Registration of Multiple Point Sets Using the EM Algorithm

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Abstract

In this paper we address the problem of global registration between multiple d-dimens-ional point patterns with a given correspondence. The actual overlapping is not necessarily between pairs. Instead, it can be between any number of patterns. It is assumed that each pattern is a portion of an image of an unobserved object under a distinct rigid transformation. We derive an iterative solution for the problem of global registration of the patterns in order to reconstruct the original object. Our solution is based on the EM algorithm and it generalizes the well known solutions for the two-pattern case. We also suggest a very efficient method to implement the proposed algorithm. Experimental results demonstrate the improved performance of the proposed method.

1 Introduction

In computer vision applications the following mathematical problem is encountered. A set of rigid transformations (i.e. rotation and translation) is simultaneously applied to an object in the d-dimensional space. Each transformation creates an image which is a reflection of the object in its own local coordinate system. A noise is added to each point of each image and only a portion of the image is observed. Our target is to reconstruct the original object from the noisy images. The problem of finding the motion parameters of a rigid object using two point patterns is a special case of this problem. This special case has been extensively studied over the past decade [1] [5] [6] [12]. Arun, Huang and Blostein [1] proposed a method that utilizes the singular value decomposition to find the transformation parameters that give the least mean square error between the two point pattern. This method is incorporated in our algorithm and it is reviewed in section (3). In the special case when $d = 3$, it is possible to exploit the isomorphism that exists between the group of rotations and the group of quaternions of unit norm. Faugeras and Hebert [5] and Horn [7] proposed methods for fitting two point sets, based on quaternions. An overview of the techniques for two sets registration can be found in [10]. It was found [11] that no one algorithm is superior in all cases to the other ones. The problem of registration of multiple patterns is much more difficult. A widely used approach is to sequentially apply a pairwise registration until all the images are combined. This scheme does not use all the available information during each registration step. It remains essentially a local approach and it can cause a cumulation of registration errors as pointed out in [2] [3]. Benjemaa and Schmitt [2] proposed an iterative global registration method for the case $d = 3$ which generalizes the quaternions based solution. Their method first finds the rotation part of the transformations. Then, the translations are found by solving a linear system.

This paper proposes a solution for the registration problem that is based on the EM algorithm [4]. We analyze the situation where there is overlapping between any number of patterns and not just the simpler situation of overlapping between pairs of patterns. In the next section we present a complete statistical model that describes the noise included in the observation as a Gaussian white noise. The unknown object and transformation set are viewed as parameters of the density function of the observed patterns. Section (3) reformulate the solution for the problem of two sets registration using the singular value decomposition. In order to implement the EM algorithm, we present in section (4) a modified model which we call "two step model". This model is closely related to the original model and it serves as a technical tool. The two step model enables us to view our problem as an estimation problem in a manner such that the EM algorithm can be easily applied to find the maximum-likelihood (ML) estimation of the model parameters. The iterative registration algorithm is presented in section (5).

Methods for efficient time computations and accelerating the convergence rate are considered in section (6). Finally, experimental results are shown in section (7).

2 A Formal Statement of the Problem

Following is a formal presentation of the problem stated above. Let $Q = \{q_1, ..., q_n\}$ be a set consisting of n points in the d-dimensional space. The set Q can be considered a discrete description of an object in the space. Let $S = \{S_1, ..., S_m\}$ be a set of rigid Euclidean transformations. Each transformation S_i consists of a rotation matrix R_i and a translation vector t_i. Denote :

$$p(j,i) = S_i(q_j) + \epsilon_{ij} = R_i q_j + t_i + \epsilon_{ij} \qquad (1)$$

$$i = 1, .., m \quad , \quad j = 1, .., n$$

The point $p(j,i)$ is a noisy image of q_j under S_i. We assume that ϵ_{ij} is a Gaussian random vector with a zero mean and a scalar covariance matrix $\sigma^2 I$ such that I is the $d \times d$ identity matrix. We further assume that the random vectors $\{\epsilon_{ij}\}$ are mutually independent. In real situations, not all the object points are reflected in all the image sets. Suppose that for each object point q_j there is a subset of $\{1, ..., m\}$, denoted by c_j, such that only the image points $\{p(j,i), i \in c_j\}$ are observed. In other words, the point q_j is only reflected in the images indexed by a member of c_j. We shall assume without any loss of generality that for each j, the size of c_j, denoted by $|c_j|$, is at least 2. Denote $P_i = \{p(j,i) \mid i \in c_j\}$. The set P_i is a noisy image of a portion of the original object Q under the transformation S_i. Denote $P = \{P_1, ..., P_m\}$. In our problem we only observe the noisy images $P_1, ..., P_m$. The original object Q and the transformation set S are unknown. Our target is to reconstruct the object Q. The solution we shall present also reconstructs the transformation set S which defines the global registration of the images.

From a statistical point of view, S and Q can be considered as parameters of the density function of the observed images. We shall find a maximum likelihood (ML) estimation for S and Q.

$$\log f(P|Q,S) = -\frac{1}{2} d \log(2\pi\sigma^2) \sum_{j=1}^{n} |c_j| \qquad (2)$$

$$-\frac{1}{2\sigma^2} \sum_{j=1}^{n} \sum_{i \in c_j} \|p(j,i) - S_i(q_j)\|^2$$

The ML estimate for the variance of the noise can be found by setting the partial derivative with respect to σ^2 to zero.

$$\hat{\sigma}^2 = \frac{\sum_{j=1}^{n} \sum_{i \in c_j} \|p(j,i) - \hat{S}_i(\hat{q}_j)\|^2}{d \sum_{j=1}^{n} |c_j|}$$

such that $\hat{S}_i$, $\hat{q}_j$ are the ML estimates of S_i and q_j. From equation (2) we derive that whether σ^2 is known or not, in order to find the ML estimates for S and Q we need only to minimize the following expression :

$$L(S,Q) = \sum_{j=1}^{n} \sum_{i \in c_j} \|(p(j,i) - S_i(q_j)\|^2 \qquad (3)$$

$$= \sum_{j=1}^{n} (|c_j| \|\hat{q}_j(S) - q_j\|^2 + \sum_{i \in c_j} \|S_i^{-1}(p(j,i)) - \hat{q}_j(S)\|^2)$$

$$\text{such that : } \quad \hat{q}_j(S) = \frac{1}{|c_j|} \sum_{i \in c_j} S_i^{-1}(p(j,i)) \qquad (4)$$

Therefore, given an estimated transformation set S, the best prediction of the original point q_j is the empirical average of the preimages of the noisy images of q_j. Denote $\hat{Q}(S) = \{\hat{q}_1(S), ..., \hat{q}_n(S)\}$. The set $\hat{Q}(S)$ is the reconstruction of Q given S. Substituting equation (4) in equation (3) yields :

$$L(S) = L(S, \hat{Q}(S)) = \sum_{j=1}^{n} \sum_{i \in c_j} \|p(j,i) - S_i(\hat{q_j})\|^2 \qquad (5)$$

Hence the ML estimate for the unknown transformation set can be found by minimization the following expression :

$$\sum_{j=1}^{n} \sum_{i \in c_j} \|S_i^{-1}(p(j,i)) - \frac{1}{|c_j|} \sum_{k \in c_j} S_k^{-1}(p(j,k))\|^2 \qquad (6)$$

Equation (6) has the following intuitive motivation. The only information conveyed in the given point sets P is that for each j the points $p(j,i), i \in c_j$ are images of the same unobserved point q_j. In a noiseless environment we could expect that for the true transformation set, for each j the pre-images of $p(j,i), i \in c_j$ will coincide to the same point. In our noisy model we expect, at least, that under the true transformation set, the empirical variance of the pre-images will be as small as possible. Equation (6) is the formal presentation of this intuition.

We are still left with the main estimation problem considered in this paper, namely how to find the ML estimation of the transformation set. In the general case this cannot be done analytically. In the special case where we have only two images of the object there is a well known solution. It shall be reviewed in the next section.

3 Fitting of Two Point Sets

The following problem is fundamental in many computer vision applications. Two noisy point patterns are given in a d-dimensional space and we want to find the transformation parameters (rotation and translation) that yield the least mean square error between these sets. This problem can be obtained from our problem as a special case where m is equal to 2. We continue the analysis performed in the previous section and show that in this special case we can obtain a closed form solution. This solution is based on the singular value decomposition [1].

Recalling equation (6), we want to minimize $L(S)$. In the case $m = 2$ we obtain :

$$L(S) = \sum_{j=1}^{n} \|S_1^{-1}(p(j,1)) - S_2^{-1}(p(j,2))\|^2 \quad (7)$$

Due to a degree of freedom in the problem statement we can assume, without loss of generality, that S_1 is the identity transformation. Hence in this case :

$$L(S) = \sum_{j=1}^{n} \|S_2(p(j,1)) - p(j,2)\|^2 \quad (8)$$

It was shown by Huang et al. [9], that the translation t_2 can be easily found. Setting the partial derivative of $L(S)$ according to t_2 to zero reveals :

$$\hat{t}_2 = \frac{1}{n}\sum_{j=1}^{n}(p(j,2) - \hat{R}_2 p(j,1)) = \bar{p}_2 - \hat{R}_2\bar{p}_1 \quad (9)$$

$$\text{such that :} \qquad \bar{p}_i = \frac{1}{n}\sum_{j=1}^{n} p(j,i) \qquad i = 1,2$$

substituting definition (9) in expression (8) yields :

$$\hat{R}_2 = \arg\max_{R_2} \sum_{j=1}^{n} (p(j,2) - \bar{p}_2)^T R_2 (p(j,1) - \bar{p}_1) \quad (10)$$

Arun et al. [1] used the singular value decomposition technique (SVD) to solve this maximization problem in the following way. Let UDV^T be the SVD of the covariance matrix $\sum_{j=1}^{n}(p(j,1) - \bar{p}_1)(p(j,2) - \bar{p}_2)^T$, then $\hat{R}_2 = VU^T$.

This algorithm is not guaranteed to return a rotation matrix, and may instead, when the data is very noisy and corrupted, return a reflection matrix. Umeyama [12] has improved the algorithm so that it always returns a rotation matrix. In [9], [1] and [12] there is an implicit assumption that the noise is only present in one of the two point sets. Goryn and Hein [6] observed that under the assumption that both sets are noisy, the estimation problem has the same form as if one of the sets was not noisy. In this section we gave a statistical explanation for this fact.

4 The Two Step Model

This section describes another modeling interpretation for the registration problem. The advantage of the new model is that the EM algorithm can be utilized in order to find the ML estimation of the model parameter. We shall first show that the ML estimates of S and Q in the two models coincide. This enables us to solve the minimization problem presented in equation (6). For each object point q_j fix an index $I_j \in c_j$ (for example define I_j to be the smallest index in c_j). Denote $P_0 = \{p(j, I_j) \,|\, j = 1, ..., n\}$. P_0 is a set constructed from the observed images in such a manner that each point from the original object is represented exactly once. In the model stated in section (2), the points q_j serve as parameters in the density function. In contrast, in the model we shall now describe, the points q_j are assumed to be Gaussian random vectors with mean $S_{I_j}^{-1}(p(j, I_j))$ and variance $\sigma^2 I$. The points in P_0 are no longer considered as observed data. Instead, $p(I_j, j)$ are parameters that define a prior distribution on the original object point q_j. Sampling the data according to this model is, therefore, composed of two steps :

1. Sampling the object Q. Each of the original object points q_j is independently sampled according to the normal distribution $N(S_{I_j}^{-1}(p(j, I_j)), \sigma^2 I)$.

2. Sampling $P \backslash P_0$. Each point $p(j,i)$ is independently sampled according to the distribution $N(S_i(q_j), \sigma^2 I)$.

We name this interpretation “Two step model”. In this model, P_0 is a known parameter and the observed data is $P \backslash P_0$. The likelihood of the observations according to the two step model is :

$$f(P\backslash P_0|P_0, S) = \int_Q f(Q|P_0, S) f(P\backslash P_0|Q, S)\, dQ \quad (11)$$

$$= \prod_j \int_{q_j} f(q_j|p(j, I_j), S) \prod_{i \in c_j \backslash I_j} f(p(j,i)|q_j, S_i)\, dq_j$$

Direct algebraic manipulation reveals that for each $j = 1, .., n$:

$$\sum_{i \in c_j} \|p(j,i) - S_i(q_j)\|^2 = |c_j| \|q_j - \hat{q}_j(S)\|^2 + \quad (12)$$

$$\sum_{i \in c_j} \|S_i^{-1}(p(j,i)) - \hat{q}_j(S)\|^2$$

$$\text{and therefore :}\quad q_j|P,S \sim N\left(\hat{q}_j(S)\,,\,\frac{\sigma^2}{|c_j|}I\right) \quad (13)$$

Relation (12) enables us to obtain the following closed from expression for the integral (11) :

$$\log f(P\backslash P_0|P_0,S) = c - \frac{1}{2\sigma^2}\sum_{j=1}^{n}\sum_{i\in c_j}\|S_i^{-1}(p(j,i)) - \hat{q}_j(S)\|^2$$

such that c is a constant that only depends on σ^2. Hence, in order to find the ML estimate of S in the two step model we have to solve the following minimization problem :

$$\min_S \sum_{j=1}^{n}\sum_{i\in c_j}\|S_i^{-1}(p(j,i)) - \frac{1}{|c_j|}\sum_{k\in c_j}S_k^{-1}(p(j,k))\|^2$$

This is exactly the same minimization problem we faced in our original model in equation (6). Therefore, the ML estimation of the transformation set S in the two models is identical.

From equation (13) it can be seen that the maximum aposteriory estimation for Q given P and S is exactly the ML estimation of Q that was computed in equation (4). From a more general point of view, there is a close relation between the two models. The two density functions, $f(P|Q,S)$ in the first model, and $f(Q,P\backslash P_0|P_0,S)$ in the two step model, are identical.

5 Applying the EM Algorithm to the Two Step Model

In this section we apply the EM algorithm to find ML estimation of the parameter set of the two step model presented in section (4). As we have already shown, this parameter set is also ML estimation for the model presented in section (2).

Using the EM terminology, we shall refer to $y = P\backslash P_0$ as the observed incomplete data. $x = Q \cup P\backslash P_0$ is the complete data. The missing data is, therefore, the original object Q which was sampled in the first step but was not reported. We shall see that parameter estimation in the complete data framework is indeed much simpler. The unknown parameter is the set S. P_0 is a known parameter. Denote the current value of the parameters set by $S_0 = \{S_{01},...,S_{0m}\}$.

$$\log f_X(x;S,P_0) = c - \frac{1}{2\sigma^2}\sum_{j=1}^{n}\sum_{i\in c_j}\|S_i^{-1}(p(j,i)) - q_j\|^2$$

such that c is a constant that depends only on the variance σ^2. c will be ignored in the sequel. The EM auxiliary function in this case is:

$$Q(\theta,\theta_0) = E(\log f(Q,P\backslash P_0;S,P_0|P\backslash P_0;S_0,P_0))$$

$$= -\frac{1}{2\sigma^2}\sum_{j=1}^{n}\sum_{i\in c_j}E(\|S_i^{-1}(p(j,i)) - q_j\|^2|S_0,P)$$

Note that given P and S_0, the only undetermined element inside the conditional expectation expression is the original object Q. Denote

$$\hat{q}_j(S_0) = \frac{1}{|c_j|}\sum_{i\in c_j}S_{0i}^{-1}(p(j,i)) \qquad j=1,...,n$$

Denote also $\hat{Q}(S_0) = \{\hat{q}_1(S_0),...,\hat{q}_n(S_0)\}$. From equation (13) we derive :

$$\begin{aligned} E(q_j|S_0,P) &= \frac{1}{|c_j|}\sum_{i\in c_j}S_{0i}^{-1}(p(j,i)) = \hat{q}_j(S_0) \\ E(q_j^T q_j|S_0,P) &= \frac{d\sigma^2}{|c_j|} + \hat{q}_j^T(S_0)\hat{q}_j(S_0) \end{aligned}$$

Therefore :

$$Q(\theta,\theta_0) = c - \frac{1}{2\sigma^2}\sum_{j=1}^{n}\sum_{i\in c_j}\|S_i^{-1}(p(j,i)) - \hat{q}_j(S_0)\|^2$$

This completes the E-step. In order to perform the maximization step we first observe that we can now maximize $Q(\theta,\theta_0)$ separately for each S_i. This is the main reason for using the EM to solve the original problem. Hence :

$$\hat{S}_i = \arg\min_{S_i}\sum_{\{j|i\in c_j\}}\|S_i^{-1}(p(j,i)) - \hat{q}_j(S_0)\|^2$$

Therefore $\hat{S}_i$ is the most likely rigid transformation from the relevant portion of $\hat{Q}(S_0)$ to P_i. In section (3) we reviewed the solution for this problem using SVD. This completes the M-step.

To summarize the EM iteration :

1. E-step : Given the current estimation of the transformation set S_0, reconstruct the original object Q :

$$\hat{q}_j(S_0) = \frac{1}{|c_j|}\sum_{i\in c_j}S_{0i}^{-1}(p(j,i)) \qquad j=1,..,n$$

2. M-step : given the current reconstructed object $\hat{Q}(S_0)$ reestimate the transformation set in the following way. Denote :

$$\begin{aligned} \bar{q}_i(S_0) &= \frac{1}{|P_i|}\sum_{\{j|i\in c_j\}}\hat{q}_j(S_0) \qquad (14) \\ \bar{p}_i &= \frac{1}{|P_i|}\sum_{\{j|i\in c_j\}}p(j,i) \\ H_i &= \frac{1}{|P_i|}\sum_{\{j|i\in c_j\}}\hat{q}_j(S_0)(p(j,i)-\bar{p}_i)^T \end{aligned}$$

Let $U_i D_i V_i^T$ be the SVD of the covariance matrix H_i. The reestimation of the transformation $S_i = (R_i, t_i)$ is :

$$R_i = V_i U_i^T \quad , \quad t_i = \bar{p}_i - R_i \bar{q}_i(S_0) \qquad i = 1, ..., m$$

As mentioned in the case of two images, there is a degree of freedom in the solution for our problem. We can apply a global transformation to all the transformations in the suggested set $\hat{S}$. This is so because we do not have any knowledge about the coordinates of the real world where the object Q is located. In other words, the reconstruction of Q is unique up to a rotation and translation of the object.

A good initialization of the EM-algorithm is crucial in order to reach the global maximum of the likelihood function. The widely used approach of sequential applying a pairwise registration until all the images are combined can be used for initial values of the transformations set. Any other global registration method that have been suggested in the literature can be used to initiate the EM iteration as well.

6 Acceleration Methods

A major problem in the implementation of the iterative algorithm, presented in the previous section, is the need to go over all the data points during each iteration. In real situations, where there are millions of data points, this cannot be done in a reasonable time. The main computational effort during one EM iteration is spent on computing the matrices H_i defined in equation (14). We shall now show that after a preproccesing on the data, the complexity of computing the matrices H_i in each iteration can be significantly reduced. Direct algebraic manipulation on the definition of H_i reveals :

$$\begin{aligned} H_i &= \frac{1}{|P_i|} \sum_{\{j|i \in c_j\}} \hat{q}_j(S_0)(p(j,i) - \bar{p}_i)^T \\ &= \frac{1}{|P_i|} \sum_{\{j|i \in c_j\}} \frac{1}{|c_j|} \sum_{k \in c_j} S_{0k}^{-1}(p(j,k))(p(j,i) - \bar{p}_i)^T \\ &= \frac{1}{|P_i|} \sum_{k=1}^{m} \sum_{\{j|i,k \in c_j\}} \frac{1}{|c_j|} S_{0k}^{-1}(p(j,k))(p(j,i) - \bar{p}_i)^T \\ &= \sum_{k=1}^{m} R_{0k}^{-1}(X_{ik}^1 - t_{0k} X_{ik}^2) \end{aligned}$$

such that :

$$X_{ik}^1 = \frac{1}{|P_i|} \sum_{\{j|i,k \in c_j\}} \frac{1}{|c_j|} p(j,k)(p(j,i) - \bar{p}_i)^T$$

$$X_{ik}^2 = \frac{1}{|P_i|} \sum_{\{j|i,k \in c_j\}} \frac{1}{|c_j|}(p(j,i) - \bar{p}_i)^T$$

It can be easily seen that X_{ik}^1 and X_{ik}^2 depend only on the observed data. Therefore, they can be computed once in advance. The rotation element of the transformation S_i can be obtained from H_i using the SVD technique. The translation element of the rigid transformation can be efficiently computed in a similar manner.

Until now we have shown how to efficiently perform one EM iteration. The iterative algorithm eventually convergences to a local maximum point of the likelihood function. It is well known, however, that the rate of convergence of the EM algorithm is very slow. We shall now propose the following modification in order to accelerate the convergence. In the EM algorithm, all the transformations are being updated simultaneously. In contrast, in the modified algorithm, after a transformation is reestimated, the new value can be immediately used in the updating of the next transformation. This enables us to propagate the influence of the updated transformation on the other ones much more rapidly. In this modified algorithm there is a monotone decrees of $L(S)$ and therefore it implies a monotone improvement of the likelihood function.

Yet another acceleration step can be done. As it was mentioned in section (3), H_i is actually a technical step in order to solve the maximization problem defined in equation (10) :

$$\hat{R}_i = \arg\max_{R_i} \sum_{\{j|i \in c_j\}} (p(j,i) - \bar{p}_i)^T R_i \hat{q}_j(S_0) \qquad (15)$$

$$= \sum_{k=1}^{m} \sum_{\{j|i,k \in c_j\}} \frac{1}{|c_j|}(p(j,i) - \bar{p}_i)^T R_i R_{0k}^{-1}(p(j,k) - t_{0k})$$

We can now plug into this expression the unknown value of R_i instead of the old value R_{0i}. In this way we do not involve the current inaccurate estimation in the reestimation step. Then, for the case $k = i$ we obtain :

$$\sum_{\{j|i \in c_j\}} \frac{1}{|c_j|}(p(j,i) - \bar{p}_i)^T R_i R_i^{-1}(p(j,i) - t_{0i}) =$$

$$\sum_{\{j|i \in c_j\}} \frac{1}{|c_j|}(p(j,i) - \bar{p}_i)^T (p(j,i) - \bar{p}_i)$$

This expression is constant and has no influence on the maximization operation. We can, therefore, eliminate the case $k = i$ from the summation (15). The updating formulae for S_i, therefore, become :

$$
\begin{aligned}
H_i &= \sum_{k \neq i} R_k^{-1}(x_{ik}^1 - t_k x_{ik}^2) \qquad (16)\\
R_i &= V_i U_i^T \quad s.t. \quad U_i D_i V_i^T \quad \text{is the SVD of} \quad H_i \\
t_i &= \bar{p}_i - R_i \sum_{k \neq i} R_k^{-1}(y_{ik}^1 - t_k y_{ik}^2)
\end{aligned}
$$

such that :

$$
\begin{aligned}
x_{ik}^1 &= \frac{1}{|P_i|} \sum_{\{j|i,k \in c_j\}} \frac{1}{|c_j - 1|} p(j,k)(p(j,i) - \bar{p}_i)^T \\
x_{ik}^2 &= \frac{1}{|P_i|} \sum_{\{j|i,k \in c_j\}} \frac{1}{|c_j - 1|} (p(j,i) - \bar{p}_i)^T \\
y_{ik}^1 &= \frac{1}{|P_i|} \sum_{\{j|i,k \in c_j\}} \frac{1}{|c_j - 1|} p(j,k) \\
y_{ik}^2 &= \frac{1}{|P_i|} \sum_{\{j|i,k \in c_j\}} \frac{1}{|c_j - 1|}
\end{aligned}
$$

These formulae can be made much simpler in cases of pairwise overlapping or full overlapping between the images. Updating the transformation S_i according to this algorithm has the following geometric intuition. Suppose a transformation set S_0 is given and we want to update the estimation of the transformation S_i. First, reconstruct the original object from all the images except P_i using the current transformation set S_0. The updated value of S_i is the best rigid transforma-

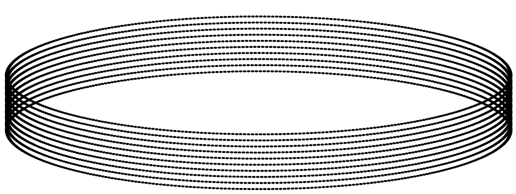

Figure 1. original object

7 Experimental Results

Experiments of global registration were performed using synthetic data. The object we wanted to reconstruct is a cylinder with radius of 1 and height of 0.1. The object was constructed from 4000 points. The original object is shown in Figure 1. The cylinder was segmented in a circular manner into 20 parts, such that each part overlaps with its two neighbors. A random

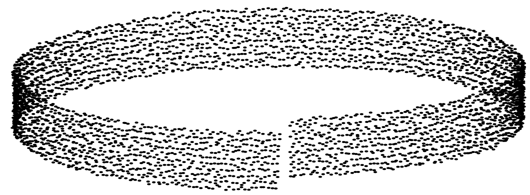

Figure 2. reconstruction results of the sequential registration.

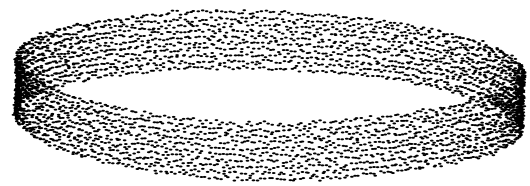

Figure 3. reconstruction results after iterative global registration.

rigid transformation was applied to each part and a noise with $\sigma = 0.01$ was added to each point of each image. First, we have applied the sequential registration algorithm. The results are shown in Figure 2. As can be seen, the quality of registration between the first part and the last one is poor. The average distance between these two parts after the registration is 0.05. The transformation set produced by the sequential registration was used as an initial guess for the iterative algorithm summarized in equations (16). In this experiment we found that 10 iterations (such that in each iteration the entire transformation set is updated) are enough to achieve convergence of the likelihood function. The results of the global registration are shown in Figure 3. In this reconstruction the registration error is homogeneously distributed.

8 Conclusion

We have presented here an iterative solution for the problem of global registration of noisy images. We have shown the stability of the iterative process by illustrating that it can be considered an example of the EM algorithm. The general theory of the EM algorithm assures that each iteration improves the quality of the reconstructed object. We have also suggested an initialization for the iterative algorithm that can improve the chance that the local maximum point, that the algorithm approaches, is actually a global one. The algorithm presented in this paper reduces the general problem of multiple registration to the simpler problem of two sets registration. In order to explicitly demonstrate our method, we adopted Arun et al. [1] method that uses the singular value decomposition to solve the problem of two set registration. Any other existing method can be plugged into the maximization step of the EM algorithm instead. A major element of the presented method is the efficient computation of the iterations.

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