showed to be NP-hard, seeks to find at least k disjoint covers among a set of subsets of a base set. Perillo and Heinzelman [15] considered a variation in which each sensor has multiple modes. They translated the problem into a generalized maximum flow graph problem, and employed linear programming to find a optimal solution. Abrams et al. [1] provided approximation algorithms for a modification of the problem in which the objective was to maximize the total area covered by the sensors. Cardei et al. [6, 7, 8] considered adjustable range sensors, but also sought to maximize the number of non-disjoint set covers over a set of target coverage points.

The work of Pach and Tóth [13, 14] also has applications in this context. They showed that a k-fold cover of translates of a centrally-symmetric open convex polygon can be decomposed into $\Omega(\sqrt{k})$ covers. Aloupis, et al. [2] improved this to the optimal $\Omega(k)$ covers, and the centrally-symmetric restriction was later lifted by Gibson and Varadarajan [11]. In each of the above cases, the concept of finding many disjoint set covers, which can be seen as shifts, is used as a proxy for maximizing network lifetime.

Finally, the general STRIP COVER problem, in which each sensor has a *different* battery charge, was studied by Bar-Noy, et al. [4]. They also the SET ONCE STRIP COVER (ONCESC) problem, in which the radius and activation time of each sensor can be set only once. They showed that ONCESC is NP-hard, and that ROUNDROBIN (sensors take turns covering the entire interval) is a $\frac{3}{2}$-approximation algorithm for both ONCESC and STRIP COVER. Bar-Noy, et al. [4] also showed that the appoximation ratio of any duty cycling algorithm, is at least $\frac{3}{2}$ for both ONCESC and STRIP COVER. Bar-Noy and Baumer [3] also analyzed non-duty cycling algorithms for STRIP COVER with identical batterie. The CONNECTED RANGE ASSIGNMENT problem studied by Chambers, et al. [9], wherein the goal is to connect a series of points in the plane using circles, is also related. They presented approximation bounds for solutions using a fixed number of circles, which is similar to limiting shift sizes.

Our results. In Section 2, we define the class of k-DUTYSC problems, and present the trivial solution to 1-DUTYSC. We present a polynomial-time algorithm, which we call MATCH, for 2-DUTYSC in Section 3. In Section 4, we compare the performance of ROUNDROBIN to an algorithm that uses only a single shift. We prove that when the sensors are equi-spaced on the coverage interval, ROUNDROBIN performs most poorly in comparison to the one shift algorithm. Then we study the performance of ROUNDROBIN on these "perfect" deployments. This study is used to analyze MATCH in k-DUTYSC, but is of independent interest, since perfect deployments are the most natural. In Section 5 we show that MATCH is a $\frac{35}{24}$-approximation algorithm for k-DUTYSC. We also give two lower bounds: $\frac{15}{11}$, for $k \geq 4$, and $\frac{6}{5}$, for $k = 3$, and provide experimental evidence suggesting that these lower bounds are tight. In Section 6, we consider a fault tolerance model, and show that if the failure rate of each sensor is sufficiently high, MATCH becomes optimal. We contend that even if the approximation ratio of k-DUTYSC for $k \geq 3$ is improved, MATCH will be of interest, due to its simplicity, performance, and fault tolerance.

2 Preliminaries

Duty cycles. Let $U = [0,1]$ be the interval that we wish to cover, and let $X = \{x_1, \ldots, x_n\} \in U^n$ be a set of n sensor locations. We assume that $x_i \leq x_{i+1}$, for every $i \in \{1, \ldots, n-1\}$. We first assume that all sensors have unit capacity batteries. We will justify this assumption later.

A pair (C, t), where $C \subseteq X$ is a subset of k sensor locations and $t \geq 0$, is called a *k-duty cycle* (or simply a *duty cycle*, or a *shift*). The sensors in C are activated at the same time and are deactivated together after t time units. A duty cycle (C, t) is feasible if the sensors in C can cover the interval $[0, 1]$ for the duration of t time units. More specifically, a sensor i such that $x_i \in C$ is assigned a radius $1/t$ and covers the range $[x_i - 1/t, x_i + 1/t]$, and the duty cycle is feasible if $[0,1] \subseteq \bigcup_{i \in C}[x_i - 1/t, x_i + 1/t]$.

Let $\textsc{All}(C)$ denote the maximum t for which (C, t) is feasible. $\textsc{All}(C)$ is called the *lifetime* of C. Given a duty cycle $C = \{x_{i_1}, \ldots, x_{i_k}\}$ define

$$d_j \triangleq \begin{cases} 2x_{i_1} & j = 0, \\ 2(1 - x_{i_k}) & j = k, \\ x_{i_{j+1}} - x_{i_j} & \text{otherwise,} \end{cases} \qquad \text{and} \qquad \Delta \triangleq \max_j \{d_j\} \ .$$

Observation 1. $\textsc{All}(C) = \frac{2}{\Delta}$.

Proof. The maximum lifetime of C is at least $\frac{2}{\Delta}$, since the radial assignment $r_i = \frac{\Delta}{2}$ covers $[0, 1]$. However, if $\textsc{All}(C) > \frac{2}{\Delta}$, then $r_i < \frac{\Delta}{2}$, for every i. Hence, $[0, 1]$ is not covered. □

In light of Observation 1, it suffices to refer to any subset $C \subseteq X$ as a shift, with a corresponding lifetime that is inferred from $\textsc{All}(C)$.

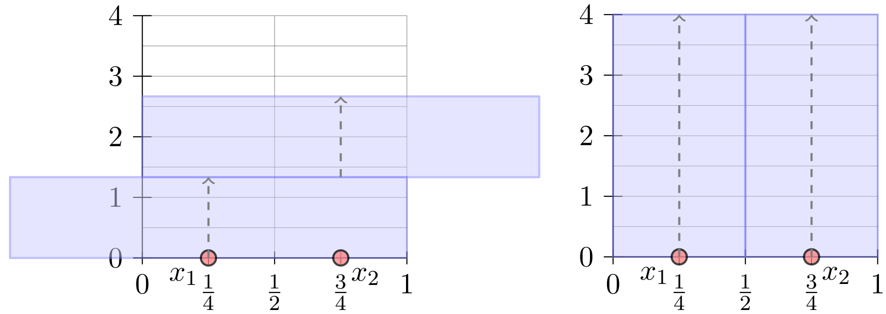
Problems. k-Duty Cycle Strip Cover (abbreviated k-DutySC) is defined as follows. The input is a set $X = \{x_1, \ldots, x_n\} \in U^n$ of n sensor locations. A solution (or *schedule*) is a partition of X into m non-empty pairwise disjoint subsets $C_1, \ldots, C_m \subseteq X$ such that $|C_j| \leq k$, for every j. The goal is to find a solution that maximizes $\sum_j \textsc{All}(C_j)$. Thus, a solution to DutySC consists of a partition of X into shifts, where each shift employs All to achieve optimal lifetime.

Note that $\textsc{All}(C)$, for any shift C, and hence the maximum lifetime are multiplied by a factor of B, if all sensors have batteries with capacity B. Hence, throughout the paper we assume that all sensors have unit capacity batteries.

The optimal lifetime for k-DutySC is denoted by $\textsc{Opt}_k$. The best possible lifetime of a k-DutySC instance X, for any k, is $2n$.

Observation 2. *$\textsc{Opt}_k(X) \leq 2n$, for every k.*

Proof. Consider a schedule $C_1, \ldots, C_m$ and let Δ_i correspond to C_i. The minimum possible value of Δ_i is $1/|C_i|$, for every i. By Observation 1 we get that $\textsc{All}(C_i) \leq \frac{2}{\Delta_i} \leq 2|C_i|$. The observation follows from the fact that each of the n sensors is used in exactly one shift. □



(a) RoundRobin$(X_2^*) = \frac{8}{3}$ (b) Opt$(X_2^*) = 4$

Figure 1: Illustration of the $\frac{3}{2}$ upper bound on the approximation guarantee of RoundRobin. For the instance $X_2^* = \{\frac{1}{4}, \frac{3}{4}\}$, RoundRobin achieves a lifetime of only $\frac{8}{3}$ time units, while Opt achieves 4.

Perfect deployment. Define

$$X_n^* = \left\{\frac{2i-1}{2n} : i \in \{1, \ldots, n\}\right\} = \left\{\frac{1}{2n}, \frac{3}{2n}, \ldots, \frac{2n-1}{2n}\right\} .$$

We refer to X_n^* as the *perfect deployment* since the n-DutySC lifetime of X_n^* is $2n$, namely All$(X_n^*) = 2n$.

Round robin In 1-DutySC each sensor must work alone, therefore there is only one possible solution: $C_i = \{i\}$, for every i. Observe that this solution is valid for k-DutySC, for every k. We refer to the algorithm that generates this solution as RoundRobin. Observe that the RoundRobin lifetime is given by $\mathrm{RR}(X) = \sum_i t_i$, where $t_i \triangleq \mathrm{All}(C_i) = \max\{1/x_i, 1/(1-x_i)\}$ by Observation 1.

Bar-Noy, et al. [4] showed that RoundRobin is a $\frac{3}{2}$-approximation algorithm for Strip Cover. Since RoundRobin schedules are duty cycle schedules and any k-DutySC schedule is also a Strip Cover schedule, it follows that

Theorem 1. RoundRobin *is a $\frac{3}{2}$-approximation algorithm for k-*DutySC*, for every $k \geq 2$.*

The above ratio is tight due to the instance $X_2^* = \left\{\frac{1}{4}, \frac{3}{4}\right\}$ as shown in [3], and illustrated in Figure 1. We note that in our figures we depict schedules graphically as space-time diagrams. In each diagram, the horizontal axis represents the interval $[0, 1]$ that is to be covered, with the sensors in each instance illustrated by red circles at the appropriate locations. The vertical axis represents time, with the maximum possible lifetime of $2n$ visible at the top. The active coverage of each sensor is represented by a blue rectangle with a vertical arrow pointing upwards.

3 Strip Cover with Shifts of Size 2

We present a polynomial-time algorithm for solving 2-DutySC. The algorithm is based on a reduction to the Maximum Weight Matching problem in bipartite graphs that can be solved in $O(n^2 \log n + nm)$ in graphs with n vertices and m edges (see, e.g., [10]).

Theorem 2. 2-DutySC *can be solved in polynomial time.*

Proof. Given a 2-DutySC instance X, with n sensors, we construct a bipartite graph $G = (L, R, E)$ as follows:

$$\begin{aligned} L &= \{v_i : i \in \{1, \ldots, n\}\} \\ R &= \left\{v'_i : i \in \{1, \ldots, n\}\right\} \\ E &= \left\{(v_i, v'_j) : i \leq j\right\} \ . \end{aligned}$$

The weight of an edge $e = (v_i, v'_j)$ is defined as follows:

$$w(e) = \begin{cases} \mathrm{RR}(x_i) & i = j, \\ \textsc{All}(\{x_i, x_j\}) & i < j. \end{cases}$$

Observe that a 2-DutySC solution $C_1, \ldots, C_m$ for X induces a (perfect) matching whose weight is the lifetime of the solution. Also, a matching $M \subseteq E$ induces a 2-DutySC solution whose lifetime is the weight of the matching. Hence, the weight of a maximum weight matching in G is the optimal 2-DutySC lifetime of X. □

The algorithm that is described in the theorem is henceforth referred to as Algorithm Match.

4 Round Robin vs. All

Assume we are given a set X of k sensors. In this section we compare $\mathrm{RR}(X)$ to $\textsc{All}(X)$. This comparison will be used in the next section to analyze Algorithm Match for k-DutySC.

Define

$$\gamma(X) \triangleq \frac{\mathrm{RR}(X)}{\textsc{All}(X)} \ .$$

In this section we look for a lower bound on $\min_{X:|X|=k} \gamma(X)$. Due to Theorem 1 it follows that $\gamma(X) \geq \frac{2}{3}$, for any set X of k sensors. In what follows, we prove the stronger result that the placement that minimizes the ratio is the perfect deployment, namely X^*_k. Notice that this is true for $k = 2$, since $\gamma(X^*_2) = \frac{2}{3}$. (Using Theorem 1 is not essential, but it may simplify the analysis.)

4.1 Stretching the Instance

Our first step is to transform X into an instance X' for which $\gamma(X') \leq \gamma(X)$ by pushing sensors away from $\frac{1}{2}$ so that all internal gaps are of size Δ. (See Section 2 for the definition of Δ.) If a sensor needs to be moved to the left of 0, it is placed at 0, and if it needs to move to the right of 1, it is placed at 1.

Definition 1. *For a given instance X, let j be the sensor whose location is closest to $\frac{1}{2}$. Then we define the* stretched *instance X' of X as follows:*

$$x'_i = \begin{cases} \max\{0, x_j - (j - i)\Delta\} & i < j, \\ x_j & i = j, \\ \min\{1, x_j + (i - j)\Delta\} & i > j. \end{cases}$$

Lemma 3. *Let X' be the stretched instance of X. Then, $\gamma(X') \le \gamma(X)$.*

Proof. Sensors only get pushed away from $\frac{1}{2}$, and thus their RoundRobin lifetime only decreases. Thus, $\mathrm{RR}(X') \le \mathrm{RR}(X)$. By definition, Δ must equal either d_0, d_n or the length of the largest internal gap in X. However neither d_0 nor d_n can be larger in X' than it was in X, since no sensors move closer to $\frac{1}{2}$. Moreover, by construction the length of the largest internal gap in X' is Δ. Hence $\Delta' \le \Delta$, and $\mathrm{All}(X') \ge \mathrm{All}(X)$. □

4.2 Perfect Deployment is the Worst

By Lemma 3, it suffices to consider only stretched instances. The next step is to show that the worst stretched instance is in fact the perfect deployment.

Given a stretched instance X' with k sensors, let k_{out} be the number of sensors located on either 0 or 1, and let $k_{in} \triangleq k - k_{out}$ be the number of sensors in $(0, 1)$. Notice that $k_{in} \ge 1$. Also notice that if $k = 1$, then $\mathrm{RR}(X') = \mathrm{All}(X')$. Recall that $\gamma(X) \ge \frac{2}{3} = \gamma(X_2^*)$, and therefore we may assume that $k = k_{in} + k_{out} \ge 3$. Nevertheless, we prove that the perfect deployment is the worst for $k \ge 2$. (We consider the case of $k = 2$ for completeness.)

Let a and b be the gaps between 0 and the leftmost sensor not at 0, and 1 and the rightmost sensor not at 1, respectively. For reasons of symmetry we assume, w.l.o.g., that $a \le b$. Hence, $\lceil k_{in}/2 \rceil$ sensors are located in $(0, \frac{1}{2}]$ and $\lfloor k_{in}/2 \rfloor$ sensors are located in $(\frac{1}{2}, 1)$.

The stretched deployment X' can be described as follows:

$$X' = \left\{ 0^{\lfloor k_{out}/2 \rfloor}, a, a + \Delta', \ldots, a + (k_{in} - 1)\Delta' = 1 - b, 1^{\lceil k_{out}/2 \rceil} \right\}$$

Note that $\Delta' = \frac{1-a-b}{k_{in}-1}$, if $k_{in} \ge 2$. Otherwise, if $k_{in} = 1$, then we have two options: if $k_{out} > 1$, then $\Delta' = b$, otherwise if $k_{out} = 1$, then $\Delta = \max\{2a, b\}$.

Since the k_{out} sensors contribute a lifetime of exactly 1, the RoundRobin lifetime of X' is:

$$\mathrm{RR}(X') = k_{out} + \sum_{i=0}^{\lceil k_{in}/2 \rceil - 1} \frac{1}{1 - (a + i\Delta')} + \sum_{i=0}^{\lfloor k_{in}/2 \rfloor - 1} \frac{1}{1 - (b + i\Delta')} . \qquad (1)$$

We distinguish three cases, illustrated in Figure 2:

1. $X' = \{a, a + \Delta', \ldots, a + (k_{in} - 1)\Delta' = 1 - b\}$, where $a \in [0, \Delta'/2]$ and $b \in (0, \Delta'/2]$.

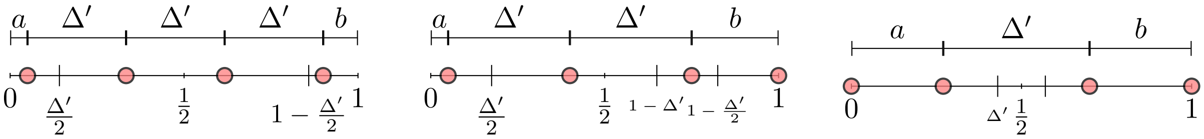
   Let Ω_0 be the set of all such instances. Note that $k_{out} = 0$, if $a > 0$, and that $k_{out} = 1$, if $a = 0$. However, notice that (1) holds if we use $k_{out} = 0$, even for the case where $a = 0$.

2. $X' = \{a, a + \Delta', \ldots, a + (k_{in} - 2)\Delta' = 1 - b, 1\}$, where $a \in [0, \Delta'/2]$, $b \in [\Delta'/2, \Delta']$.

   Let Ω_1 be the set of all such instances. Note that $k_{out} = 1$, if $a > 0$, and that $k_{out} = 2$, if $a = 0$. However, notice that (1) holds if we use $k_{out} = 1$, even for the case where $a = 0$.

3. $X' = \left\{ 0^{\lfloor k_{out}/2 \rfloor}, a, a + \Delta', \ldots, a + (k_{in} - 1)\Delta' = 1 - b, 1^{\lceil k_{out}/2 \rceil} \right\}$, where $a, b \in [0, \Delta']$ and $k_{out} \ge 2$.

   Let $\Omega_{k_{out}}$ be the set of all such instances that correspond to k_{out}. Note that if $a = 0$, (1) holds if we use $k_{out} + 1$ in place of k_{out} and $a = \Delta'$.



(a) In Ω_0, $a \in [0, \frac{\Delta'}{2}]$, but $b \in (0, \frac{\Delta'}{2}]$ (b) In Ω_1, $a \in [0, \frac{\Delta'}{2}]$, but $b \in [\frac{\Delta'}{2}, \Delta']$ (c) In Ω_2, $a, b \in [0, \Delta']$

Figure 2: Illustration of $\Omega_{k_{out}}$. The the first case, at most one sensor can be located on 0 or 1. In the second case, 1 or 2 sensors can be located on 0 or 1. In the third case, k_{out} or $k_{out}+1$ sensors are located on 0 or 1.

Lemma 4. *$\gamma(X')$ has no local minima in $\Omega_{k_{out}}$, for any k_{out}.*

Proof. First assume that $k_{in} \geq 2$. Due to (1) and Observation 1 we have that

$$\begin{aligned}\gamma(X') &= \frac{\Delta'}{2}\left[k_{out} + \sum_{i=0}^{\lceil k_{in}/2\rceil-1} \frac{1}{1-(a+i\Delta')} + \sum_{i=0}^{\lfloor k_{in}/2\rfloor-1} \frac{1}{1-(b+i\Delta')}\right] \\ &= \frac{k_{out}}{2}\cdot\Delta' + \sum_{i=0}^{\lceil k_{in}/2\rceil-1} f_{k_{in}}^{(i)}(a,b) + \sum_{i=0}^{\lfloor k_{in}/2\rfloor-1} f_{k_{in}}^{(i)}(b,a)\ ,\end{aligned}$$

where $f_{k_{in}}^{(i)}(a,b) = \frac{\Delta'}{2-2(a+i\Delta')}$. Since $\frac{\partial\Delta'}{\partial a} = -\frac{1}{k_{in}-1}$, it follows that

$$\begin{aligned}\frac{\partial f_{k_{in}}^{(i)}(a,b)}{\partial a} &= \frac{(2-2(a+i\Delta'))(\frac{\partial\Delta}{\partial a}) - \Delta(-2-2i\frac{\partial\Delta'}{\partial a})}{(2-2(a+i\Delta'))^2} \\ &= \frac{2\left(\frac{\partial\Delta'}{\partial a}(1-a)+\Delta'\right)}{4(1-(a+i\Delta'))^2} = \frac{1}{2(k_{in}-1)}\cdot\frac{-b}{(g_{k_{in}}^{(i)}(a,b))^2}\ ,\end{aligned}$$

where $g_{k_{in}}^{(i)}(a,b) = 1-(a+i\Delta')$. Thus,

$$\frac{\partial\gamma(X')}{\partial a} = \frac{-k_{out}}{2(k_{in}-1)} + \frac{1}{2(k_{in}-1)}\left[\sum_{i=0}^{\lceil k_{in}/2\rceil-1}\frac{-b}{(g_{k_{in}}^{(i)}(a,b))^2} + \sum_{i=0}^{\lfloor k_{in}/2\rfloor-1}\frac{-a}{(g_{k_{in}}^{(i)}(b,a))^2}\right]\ .$$

Since $a=b=k_{out}=0$ is not possible for any domain $\Omega_{k_{out}}$, we have that $\frac{\partial\gamma(X')}{\partial a} < 0$. Hence $\gamma(X')$ decreases as a increases. An analogous calculation shows that the same is true for $\frac{\partial\gamma(X')}{\partial b}$. Thus, since neither $\frac{\partial\gamma(X')}{\partial a}$ nor $\frac{\partial\gamma(X')}{\partial b}$ can be zero at any point in the interior of the domain $\Omega_{k_{out}}$, $\gamma(X')$ has no local minima. Finally, any minima must occur when both a and b are as large as possible within the domain $\Omega_{k_{out}}$.

It remains to consider the case where $k_{in}=1$. If $k_{out}>1$, we have that $\Delta'=b$. Hence,

$$\gamma(X') = \frac{b}{2}\left(k_{out}+\frac{1}{b}\right) = \frac{1}{2}(bk_{out}+1)\ ,$$

which means that $\frac{\partial\gamma(X')}{\partial b} > 0$. Hence, $\gamma(X')$ decreases as b decreases. It follows that the minima occurs when $a = b = \frac{1}{2}$.

If $k_{out} = 1$, then $\Delta' = \max\{2a, b\}$. If $b > 2a$, $\gamma(X')$ decreases as b decreases as shown above. However, if $b < 2a$, we have that

$$\gamma(X') = \frac{2a}{2}\left(k_{out} + \frac{1}{1-a}\right) = a + \frac{a}{1-a} \ ,$$

which means that

$$\frac{\partial\gamma(X')}{\partial a} = 1 + \frac{1-a+a}{(1-a)^2} = 1 + \frac{1}{(1-a)^2} > 0 \ .$$

Hence, $\gamma(X')$ decreases as a decreases. It follows that the minima occurs then $2a = b = \frac{2}{3}$. □

Let $\gamma_k^* = \gamma(X_k^*)$. We show that, for any fixed k, $\gamma(X)$ reaches its minimum at $X = X_k^*$.

Theorem 3. $\min_{X:|X|=k} \gamma(X) = \gamma_k^*$.

Proof. We prove that $\min_{X\in\Omega_{k_{out}}} \gamma(X) \geq \gamma_k^*$ by induction on k_{out}.

For the base case, if $X' \in \Omega_0$, then by Lemma 4, $\gamma(X')$ achieves its minimum on the boundary of Ω_0, when a and b are as large as possible, namely for $a = b = \Delta/2$. In this case, $X' = X_k^*$. Thus, for all $X' \in \Omega_0$, $\gamma(X') > \gamma_k^*$ if $X \neq X_k^*$.

For the inductive step, let $X \in \Omega_{k_{out}}$, for $k_{out} \geq 1$, and assume that $\min_{X\in\Omega_{k_{out}-1}} \gamma(X) \geq \gamma_k^*$. By Lemma 4 it follows that $\gamma(X')$ achieves its minimum on the boundary of $\Omega_{k_{out}}$. If $k_{out} = 1$ (and $k_{in} \geq 1$), then the minimum is when $a = \Delta/2$ and $b = \Delta$ (If $k_{in} = 1$, then the minimum is when $2a = b = \frac{2}{3}$.), namely for $X' = \{\frac{\Delta}{2}, \frac{3\Delta}{2}, \ldots, 1-\Delta, 1\}$. By symmetry, this instance has the same ratio as the instance $X'' = \{1 - x : x \in X'\}$, which is in Ω_0 with parameters $a = 0$ and $b = \frac{\Delta}{2}$. If $k_{out} > 1$, then the minimum is when $a = \Delta$ and $b = \Delta$. In this case $X' \in \Omega_{k_{out}-1}$ with parameters $a = 0$ and $b = \Delta$. Hence by the induction hypothesis we have that $\gamma(X') \geq \gamma_k^*$. □

4.3 Properties of γ_k^*

In this section we explore γ_k^* as a function of k. Observe that for even k we have that

$$\gamma_k^* = \frac{1}{2k} \cdot 2\sum_{i=1}^{k/2} \frac{2k}{2k+1-2i} = 2\sum_{i=1}^{k/2} \frac{1}{2k+1-2i} = 2\sum_{i=k/2+1}^{k} \frac{1}{2i-1} \ ,$$

and for odd k we have that

$$\gamma_k^* = \frac{1}{2k}\left[2 + 2\sum_{i=1}^{(k-1)/2} \frac{2k}{2k+1-2i}\right] = \frac{1}{k} + 2\sum_{i=(k+1)/2}^{k-1} \frac{1}{2i+1} \ .$$

Lemma 5. *γ_k^* satisfies the following: (i) $\gamma_k^* \leq \gamma_{k+2}^*$, for every even k. (ii) $\gamma_k^* \geq \gamma_{k+2}^*$, for every odd k. (iii) $\gamma_k^* \geq \gamma_{k+1}^*$, for every odd k.*

Proof. Due to the convexity of the function $f(z) = \frac{1}{z}$, we have that for even k,

$$\gamma^*_{k+2} - \gamma^*_k = 2\sum_{i=k/2+2}^{k+2} \frac{1}{2i-1} - 2\sum_{i=k/2+1}^{k} \frac{1}{2i-1} = \frac{2}{2k+3} + \frac{2}{2k+1} - \frac{2}{k+1} > 0 \ .$$

By the same rationale, for odd k,

$$\begin{aligned}
\gamma^*_k - \gamma^*_{k+2} &= \left(\frac{1}{k} + 2\sum_{i=(k+1)/2}^{k-1} \frac{1}{2i+1}\right) - \left(\frac{1}{k+2} + 2\sum_{i=(k+1)/2+1}^{k+1} \frac{1}{2i+1}\right) \\
&= \frac{1}{k} + \frac{2}{k+2} - \frac{1}{k+2} - \frac{2}{2k+1} - \frac{2}{2k+3} \\
&= \frac{1}{k} + \frac{1}{k+2} - \frac{2}{2k+1} - \frac{2}{2k+3} \\
&> 0 \ .
\end{aligned}$$

Finally, for odd k,

$$\gamma^*_k - \gamma^*_{k+1} = \frac{1}{k} + 2\sum_{i=(k+1)/2}^{k-1} \frac{1}{2i+1} - 2\sum_{i=(k+1)/2+1}^{k+1} \frac{1}{2i-1} = \frac{1}{k} - \frac{2}{2k+1} > 0 \ ,$$

as required. □

Lemma 6. $\lim_{k\to\infty} \gamma^*_k = \ln 2$.

Proof. Observe that for both even and odd k's we have that

$$\gamma^*_k \geq \sum_{i=k+1}^{2k} \frac{1}{i} = H_{2k} - H_k \qquad \text{and} \qquad \gamma^*_k \leq \sum_{i=k}^{2k-1} \frac{1}{i} = H_{2k-1} - H_{k-1},$$

where H_k the kth Harmonic number. It follows that $\lim_{k\to\infty} \gamma^*_k = \lim_{k\to\infty}(H_{2k} - H_k) = \ln 2$. □

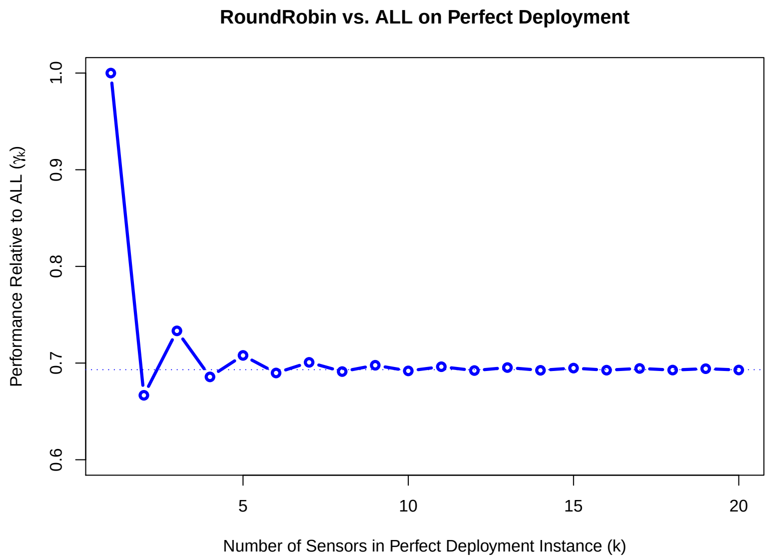
The table in Figure 3(a) contains several values of γ^*_k, whose convergence is also depicted graphically in Figure 3(b).

5 Strip Cover with Shifts of Size k

In this section we analyze the performance of Match in k-DutySC for $k \geq 3$. Recall that Algorithm Match finds the best solution among those using shifts of size at most 2. Since Match is more powerful than RoundRobin, its approximation ratio is at most $\frac{3}{2} = 1.5$ (by Theorem 1). We show that the approximation ratio of Match is at most $\frac{35}{24} \approx 1.458$. We also provide lower bounds: $\frac{15}{11} \approx 1.364$ for $k \geq 4$, and $\frac{6}{5} = 1.2$, for $k = 3$. At the end of the section we discuss ways to improve the analysis of Match.

k	γ_k^*	Approx
1	1	1
2	$\frac{2}{3}$	0.6667
3	$\frac{11}{15}$	0.7333
4	$\frac{24}{35}$	0.6857
5	$\frac{223}{315}$	0.7079
6	$\frac{478}{693}$	0.6898
7	$\frac{6313}{9009}$	0.7007
8	$\frac{4448}{6435}$	0.6912
$\vdots$	$\vdots$	$\vdots$
∞	$\ln 2$	0.6931

(a) Exact and approximate values of γ_k^*.



(b) γ_k^* is an alternating sequence that converges to $\ln 2$

Figure 3: Tabular and graphical representation of small values of γ_k^*.

5.1 Upper Bound

We use our analysis of RoundRobin vs. All to obtain an upper bound on the performance of Match for k-DutySC.

Theorem 4. *Algorithm* Match *is a* $\frac{35}{24}$*-approximation algorithm for* k-DutySC*, for every* $k \geq 3$.

Proof. Let X be a k-DutySC instance and let $C_1, \ldots, C_m$ be an optimal solution for X. Since for every C_j with $|C_j| \leq 2$, $\text{Match}(C_j) = \text{Opt}_k(C_j)$, we construct an alternative solution by splitting to singletons every subset C_j such that $|C_j| > 2$. For any such C_j, by Theorem 3 we know that $\text{RR}(C_j) \geq \gamma_{C_j}^* \text{All}(C_j)$, and by Lemma 5 we know that $\gamma_{C_j}^* \geq \min_{k \geq 3} \gamma_k^* = \gamma_4^* = \frac{24}{35}$. The lemma follows. □

We can generalize this approach to find upper bounds on the performance of Opt_k in n-DutySC, for $k \leq n$.

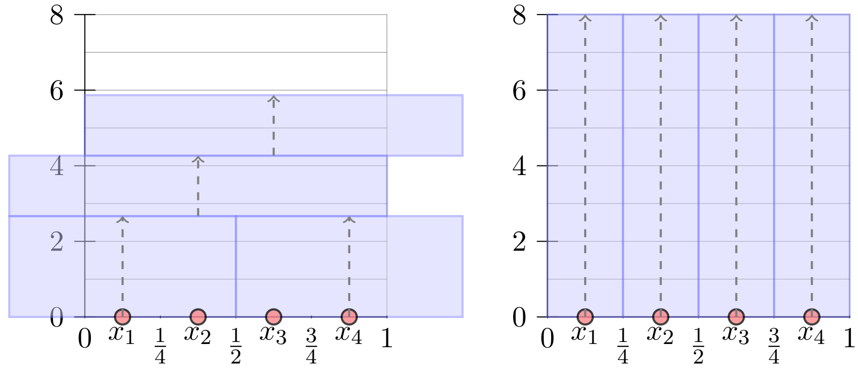
Lemma 7. $\text{Opt}_k(X) \geq \gamma_\ell^* \text{Opt}_n(X)$*, where* ℓ *is the smallest even integer larger than* k.

Hence, the approximation ratio of an algorithm that solves k-DutySC is at most $1/\gamma_\ell^*$. Lemma 6 implies that the least upper bound achievable via this technique is $1/\ln 2 \approx 1.4427$.

5.2 Lower Bounds

We show that the approximation ratio of Match is at least $\frac{15}{11}$, for $k \geq 4$, and at least $\frac{6}{5}$, for $k = 3$.

Lemma 8. $\text{Match}(X_4^*) = \frac{11}{15}\text{Opt}_k(X_4^*)$*, for every* $k \geq 4$.



(a) $\text{MATCH}(X_4^*) = \frac{88}{15}$

(b) $\text{OPT}(X_4^*) = 8$

Figure 4: Illustration of the $\frac{15}{11}$ upper bound for the approximation ratio of MATCH for $k \geq 4$. For the instance $X_4^* = \{\frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}\}$, MATCH achieves a lifetime of $\frac{88}{15}$ while OPT achieves 8.

Proof. Consider the instance $X_4^* = \{\frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}\}$. Observe that the ROUNDROBIN lifetime of the sensors on the outside is $\text{RR}(\frac{1}{8}) = \text{RR}(\frac{7}{8}) = \frac{8}{7}$, while the lifetime is $\text{RR}(\frac{3}{8}) = \text{RR}(\frac{5}{8}) = \frac{8}{5}$ for the sensors in the middle. Perhaps surprisingly, any sensible pairing of the sensors achieves a lifetime of $8/3$. Thus, one MATCH solution is to pair the outside sensors for a lifetime of $8/3$, and then run ROUNDROBIN on the middle sensors successively, for an additional lifetime of $2 \cdot \frac{8}{5}$, as depicted in Figure 4. Thus, the total lifetime is $\text{MATCH}(X_4^*) = \frac{8}{3} + 2 \cdot \frac{8}{5} = \frac{88}{15}$. The lemma follows, since $\text{OPT}_k(X_4^*) = \text{ALL}(X_4^*) = 8$, for every $k \geq 4$. □

We have a weaker lower bound for $k = 3$.

Lemma 9. $\text{MATCH}(X_3^*) = \frac{5}{6}\text{OPT}_3(X_3^*)$.

Proof. MATCH has 2 duty cycles: $\{\frac{1}{6}, \frac{5}{6}\}, \{\frac{1}{2}\}$. Hence, $\text{MATCH}(X_3^*) = 5$, while $\text{ALL}(X_3^*) = 6$. □

We conjecture that both Lemma 8 and Lemma 9 are tight.

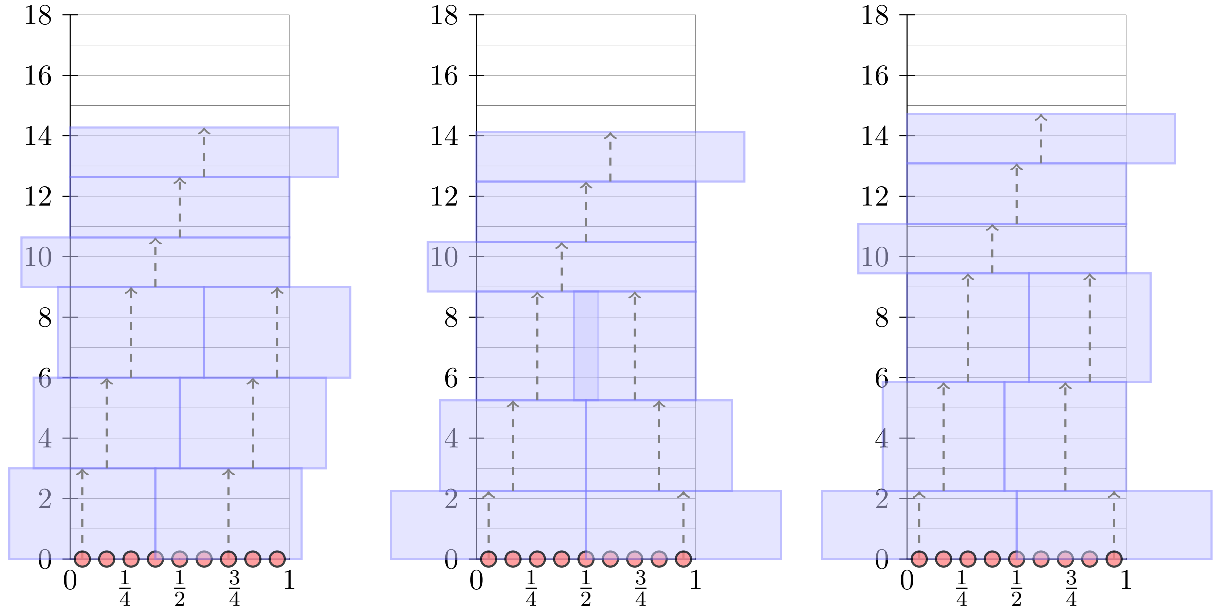
Conjecture 1. $\text{MATCH}(X) \geq \frac{11}{15}\text{OPT}_k(X)$, *for every* $k \geq 4$, *and* $\text{MATCH}(X) \geq \frac{5}{6}\text{OPT}_3(X)$.

For some positive integer d, let $D_d = \{\frac{i}{d} : i \in \{0, 1, \ldots, d\}\}$ be a discretization of $[0, 1]$. Clearly, as $d \to \infty$, D becomes a close approximation of $[0, 1]$. Using brute force, we checked all 680 possible instances in D_{16}^3 and all 2380 possible instances in D_{16}^4, and found no instance X for which $\text{MATCH}(X) < \frac{5}{6}\text{OPT}_3(X)$ in the first case, nor any for which $\text{MATCH}(X) < \frac{11}{15}\text{OPT}_4(X)$ in the second case.

Again, due to Lemma 5, we can generalize Lemma 8 to find lower bounds on the performance of OPT_k in n-DUTYSC.

Lemma 10. *For every* $\ell \leq n$, $\text{OPT}_k(X) \geq \frac{\text{OPT}_k(X_\ell^*)}{2\ell} \cdot \text{OPT}_n(X)$.

Note that For $k = 1, 2$ we recover the $\frac{2}{3}$ and $\frac{11}{15}$ bounds demonstrated previously.



(a) $\textsc{Sweep}(X_9^*) = 14.27273$ (b) $\textsc{Nest}(X_9^*) = 14.12273$ (c) $\textsc{Match}(X_9^*) = 14.72273$

Figure 5: Illustration of the performance of the two heuristics, SWEEP and NEST, and Algorithm MATCH. For the instance X_9^*, each procedure returns a different solution. The lifetimes achieved by SWEEP and NEST represent lower bounds on the performance of MATCH.

5.3 Asymptotics

One way to improve the analysis of Algorithm MATCH would be to first prove that perfect deployments are worst with respect to MATCH (as they are with respect to ROUNDROBIN), and then to analyze $\gamma_k^2 = \textsc{Match}(X_k^*)/\textsc{All}(X_k^*)$.

Our experiments show that γ_k^2 seems to converge to approximately 0.816, which is significantly higher than $\lim_{k\to\infty} \gamma_k^* = \ln 2 \approx 0.693$. See Figure 6.

We would like to evaluate $\lim_{k\to\infty} \gamma_k^2$. However, since MATCH explicitly evaluates each pair of sensors, it is not trivial to extrapolate its behavior for large k. Nevertheless, we obtain lower bounds on the limit by analyzing simple heuristics. For a given instance X, let $L = X \cap [0, \frac{1}{3}]$, $M = X \cap (\frac{1}{3}, \frac{2}{3})$, and $R = X \cap [\frac{2}{3}, 1]$. Note that $|L| = |R|$ for the perfect deployment instance X_k^* of any size. Consider the following heuristics:

- SWEEP: Sensors in L and R are paired to create shifts of size two, starting with the leftmost sensor in L and the *leftmost* sensor in R. Any remaining sensors are put in size one shifts.

- NEST: Sensors in L and R are paired to create shifts of size two, starting with the leftmost sensor in L and the *rightmost* sensor in R. Any remaining sensors are put in size one shifts.

Note that these heuristics return valid (albeit suboptimal) 2-DUTYSC solutions. Examples of these solutions shown in Figure 5.

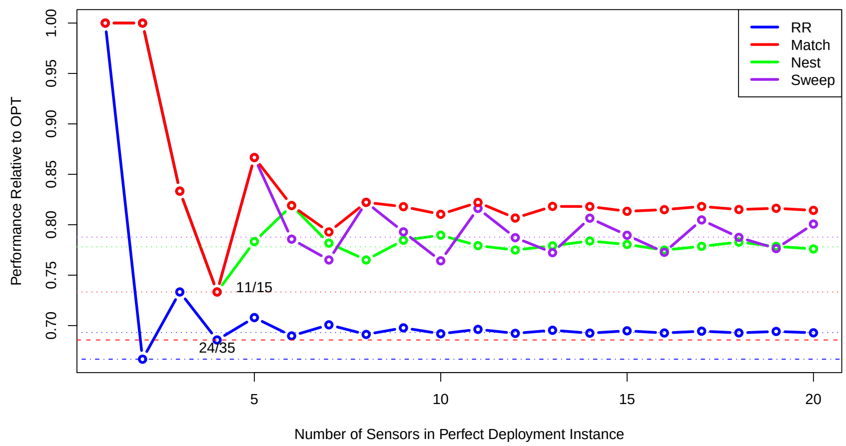


Figure 6: Performance of RoundRobin, Match, Sweep, and Nest compared to Opt on perfect deployments.

Using similar arguments as in Section 4.3 we can show that:

$$\frac{\text{Sweep}(X_n^*)}{\text{All}(X_n^*)} \approx 1/2 + H_{4n/3} - H_n$$
$$\frac{\text{Nest}(X_n^*)}{\text{All}(X_n^*)} \approx (H_n - H_{n/2})/2 + (H_{2k/3} - H_{k/2})/2 + (H_{4n/3} - H_n)$$

which means that

$$\lim_{n\to\infty} \frac{\text{Sweep}(X_n^*)}{\text{All}(X_n^*)} = \frac{1}{2} + \ln(4/3) \approx 0.788$$
$$\lim_{n\to\infty} \frac{\text{Nest}(X_n^*)}{\text{All}(X_n^*)} = \frac{1}{2}\ln 2 + \frac{1}{2}\ln(4/3) + \ln(4/3) \approx 0.778$$

A comparison of the performance of RoundRobin, Match, Sweep and Nest on perfect deployments to the performance of Opt is given in Figure 6.

In Figure 7, we show the density of the per sensor network lifetime for RoundRobin, All, and Match relative to Opt (computed via brute force), for all 2380 possible instances with 4 sensors over a grid of size 16. X_4^* was the only instance found for which $\text{Match}(X) \leq \frac{11}{15}\text{Opt}_4(X)$. Moreover, for about 82% of the instances, $\text{Match}(X) = \text{Opt}(X)$, and the mean approximation ratio was 0.9923. Meanwhile, the average per sensor lifetime for Match was 1.483, which easily surpasses the corresponding figure for RoundRobin of $2\ln 2 = 1.386$.

6 Fault Tolerance

In this section we extend our analysis to incorporate a fault tolerance model, in which each sensor may fail to activate with probability $p \in [0, 1]$. We assume that failures occur randomly and independently. If any sensor in a shift fails to activate, then the entire coverage lifetime of that

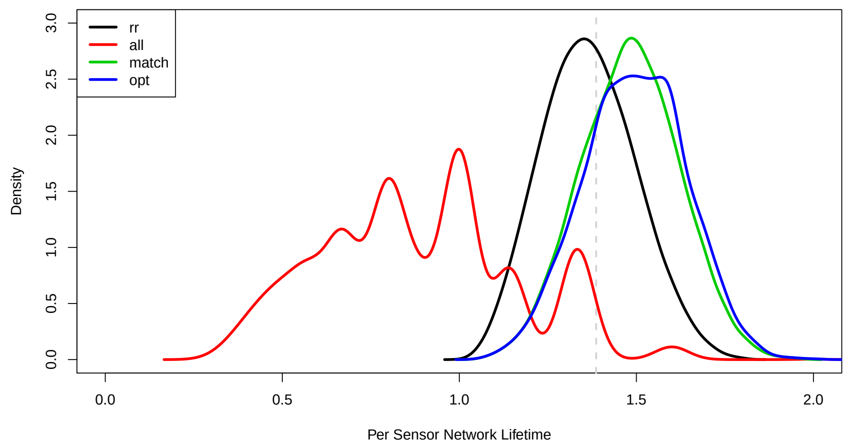


Figure 7: Comparison of the Distribution of Per Sensor Network Lifetime for various algorithms in 4-DutySC. The lifetime of each algorithm was computed for all 2380 possible instances of 4 sensors on a grid of size 16.

shift is lost. Under these assumptions, we can make the following observation about the expected lifetime of an algorithm.

Observation 11. *For any shift $C \subseteq X$, the expected lifetime of the shift is $(1-p)^{|C|} \cdot \textsc{All}(C)$.*

The expected lifetime of a solution $C_1, \ldots, C_m \subseteq X$ is $\sum_i (1-p)^{|C_i|} \cdot \textsc{All}(C_i)$. In the fault tolerant version of k-DutySC our goal is to find a solution $C_1, \ldots, C_m \subseteq X$ such that $|C_i| \leq k$ with maximum expected lifetime. Let $\textsc{Opt}_k^p(X)$ denote the expected lifetime of an optimal k-DutySC solution of X.

Theorem 5. *Fault tolerant* 2-DutySC *can be solved in polynomial time.*

Proof. The proof is similar to the proof of Theorem 2. The only difference is that the weight of an edge $e = (v_i, v'_j)$ is defined as follows:

$$w(e) = \begin{cases} (1-p) \cdot \max\{x_i, 1-x_i\} & i = j, \\ (1-p)^2 \cdot \textsc{All}(\{x_i, x_j\}) & i < j. \end{cases}$$

□

We show that if the probability of failure is high enough, Match, or even RoundRobin, compute optimal solutions.

Lemma 12. *If $p \geq \frac{1}{3}$, then $\mathbb{E}[\mathrm{RR}(X)] = \textsc{Opt}_k^p(X)$, for every X.*

Proof. Let $C_1, \ldots, C_m$ be an optimal schedule. Consider any shift C_j for which $|C_j| \geq 2$. By Observation 11, the expected lifetime of that shift is at most $(1-p)^2 \cdot \textsc{All}(C_j)$. Since $\gamma(C_j) \geq \frac{2}{3}$ for every C_j (due to [4]), we have that $\mathrm{RR}(C_j) \geq \frac{2}{3}\textsc{All}(C_j)$. It follows that $(1-p)^2 \cdot \textsc{All}(C_j) \leq \frac{4}{9}\textsc{All}(C_j) \leq \frac{2}{3}\mathrm{RR}(C_j)$. □

Lemma 13. *If* $p \geq 1 - \sqrt{\gamma_3^*} \approx 0.144$*, then* $\mathbb{E}[\textsc{Match}(X)] = \textsc{Opt}_k^p(X)$*, for every* X*.*

Proof. Let $C_1, \ldots, C_m$ be an optimal schedule. Consider any shift C_j for which $|C_j| \geq 3$. By Observation 11, the expected lifetime of that shift is $(1-p)^{|C_j|} \cdot \textsc{All}(C_j)$. Due to Theorem 3 we have that

$$(1-p)^{|C_j|}\textsc{All}(C_j) \leq \frac{(1-p)^{|C_j|-1}}{\gamma_k^*} \cdot (1-p)\mathrm{RR}(C_j) \ .$$

If $|C_j| = 3$, we have that $(1-p)^{|C_j|-1}/\gamma_k^* \leq 1$, since $p \geq 1 - \sqrt{\gamma_3^*} \approx 0.144$. Also, for $|C_j| > 3$ we have that $(1-p)^{|C_j|-1}/\gamma_k^* \leq (1-p)^3/\gamma_4^* < 1$, since $p > 1 - \sqrt[3]{\gamma_4^*} \approx 0.118$. It follows that if $p \geq 1 - \sqrt{\gamma_3^*}$, then there exists an optimal schedule that does not use shifts of size larger than 2. Hence, Algorithm Match computes an optimal solution. □

7 Discussion and Open Problems

While 1-DutySC can be solved trivially by RoundRobin, and we have shown that 2-DutySC can be solved in polynomial time using Algorithm Match, it remains open whether k-DutySC is NP-hard, for $k \geq 3$. It would also be interesting to close the gap between the upper and lower bounds on the approximation ratio of Match, for $k \geq 3$. We offered one possible direction to improving the upper bound in Section 5.3.

In this paper, we have assumed that: (i) the initial battery charge of each sensor is identical; and (ii) the battery charge in each sensor drains in inverse linear proportion to its assigned radius. Two natural extensions of our work would be to allow the initial battery charges to differ, and to allow the latter proportion to vary according to some exponent $\alpha \neq 1$.

Finally, while we have restricted our attention to a one-dimensional coverage region, one could consider a variety of similar problems in higher dimensions. For example, one might keep the sensor locations restricted to the line, but consider a two-dimensional coverage region. Conversely, the sensors could be located in the plane, while the coverage region remains one-dimensional. Of course, an even more general problem would allow both the sensor locations and the coverage region to be two-dimensional.

References

[1] Z. Abrams, A. Goel, and S. A. Plotkin. Set k-cover algorithms for energy efficient monitoring in wireless sensor networks. In *IPSN*, pages 424–432, 2004.

[2] G. Aloupis, J. Cardinal, S. Collette, S. Langerman, D. Orden, and P. Ramos. Decomposition of multiple coverings into more parts. *Discrete & Computational Geometry*, 44(3):706–723, 2010.

[3] A. Bar-Noy and B. Baumer. Maximizing network lifetime on the line with adjustable sensing ranges. In *ALGOSENSORS*, pages 28–41, 2011.

[4] A. Bar-Noy, B. Baumer, and D. Rawitz. Set It and Forget It: Approximating the Set Once Strip Cover Problem. *Arxiv preprint arXiv:1204.1082v1*, 2012.

[5] A. Buchsbaum, A. Efrat, S. Jain, S. Venkatasubramanian, and K. Yi. Restricted strip covering and the sensor cover problem. In *SODA*, pages 1056–1063, 2007.

[6] M. Cardei and D. Du. Improving wireless sensor network lifetime through power aware organization. *Wireless Networks*, 11(3):333–340, 2005.

[7] M. Cardei, J. Wu, and M. Lu. Improving network lifetime using sensors with adjustable sensing ranges. *Int. J. Sensor Networks*, 1(1/2):41–49, 2006.

[8] M. Cardei, J. Wu, M. Lu, and M. Pervaiz. Maximum network lifetime in wireless sensor networks with adjustable sensing ranges. In *WiMob (3)*, pages 438–445, 2005.

[9] E. Chambers, S. Fekete, H. Hoffmann, D. Marinakis, J. Mitchell, V. Srinivasan, U. Stege, and S. Whitesides. Connecting a set of circles with minimum sum of radii. In *WADS*, pages 183–194. Springer, 2011.

[10] W. J. Cook, W. H. Cunningham, and A. S. William R. Pulleyblank. *Combinatorial Optimization*. John Wiley & Sons, 1998.

[11] M. Gibson and K. Varadarajan. Decomposing coverings and the planar sensor cover problem. In *FOCS*, pages 159–168. IEEE, 2009.

[12] N. Lev-Tov and D. Peleg. Polynomial time approximation schemes for base station coverage with minimum total radii. *Computer Networks*, 47(4):489–501, 2005.

[13] J. Pach. Covering the plane with convex polygons. *Discrete & Computational Geometry*, 1(1):73–81, 1986.

[14] J. Pach and G. Tóth. Decomposition of multiple coverings into many parts. *Computational Geometry*, 42(2):127–133, 2009.

[15] M. Perillo and W. Heinzelman. Optimal sensor management under energy and reliability constraints. In *Wireless Communications and Networking*, volume 3, pages 1621–1626. IEEE, 2003.

[16] S. Slijepcevic and M. Potkonjak. Power efficient organization of wireless sensor networks. In *IEEE Intl. Conf. on Communications*, volume 2, pages 472–476, 2001.