APPENDIX A

PROBLEM FORMULATION

A.1 Justification for Location Assumption

In our problem formulation, we ignore the cost of sending periodic location updates to the director. In practice, this may be a reasonable assumption for three reasons. First, the cost of location updates may be amortized over other context aware applications that may be executing on the smart phone. Second, although this cost may be significant, it adds a fixed cost to our formulations and does not affect the results we present in the paper. Finally, the absolute cost of the location updates themselves is significantly less than the cost of video transmissions, for example.

APPENDIX B

COMPLEXITY ANALYSIS

B.1 Proof of Theorem 3.1

Proof: We first show a reduction from the Knapsack Problem to MaxCRED. Suppose we are given an instance of the decision version of the Knapsack Problem with $N$ distinct elements, element $i$ has value $v_i$ and weight $w_i$. Assume the maximum weight of the knapsack is $B$. Is there a collection of items with weight at most $B$ and with value at least $K$? Given an instance of the Knapsack Problem, we shall construct an instance of MaxCRED given a budget $B$ in polynomial time. Let $c_{i,j} = v_i$ for $i = j$ and 0 otherwise. Let $c_{i,j} = w_i$ for all $i \in \{1, ..., N\}$. It is now clear that the maximum value of the Knapsack is at least $K$ iff the maximum credibility is at least $K$.

Furthermore, the Knapsack Problem can be cast as a minimization problem: selecting a set of items with weight at most $B$ and value at least $K$, is equivalent to selecting a set, the complement, with weight at least $C = \sum_{i=1}^{n} w_i - B$, and value at most $V = \sum_{i=1}^{n} v_i - K$. We shall reduce this version of the problem to MinCost in the following way: Let $e_i = v_i$ for all $i \in \{1, ..., N\}$, let $c_{i,j} = w_i$ for $i = j$ and 0 otherwise. It is now clear that the minimum value of the Knapsack is at most $V$ with weight at least $C$ iff the minimum cost is at most $V$ with credibility at least $C$.

B.2 Much Stronger Result on the Complexity

A much stronger result can also be proven. Assume the credibility values are given by Equation 1. MaxCRED and MinCost are even NP-Hard for this instance, which we shall refer to as MaxCRED-G and MinCost-G respectively.

The proofs of NP-Hardness for the Partition Problem [20].

Given an instance of Partition: a set of integers $\{x_1 \leq x_2 \leq \cdots \leq x_N\}$ and $P = \frac{1}{2} \sum_{i=1}^{N} x_i$, construct the following instance of MaxCRED-G and MinCost-G. Let there be $N$ reporters with $d(p_i, E) = 1$ for all $i \in \{1, ..., N\}$ and $N = 2$ formats for each reporter to choose from. Note that since $d(p_i, E) = 1$ for all $i \in \{1, ..., N\}$, the credibility values per format are equivalent for each reporter and furthermore we may allow $\delta_1, ..., \delta_2N$ to be arbitrary positive integers satisfying $\delta_1 > \delta_2, \ldots > \delta_{2N}$. Let the credibility of format $2i-1$ be $\delta_{2i-1} = \gamma_{2i-1} = 2^{2i}/2i$ with cost $e_{2i-1} = 2^{2i}$, and the credibility of format $2i$ be $\delta_{2i} = \gamma_{2i} = 2^{2i+1} + x_i$ with cost $e_{2i} = 2^{2i} + x_i$, $i \in \{1, ..., N\}$, where $w = \log_2 2P + 1$.

Note that $\gamma_1 \leq \gamma_2, \ldots, \leq \gamma_N$ since $2^{2i} + x_i < 2 \cdot 2^{2i+1} = 2^{2i+2} < 2^{2i+2} + x_i, i \in \{1, ..., 2N-1\}$ where the first inequality uses that $x_i < 2P < 2 < 2^{2i}, i \in \{1, ..., N\}$. To complete the reduction, define $B = 2^i \sum_{i=1}^{N} 2^i + P$ and $C = 2^i \sum_{i=1}^{N} 2^i + P$.

This reduction can be done in polynomial time. Let $n$ be the size of the Partition problem in bits. Clearly $N \leq n$ and $|x_i| \leq n$, $i \in \{1, ..., N\}$, where $|x|$ denotes the size of $x$ in bits. Now, $|P| < |\sum_{i=1}^{N} x_i| = O(n + N) = O(n)$ where the first equality uses $|a + b| \leq \max(|a|, |b|) + 1$. Since $t \leq 4P$, $v_{\max} = O(n)$. Since $c_{i,j} = \delta_{2N} + \max_{i,j \in \mathcal{X}} x_i, |c_{i,j}| = O(t + 2N + n) = O(n)$, $i \in \{1, ..., 2N\}$. Furthermore, $B \leq C = 2^i \sum_{i=1}^{N} 2^i + P$, thus $|B|, |C| = O(n)$. The reduction uses $4N + 2 = O(n)$ integers each of size $O(n)$ and hence is done in polynomial time.

The proofs of NP-Hardness also make use of the following lemma.

Lemma B.1: In the reduction, if credibility at least $C = 2^i \sum_{i=1}^{N} 2^i + P$ can be obtained with cost at most $B = 2^i \sum_{i=1}^{N} 2^i + P$, then exactly one format from $\{2i-1, 2i\}$ is selected for each $i \in \{1, ..., N\}$.

Proof: Assume this is not the case. Consider the last index $j$ for which exactly one format from $\{2j-1, 2j\}$ is not selected. Assuming neither format $2j-1$ nor $2j$ is selected, credibility $C$ can not be obtained: the cost effectiveness of format $i < 2j - 1$ is at most $2^{i+2} - 2^{i-1} = 2^{i+1}$. Any assignment of formats $\{1, ..., 2j\}$ to $j$ reporters which does not use formats $\{2j-1, 2j\}$ can gain at most

$$2^{i} \sum_{i=1}^{N} 2^i + P < 2^{i} (2^{i+1} - 2) = 2^{i+1}$$

Hence any assignment of formats $\{1, ..., 2N\}$ to $N$ reporters for which neither format $2j - 1$ nor $2j$ is selected, and where exactly one format from $\{2i-1, 2i\}$ is selected for each $i \in \{j+1, ..., N\}$ can gain at most $2^{i+j} - 2^i \sum_{i=1}^{j} 2^{i+1} + 2^i \sum_{i=j+1}^{N} x_i$ which is less than $C$, a contradiction.

If 2 or more formats are selected from $\{2j-1, 2j\}$, then the cost of such an assignment is greater than $B$: the cost is at least

$$2^{i} \sum_{i=1}^{N} 2^i + P < 2^{i} (2^{i+1} - 1) < 2^{i+1}$$

since $B = 2^i \sum_{i=1}^{N} 2^i + P < 2^{i} (2^{i+1} - 1) = 2^{i+1} - 1$, we have a contradiction.
Theorem B.2: MaxCred-G is NP-Hard

Proof: \( x_1, \ldots, x_N \in \text{Partition} \) if and only if credibility \( C = 2^t \sum_{i=1}^{N} 2^i + P \) can be obtained with cost at most \( B \): Assume \( x_1, \ldots, x_N \in \text{Partition} \). Then there is a set \( I \) such that \( \sum_{i \in I} x_i = P \). Select format \( 2i \) for every \( i \in I \) and format \( 2i+1 \) otherwise. Then the credibility obtained for this assignment is \( 2^t \sum_{i=1}^{N} 2^i + P \) with cost \( 2^t \sum_{i=1}^{N} 2^i + P = B \).

Assume there is an assignment for which credibility \( 2^t \sum_{i=1}^{N} 2^i + P \) can be obtained with cost at most \( B = 2^t \sum_{i=1}^{N} 2^i + P \). By Lemma B.1 exactly one format from \( \{2i-1, 2i\} \) is selected for each \( i \in \{1, \ldots, N\} \). Define \( I = \{ i : \text{format } 2i \text{ was selected} \} \). Then \( \sum_{i \in I} x_i = P \) and hence \( x_1, \ldots, x_N \in \text{Partition} \).

Theorem B.3: MinCost-G is NP-Hard

Proof: \( x_1, \ldots, x_N \in \text{Partition} \) if and only if cost \( B = 2^t \sum_{i=1}^{N} 2^i + P \) can be obtained with credibility at least \( C = 2^t \sum_{i=1}^{N} 2^i + P \).

Assume \( x_1, \ldots, x_N \in \text{Partition} \). Then there is a set \( I \) such that \( \sum_{i \in I} x_i = P \). Select format \( 2i \) for every \( i \in I \) and format \( 2i+1 \) otherwise. Then the cost obtained for this assignment is \( 2^t \sum_{i=1}^{N} 2^i + P = B \) with credibility \( 2^t \sum_{i=1}^{N} 2^i + P = C \).

Assume cost \( B = 2^t \sum_{i=1}^{N} 2^i + P \) can be obtained with credibility at least \( C = 2^t \sum_{i=1}^{N} 2^i + P \). By Lemma B.1 exactly one format from \( \{2i-1, 2i\} \) is selected for each \( i \in \{1, \ldots, N\} \). Define \( I = \{ i : \text{format } 2i \text{ was selected} \} \). Then \( \sum_{i \in I} x_i = P \) and hence \( x_1, \ldots, x_N \in \text{Partition} \).

Appendix C

Optimal Solutions

C.1 Proof of Theorem 3.4

Proof: MinCost-2F loops through all possible \( (i, Y) \) pairs. For each possible \( (i, Y) \) pair, it finds an assignment of maximum credibility using the same routine as in MaxCred-2F which was shown to be optimal. If the maximum credibility of this assignment exceeds the threshold \( C \), the cost of such an assignment is computed, otherwise it is set to \( \infty \). The algorithm chooses the assignment of minimum cost whose maximum credibility exceeds the threshold \( C \) as the minimizer and hence is optimal.

Appendix D

Renewals Problem

D.1 Proof of Theorem 4.2

Proof: To prove (29), we use induction. Assume that \( Z_i[k] \leq V \beta + \epsilon_{\max} \) for some frame \( k \) (it is true by assumption for \( k = 1 \)). We prove it also holds for frame \( k + 1 \). Consider the case when \( Z_i[k] \leq V \beta \). Then \( Z_i[k + 1] \leq V \beta + \epsilon_{\max} \) (because \( Z_i[k] \) can increase by at most \( \epsilon_{\max} \) on any frame, see dynamics (24)). Now consider the opposite case when \( Z_i[k] > V \beta \). Then for any \( i, j \) with \( e_{ij}[k] > 0 \), the variable \( x_{ij}[k] \) has weight:

\[
Vc_{ij}[k] - Z_i[k] e_{ij}[k] = e_{ij}[k] (Vc_{ij}[k] - Z_i[k]) \leq e_{ij}[k] (V \beta + Z_i[k]) \leq 0
\]

It follows that for all reporters \( i \), the above algorithm chooses a format \( j \) such that either \( e_{ij}[k] = 0 \), or chooses