Multi-Microphone Speech Dereverberation using Eigen-decomposition

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The Reverberation Phenomenon

(a) Clean signal

(b) Reverberant signal ($T_{60} = 0.4s$)
The Room impulse Response (RIR)

The talk is based on:

Outline

1. Problem Formulation
Outline

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2. Preliminaries
Outline

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2. Preliminaries
3. RIR Estimation - Algorithm Derivation
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4. Extensions of the Basic Algorithm
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3. RIR Estimation - Algorithm Derivation
4. Extensions of the Basic Algorithm
5. RIR Estimation in Subbands
6. Signal Reconstruction
7. Experimental Study

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Speech Dereverberation using EVD
Outline

1. Problem Formulation
2. Preliminaries
3. RIR Estimation - Algorithm Derivation
4. Extensions of the Basic Algorithm
5. RIR Estimation in Subbands
6. Signal Reconstruction
7. Experimental Study
8. Summary and Conclusions
Problem Formulation

Problem Formulation

\[ x_m(n) = y_m(n) + \nu_m(n) = \sum_{k=0}^{n_h} h_m(k)s(n - k) + \nu_m(n) \]

\[ H_m(z) = \sum_{k=0}^{n_h} h_m(k)z^{-k}; \quad m = 1, 2, \ldots, M. \]
Goal

Use a Two Stage Approach

Estimation

RIR

$\hat{H}_1(z)$

$\hat{H}_2(z)$

$\hat{H}_M(z)$

$x_1(n)$

$x_2(n)$

$x_M(n)$

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Speech Dereverberation using EVD
Goal

Use a Two Stage Approach

- Estimate the Acoustic Transfer Function (ATFs) $H_m(z)$. 

\[ x_1(n) \quad \rightarrow \quad \hat{H}_1(z) \]
\[ x_2(n) \quad \rightarrow \quad \hat{H}_2(z) \]
\[ x_M(n) \quad \rightarrow \quad \hat{H}_M(z) \]
Use a Two Stage Approach

- Estimate the Acoustic Transfer Function (ATFs) $H_m(z)$.
- Use $\hat{H}_m(z)$; $m = 1, \ldots, M$ to extract $s(n)$. 

Goal

Use a Two Stage Approach

- Estimate the Acoustic Transfer Function (ATFs) $H_m(z)$.
- Use $\hat{H}_m(z)$; $m = 1, \ldots, M$ to extract $s(n)$. 

Two Microphone, Noiseless Case

\[ y_1(n) = h_1(n) \ast s(n) \]
\[ y_2(n) = h_2(n) \ast s(n) \]

ATFs Nullifying filters

\[ y_1(n) \]
\[ H_1(z) \]
\[ s(n) \]

\[ y_2(n) \]
\[ H_2(z) \]
\[ E_\ell(z) \]

\[ -H_1(z) \]
\[ E_\ell(z) \]

Nullifying Filters

\[ [y_2(n) \ast h_1(n) - y_1(n) \ast h_2(n)] \ast e_\ell(n) = 0 \]
\[ \tilde{h}_{m,\ell}(n) = h_m(n) \ast e_\ell(n); \ m = 1, 2 \]
Data Matrix

$$Y_m^T = \begin{bmatrix}
y_m(0) & 0 & \cdots & 0 \\
y_m(1) & y_m(0) & & \\
: & & y_m(1) & \ddots & 0 \\
y_m(\hat{n}_h-1) & \vdots & \ddots & y_m(0) \\
y_m(\hat{n}_h) & y_m(\hat{n}_h-1) & y_m(1) \\
: & \vdots & \ddots & \ddots \\
y_m(N) & \vdots & \ddots & y_m(\hat{n}_h-1) \\
0 & y_m(N) & \ddots & y_m(\hat{n}_h) \\
: & \vdots & \ddots & \ddots \\
0 & \cdots & 0 & y_m(N)
\end{bmatrix}$$
Filtered Room Impulse Responses (RIRs)

Define:

\[
\tilde{h}^T_{m,\ell} = [\tilde{h}_{m,\ell}(0) \, \tilde{h}_{m,\ell}(1) \, \ldots \, \tilde{h}_{m,\ell}(\hat{n}_h)] \; ; \; m = 1, 2
\]

Concatenate:

\[
\tilde{h}_\ell = \begin{bmatrix} \tilde{h}_{1,\ell} \\ \tilde{h}_{2,\ell} \end{bmatrix} ; \quad Y = \begin{bmatrix} Y_2 \\ -Y_1 \end{bmatrix}
\]

Nullifying Filters:

\[
Y^T \tilde{h}_\ell = 0 ; \; \forall \ell.
\]

Therefore:

\[
\tilde{h}_\ell Y Y^T \tilde{h}_\ell = 0 \Rightarrow \tilde{h}_\ell \hat{R}_y \tilde{h}_\ell = 0 ; \; \forall \ell
\]
Null Subspace

Eigenvalue (or Singular Value) Decomposition

\[ \lambda_\ell = 0 \quad \ell = 0, 1, \ldots, \hat{n}_h - n_h \]
\[ \lambda_\ell > 0 \quad \text{otherwise} \]
Null Subspace

**Eigenvalue (or Singular Value) Decomposition**

\[ \lambda_\ell = 0 \quad \ell = 0, 1, \ldots, \hat{n}_h - n_h \]
\[ \lambda_\ell > 0 \text{ otherwise} \]

**Null Subspace Vectors**

\[ V = [ v_0 \ v_1 \ \cdots \ v_{\hat{n}_h-n_h} ] = \begin{bmatrix} \tilde{h}_{1,0} & \tilde{h}_{1,1} & \cdots & \tilde{h}_{1,\hat{n}_h-n_h} \\ \tilde{h}_{2,0} & \tilde{h}_{2,1} & \cdots & \tilde{h}_{2,\hat{n}_h-n_h} \end{bmatrix} \]
Over-Estimated Room Impulse Responses

**Acoustical Transfer Functions**

For $\ell = 0, 1, \ldots, \hat{n}_h - n_h$, $m = 1, 2$:

\[
\tilde{h}_\ell \iff \tilde{H}_{m,\ell}(z)
\]

\[
\tilde{H}_{m,\ell}(z) = H_m(z)E_\ell(z)
\]
Over-Estimated Room Impulse Responses

Acoustical Transfer Functions
For \( \ell = 0, 1, \ldots, \hat{n}_h - n_h, \ m = 1, 2: \)

\[
\hat{h}_\ell \Leftrightarrow \hat{H}_{m, \ell}(z) \\
\hat{H}_{m, \ell}(z) = H_m(z)E_\ell(z)
\]

Fundamental Lemma
Over-Estimated Room Impulse Responses

Acoustical Transfer Functions

For $\ell = 0, 1, \ldots, \hat{n}_h - n_h$, $m = 1, 2$:

$$\tilde{h}_\ell \Leftrightarrow \tilde{H}_{m,\ell}(z)$$

$$\tilde{H}_{m,\ell}(z) = H_m(z)E_\ell(z)$$

Fundamental Lemma

- For $m = 1, 2, \ldots, M$:

  $\tilde{H}_{m,\ell}(z)$ have $\hat{n}_h - n_h$ common roots $\Rightarrow E_\ell(z)$. 

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Over-Estimated Room Impulse Responses

Acoustical Transfer Functions
For $\ell = 0, 1, \ldots, \hat{n}_h - n_h$, $m = 1, 2$:

$$\tilde{h}_\ell \iff \tilde{H}_{m,\ell}(z)$$
$$\tilde{H}_{m,\ell}(z) = H_m(z)E_{\ell}(z)$$

Fundamental Lemma
- For $m = 1, 2, \ldots, M$:
  $\tilde{H}_{m,\ell}(z)$ have $\hat{n}_h - n_h$ common roots $\Rightarrow E_{\ell}(z)$.
- For $\ell = 0, 1, \ldots, \hat{n}_h - n_h$:
  $\tilde{H}_{m,\ell}(z)$ have $n_h$ common roots $\Rightarrow H_m(z)$.
RIR Estimation - Algorithm Derivation

Filtering (Silvester) Matrix:

\[ H_m = \begin{bmatrix}
  h_m(0) & 0 & 0 & \ldots & 0 \\
  h_m(1) & h_m(0) & 0 & \ldots & 0 \\
  \vdots & \ddots & \ddots & \ddots & \vdots \\
  h_m(n_h) & \ddots & \ddots & \ddots & 0 \\
  0 & h_m(n_h) & \ddots & h_m(0) & \vdots \\
  \vdots & 0 & h_m(1) & \ddots & \vdots \\
  \vdots & \ddots & \ddots & \ddots & \vdots \\
  0 & 0 & \ldots & 0 & h_m(n_h)
\end{bmatrix}_{\hat{n}_h-n_h+1} \]
Over-Estimated Room Impulse Responses
Matrix Form

Define:
\[ e^T_\ell = [ e_\ell(0) \; e_\ell(1) \; \ldots \; e_\ell(\hat{n}_h - n_h) ] \]

Extraneous Filters:
\[ E = [ e_0 \; e_1 \; \cdots \; e_{\hat{n}_h - n_h} ] . \]

Null Subspace Vectors (Over-estimated RIRs):
\[ V = \begin{bmatrix} \tilde{h}_{1,0} & \tilde{h}_{1,1} & \cdots & \tilde{h}_{1,\hat{n}_h - n_h} \\ \tilde{h}_{2,0} & \tilde{h}_{2,1} & \cdots & \tilde{h}_{2,\hat{n}_h - n_h} \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} E \triangleq HE \]

Define \( E^i \triangleq \text{inv}(E) = [ e^i_0 \; e^i_1 \; \cdots \; e^i_{\hat{n}_h - n_h} ] \)

Then:
\[ H = VE^i \]
RIR Extraction
Exploiting the Silvester Structure

\[
\begin{bmatrix}
V \ O \ & \cdots \ & \cdots \ & \cdots \ & O \\
O \ & \ V \ & O \ & \cdots \ & O \\
\vdots \ & \vdots \ & \vdots \ & \ddots \ & \vdots \\
O \ & \cdots \ & \cdots \ & \cdots \ & \ O \\
O \ & \cdots \ & \cdots \ & \cdots \ & \ O \\
\end{bmatrix}
- S^{(0)}
\]

\[
\begin{bmatrix}
\vdots \\
0 \\
\vdots \\
\vdots \\
\end{bmatrix}
- S^{(1)}
\]

\[
\begin{bmatrix}
e_i^0 \\
\cdots \\
e_i^j \\
\cdots \\
e_i^j_{\tilde{n}_h-n_h} \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
0 \\
0 \\
\vdots \\
0 \\
0 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
\theta
\end{bmatrix}
\]

- all-zeros matrix
- \( S^{(\ell)} \) - shift by \( \ell \) matrix (\( \ell = 0, 1, \ldots, \tilde{n}_h - n_h \))
RIR Estimation - Basic Case

\[ \tilde{V}_\theta = 0 \]

Find eigenvector of \( \tilde{V} \) corresponding to eigenvalue 0

Extract \( h_1, h_2 \) from the eigenvector
Algorithm Summary

RIR Estimation - Basic Case

\[ \tilde{V} \theta = 0 \]
Algorithm Summary

RIR Estimation - Basic Case

\[ \tilde{V} \theta = 0 \]

Find eigenvector of \( \tilde{V} \) corresponding to eigenvalue 0
Algorithm Summary

RIR Estimation - Basic Case

- $\tilde{V} \theta = 0$
- Find eigenvector of $\tilde{V}$ corresponding to eigenvalue 0
- Extract $h_1, h_2$ from the eigenvector
Extensions

- Two Microphone Noisy Case
Extensions

- Two Microphone Noisy Case
  - White Noise Case
Extensions

- Two Microphone Noisy Case
  - White Noise Case
  - Colored Noise Case
Extensions

- Two Microphone Noisy Case
  - White Noise Case
  - Colored Noise Case

- Multi-Microphone Case ($M > 2$)
Extensions

- **Two Microphone Noisy Case**
  - White Noise Case
  - Colored Noise Case

- **Multi-Microphone Case** \((M > 2)\)

- **Partial Knowledge of the Null Subspace**
Two Microphone Noisy Case

\[ \mathbf{X} = \mathbf{Y} + \mathbf{\Upsilon}, \]

\( \mathbf{X} \) - noisy signal data matrix
\( \mathbf{\Upsilon} \) - noise-only data matrix

\[ \hat{\mathbf{R}}_x \approx \hat{\mathbf{R}}_y + \hat{\mathbf{R}}_\nu \]

\[ \hat{\mathbf{R}}_x = \frac{\mathbf{X}\mathbf{X}^T}{N+1} \) - noisy signal correlation matrix
\[ \hat{\mathbf{R}}_\nu = \frac{\mathbf{\Upsilon}\mathbf{\Upsilon}^T}{N+1} \) - noise-only signal correlation matrix
White Noise
\( \hat{R}_\nu \approx \sigma_\nu^2 I \)

RIR Estimation - White Noise

1. Find eigenvector of \( \tilde{V} \) corresponding to the smallest eigenvalue
2. Total Least Squares
3. Extract \( h_1, h_2 \) from the eigenvector
White Noise
\( \hat{R}_\nu \approx \sigma^2_\nu I \)

RIR Estimation - White Noise

- \( \mathbf{V} \) - eigenvectors corresponding to eigenvalue \( \sigma^2_\nu \)
  (remains intact)
White Noise
\( \hat{R}_\nu \approx \sigma^2_{\nu} I \)

**RIR Estimation - White Noise**

- \( \mathbf{V} \) - eigenvectors corresponding to eigenvalue \( \sigma^2_{\nu} \) (remains intact)
- \( \tilde{\mathbf{V}} \theta = \epsilon \)
White Noise
\[ \hat{R}_\nu \approx \sigma^2_\nu \mathbf{I} \]

RIR Estimation - White Noise
- \( \mathbf{V} \) - eigenvectors corresponding to eigenvalue \( \sigma^2_\nu \)
  (remains intact)
- \( \tilde{\mathbf{V}} \theta = \epsilon \)
- Find eigenvector of \( \tilde{\mathbf{V}} \) corresponding to the smallest eigenvalue \( \Rightarrow \) Total Least Squares
White Noise
\[ \hat{R}_\nu \approx \sigma^2 \nu I \]

RIR Estimation - White Noise
- \( \mathbf{V} \) - eigenvectors corresponding to eigenvalue \( \sigma^2_\nu \)
  (remains intact)
- \( \tilde{\mathbf{V}} \theta = \epsilon \)
- Find eigenvector of \( \tilde{\mathbf{V}} \) corresponding to the smallest
  eigenvalue \( \Rightarrow \text{Total Least Squares} \)
- Extract \( \mathbf{h}_1, \mathbf{h}_2 \) from the eigenvector
Colored Noise

RIR Estimation - Colored Noise

Calculate generalized EVD of $\hat{R}_x$ and $\hat{R}_\nu$ (or generalized SVD of $X$ and $\Upsilon$).

$V$ - generalized eigenvectors corresponding to generalized eigenvalue 1

$\tilde{V}_\theta = \epsilon$

Find eigenvector of $\tilde{V}$ corresponding to the smallest eigenvalue $\Rightarrow$ Total Least Squares

Extract $h_1, h_2$ from the eigenvector.

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Speech Dereverberation using EVD
Colored Noise

RIR Estimation - Colored Noise

- Calculate generalized EVD of $\hat{R}_x$ and $\hat{R}_\nu$
  (or generalized SVD of $X$ and $\Upsilon$)
Colored Noise

**RIR Estimation - Colored Noise**

- Calculate generalized EVD of $\hat{R}_x$ and $\hat{R}_\nu$
  (or generalized SVD of $X$ and $\Upsilon$)
- $V$ - generalized eigenvectors corresponding to generalized eigenvalue 1
Colored Noise

RIR Estimation - Colored Noise

- Calculate generalized EVD of $\hat{R}_x$ and $\hat{R}_\nu$
  (or generalized SVD of $X$ and $\Upsilon$)
- $V$ - generalized eigenvectors corresponding to generalized eigenvalue 1
- $\tilde{V}\theta = \epsilon$
Colored Noise

RIR Estimation - Colored Noise

- Calculate generalized EVD of $\hat{R}_x$ and $\hat{R}_\nu$
  (or generalized SVD of $X$ and $Y$)
- $V$ - generalized eigenvectors corresponding to generalized eigenvalue 1
- $\tilde{V}\theta = \epsilon$
- Find eigenvector of $\tilde{V}$ corresponding to the smallest eigenvalue $\Rightarrow$ Total Least Squares
Colored Noise

### RIR Estimation - Colored Noise

- Calculate generalized EVD of $\hat{R}_x$ and $\hat{R}_\nu$
  (or generalized SVD of $X$ and $\Upsilon$

- $V$ - generalized eigenvectors corresponding to generalized eigenvalue 1

- $\tilde{V}\theta = \epsilon$

- Find eigenvector of $\tilde{V}$ corresponding to the smallest eigenvalue $\Rightarrow$ **Total Least Squares**

- Extract $h_1, h_2$ from the eigenvector
Multi-Microphone Case \((M > 2)\)

Pairing \(\frac{M \times (M-1)}{2}\) channels:

\[
[y_i(n) * h_j(n) - y_j(n) * h_i(n)] * e_l(n) = 0
\]

\(i, j = 1, 2, \ldots, M; \ l = 0, 1, \ldots, \hat{n}_h - n_h\)

Construct an extended data matrix:

\[
X = \begin{bmatrix}
X_2 & X_3 & \cdots & X_M & O & \cdots & O & \cdots & O \\
-X_1 & O & \cdots & X_3 & \cdots & X_M & O \\
O & -X_1 & \cdots & -X_2 & O & \cdots & \vdots \\
\vdots & O & \cdots & \vdots & \vdots & \vdots & O \\
\vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \vdots \\
O & O & \cdots & -X_1 & \cdots & -X_2 & \cdots & -X_{M-1}
\end{bmatrix}
\]
Algorithm

RIR Estimation - Multi-Microphone

Calculate generalized EVD of new $\hat{R}$ and $\hat{R}_\nu$ (or generalized SVD of new $X$ and $\Upsilon$)

$V_{\text{new null subspace}}$

$\tilde{V}_\theta = \epsilon_1, \epsilon_2, \ldots, \epsilon_n$

Find eigenvector of $\tilde{V}$ corresponding to the smallest eigenvalue $\Rightarrow$ Total Least Squares

Extract $h_1, h_2, \ldots, h_M$ from the eigenvector

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Speech Dereverberation using EVD


Algorithm

\textbf{RIR Estimation - Multi-Microphone}

- Calculate generalized EVD of new $\hat{R}_x$ and $\hat{R}_y$
  (or generalized SVD of new $X$ and $Y$)
## Algorithm

**RIR Estimation - Multi-Microphone**

- Calculate generalized EVD of new $\hat{R}_x$ and $\hat{R}_\nu$
  (or generalized SVD of new $X$ and $Y$)
- $\mathbf{V}$ - new null subspace
Algorithm

**RIR Estimation - Multi-Microphone**

- Calculate generalized EVD of new $\hat{R}_x$ and $\hat{R}_\nu$
  (or generalized SVD of new $X$ and $\mathcal{Y}$)
- $\tilde{V}$ - new null subspace
- $\tilde{V}\hat{\theta} = \epsilon$, where:
  \[
  \hat{\theta}^T = \begin{bmatrix}
  (e_0^i)^T & (e_1^i)^T & \cdots & (e_{\hat{n}_h-n_h}^i)^T & h_1^T & h_2^T & \cdots & h_M^T
  \end{bmatrix}
  \]
Algorithm

RIR Estimation - Multi-Microphone

- Calculate generalized EVD of new \( \hat{R}_x \) and \( \hat{R}_\nu \)
  (or generalized SVD of new \( X \) and \( \nu \))
- \( \tilde{V} \) - new null subspace
- \( \tilde{V} \theta = \epsilon \), where:
  \[
  \theta^T = \begin{bmatrix}
  (e_0^i)^T & (e_1^i)^T & \cdots & (e_{\hat{n}_h-n_h}^i)^T & h_1^T & h_2^T & \cdots & h_M^T
  \end{bmatrix}
  \]
- Find eigenvector of \( \tilde{V} \) corresponding to the smallest eigenvalue \( \Rightarrow \) Total Least Squares
Algorithm

**RIR Estimation - Multi-Microphone**

- Calculate generalized EVD of new $\hat{R}_x$ and $\hat{R}_v$ (or generalized SVD of new $X$ and $Y$)
- $V$ - new null subspace
- $\tilde{V} \theta = \epsilon$, where:
  
  $\theta^T = \begin{bmatrix} (e_i^0)^T & (e_i^1)^T & \cdots & (e_i^{\hat{n}_h-n_h})^T & h_1^T & h_2^T & \cdots & h_M^T \end{bmatrix}$
- Find eigenvector of $\tilde{V}$ corresponding to the smallest eigenvalue $\Rightarrow$ **Total Least Squares**
- Extract $h_1, h_2, \ldots, h_M$ from the eigenvector
Partial Knowledge of the Null Subspace

Augmented Null Subspace:

\[ \bar{V} = \begin{bmatrix} V & 0^T & 0^T & 0^T \\ 0^T & V & 0^T & 0^T \\ \vdots & \vdots & \ddots & \vdots \\ 0^T & 0^T & \cdots & V \end{bmatrix} = \bar{H} \begin{bmatrix} E & 0^T & 0^T & 0^T \\ 0^T & E & 0^T & 0^T \\ \vdots & \vdots & \ddots & \vdots \\ 0^T & 0^T & \cdots & E \end{bmatrix} \]

\[ L > \hat{n}_h - n_h + \hat{\ell} \]

\[ E^{Pi} = \text{Pinv}\{\bar{E}\} = \bar{E}^T (\bar{E}\bar{E}^T)^{-1} \]

\[ \Rightarrow \bar{V}E^{Pi} = \bar{H} \]
Algorithm

RIR Estimation - Multi-Microphone

Calculate $\bar{V}$ - augmented null subspace

$\tilde{\bar{V}}_{\theta} = \epsilon$

Find eigenvector of $\tilde{\bar{V}}$ corresponding to the smallest eigenvalue

$\Rightarrow$ Total Least Squares

Extract $h_1, h_2$ from the eigenvector

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Speech Dereverberation using EVD
Algorithm

RIR Estimation - Multi-Microphone

- Calculate $\tilde{V}$ - augmented null subspace
Algorithm

RIR Estimation - Multi-Microphone

- Calculate $\tilde{V}$ - augmented null subspace
- $\tilde{V} \theta = \epsilon$
Algorithm

RIR Estimation - Multi-Microphone

- Calculate $\tilde{V}$ - augmented null subspace
- $\tilde{V} \theta = \epsilon$
- Find eigenvector of $\tilde{V}$ corresponding to the smallest eigenvalue $\Rightarrow$ Total Least Squares
Algorithm

RIR Estimation - Multi-Microphone

- Calculate $\tilde{\mathbf{V}}$ - augmented null subspace
- $\tilde{\mathbf{V}} \theta = \epsilon$
- Find eigenvector of $\tilde{\mathbf{V}}$ corresponding to the smallest eigenvalue $\Rightarrow$ Total Least Squares
- Extract $\mathbf{h}_1, \mathbf{h}_2$ from the eigenvector
Subband Filters

![Graph showing the amplitude of different subbands across frequency.

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RIR Estimation in Subbands

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Speech Dereverberation using EVD
Signal Reconstruction (general)

\[ g_m(n); \ m = 1, 2, \ldots, M - \text{set of} \ M \text{ equalizers.} \]

Estimated speech signal:

\[
\hat{s}(n) = \sum_{m=1}^{M} g_m(n) \ast x_m(n) = \\
\sum_{m=1}^{M} g_m(n) \ast h_m(n) \ast s(n) + \sum_{m=1}^{M} g_m(n) \ast \nu_m(n)
\]

Equalization:

\[
\sum_{m=1}^{M} g_m(n) \ast h_m(n) = \delta(n) \iff \sum_{m=1}^{M} G_m(z)H_m(z) = 1
\]

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Speech Dereverberation using EVD
Multi-channel Inverse Filter Theorem (MINT)

FIR Equalizers:

\[ g_m^T = \begin{bmatrix} g_m(0) & g_m(1) & \cdots & g_m(L_g) \end{bmatrix} \]

Causal equalization:

\[
\begin{bmatrix}
H_1 & H_2 & \cdots & H_M
\end{bmatrix}
\begin{bmatrix}
g_1 \\
g_2 \\
\vdots \\
g_M
\end{bmatrix}
= 
\begin{bmatrix}
1 \\
0 \\
0 \\
\vdots \\
0
\end{bmatrix}
\]

\[
\hat{g} = \arg\min_g \| Hg - d \|^2 = \left( H^T H \right)^{-1} H^T d
\]
Non-Causal Equalizers

**Matched Beamformer (MBF)**

\[
G_m(z) = \frac{H_m^*(1/z^*)}{\sum_{m=1}^{M} H_m(z) H_m^*(1/z^*)} \iff G_m(e^{j\omega}) = \frac{H_m^*(e^{j\omega})}{\sum_{m=1}^{M} |H_m(e^{j\omega})|^2}.
\]
Non-Causal Equalizers

**Matched Beamformer (MBF)**

\[
G_m(z) = \frac{H_m^*(1/z^*)}{\sum_{m=1}^{M} H_m(z)H_m^*(1/z^*)} \iff G_m(e^{j\omega}) = \frac{H_m^*(e^{j\omega})}{\sum_{m=1}^{M} |H_m(e^{j\omega})|^2}.
\]

**Inverse Filter**

\[
G_m(z) = \frac{1}{H_m(z)} \iff G_m(e^{j\omega}) = \frac{1}{H_m(e^{j\omega})}.
\]
Experimental Study

Figures of Merit

- Inspection of the estimated RIR and ATF
Experimental Study

Figures of Merit

- Inspection of the estimated RIR and ATF
- Comparison of the input speech signal, the reverberant signal, and the processed signal
Experimental Study
Figures of Merit

- Inspection of the estimated RIR and ATF
- Comparison of the input speech signal, the reverberant signal, and the processed signal
- Normalized Projection Misalignment (NPM)

\[
\text{NPM [dB]} = 20 \log_{10} \left( \frac{1}{\|h\|^2} \left| h - \frac{(h^T \hat{h}) \hat{h}}{\|\hat{h}\|^2} \right|^2 \right) \\
= 20 \log_{10} \left( 1 - \left( \frac{h^T \hat{h}}{\|h\| \|\hat{h}\|} \right)^2 \right)
\]
Full-band Version - Results

NPM vs. SNR

Scenario

\[ M = 2, \quad n_h = 16, \quad \hat{n}_h = 21, \quad F_s = 8000\text{Hz}, \quad T = 0.5\text{s}, \quad \text{Discrete uniform distributed RIR coefficients, 50 “Monte Carlo” trials.} \]
Full-band Version - Results

NPM vs. SNR

Scenario

\( M = 2, n_h = 16, \hat{n}_h = 21, Fs = 8000Hz, T = 0.5s, \) Discrete uniform distributed RIR coefficients, 50 “Monte Carlo” trials.

White Noise Input

<table>
<thead>
<tr>
<th>SNR</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPM</td>
<td>-3.5</td>
<td>-8.6</td>
<td>-16.5</td>
<td>-28.0</td>
<td>-35.0</td>
<td>-44.0</td>
<td>-53</td>
</tr>
</tbody>
</table>

Speech Input

<table>
<thead>
<tr>
<th>SNR</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
<th>55</th>
<th>60</th>
<th>65</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPM</td>
<td>0.0</td>
<td>0.0</td>
<td>-2.0</td>
<td>-10.0</td>
<td>-11.0</td>
<td>-24.5</td>
<td>-38.0</td>
</tr>
</tbody>
</table>
Full-band Version - Results

NPM vs. SNR

**Scenario**

\( M = 2, n_h = 16, \hat{n_h} = 21, Fs = 8000Hz, T = 0.5s, \) Discrete uniform distributed RIR coefficients, 50 “Monte Carlo” trials.

<table>
<thead>
<tr>
<th>White Noise Input</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>SNR</td>
<td>15</td>
<td>20</td>
<td>25</td>
<td>30</td>
<td>35</td>
<td>40</td>
<td>45</td>
</tr>
<tr>
<td>NPM</td>
<td>-3.5</td>
<td>-8.6</td>
<td>-16.5</td>
<td>-28.0</td>
<td>-35.0</td>
<td>-44.0</td>
<td>-53</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Speech Input</th>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>SNR</td>
<td>35</td>
<td>40</td>
<td>45</td>
<td>50</td>
<td>55</td>
<td>60</td>
<td>65</td>
</tr>
<tr>
<td>NPM</td>
<td>0.0</td>
<td>0.0</td>
<td>-2.0</td>
<td>-10.0</td>
<td>-11.0</td>
<td>-24.5</td>
<td>-38.0</td>
</tr>
</tbody>
</table>
Full-band Version - Results

NPM vs. filter order

Scenario

$M = 2$, SNR=50dB, $\hat{n}_h - n_h = 5$, $Fs = 8000$Hz, $T = 0.5$s, Gaussian distributed with decaying envelope RIR coefficients, 50 “Monte Carlo” trials.
Scenario

$M = 2$, SNR$=50$dB, $\hat{n}_h - n_h = 5$, $Fs = 8000$Hz, $T = 0.5$s, Gaussian distributed with decaying envelope RIR coefficients, 50 “Monte Carlo” trials.

White Noise Input

<table>
<thead>
<tr>
<th>$n_h$</th>
<th>16</th>
<th>32</th>
<th>64</th>
<th>128</th>
<th>256</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPM</td>
<td>-60.0</td>
<td>-49.5</td>
<td>-33.0</td>
<td>-18.0</td>
<td>-0.5</td>
</tr>
</tbody>
</table>
Full-band Version - Results

Truncated Simulated RIR

**Scenario**

\[ M = 2, \text{SNR}=50\text{dB}, \hat{n}_h - n_h = 5, F_s = 8000\text{Hz}, T = 0.5\text{s}, T_{60} = 0.7\text{s}, \text{RIR truncated to } n_h = 600. \text{NPM}=-26\text{dB}. \]
Full-band Version - Results

Sonograms

(a) Clean signal

(b) Reverberant signal (500 taps)

(a) Dereverberated signal (MINT)

(b) Dereverberated signal (MBF)

Sharon Gannot
Speech Dereverberation using EVD
Subband Version - Results

Scenario

$M = 2$, $\text{SNR}=120\text{dB}$, $n_h = 24$, 6 bands, $\hat{n}_h^k - n_h^k = 2$ per-band, $T=4000$, Gaussian distributed with decaying envelope RIR coefficients, white noise input, gain ambiguity compensated.
Limitations of the Proposed Methods

- Noise Robustness
Limitations of the Proposed Methods

- Noise Robustness
- Null Subspace
Limitations of the Proposed Methods

- Noise Robustness
  - Null Subspace
  - MINT
Limitations of the Proposed Methods

- Noise Robustness
- Null Subspace
- MINT
- Common Zeros
Limitations of the Proposed Methods

- Noise Robustness
  - Null Subspace
  - MINT
- Common Zeros
  - Room Impulse Responses
Limitations of the Proposed Methods

- Noise Robustness
  - Null Subspace
  - MINT

- Common Zeros
  - Room Impulse Responses
  - Extraneous zeros resulting in from the overestimation
Limitations of the Proposed Methods

- Noise Robustness
  - Null Subspace
  - MINT

- Common Zeros
  - Room Impulse Responses
  - Extraneous zeros resulting in from the overestimation

- The Demand for the Entire RIR Compensation
Limitations of the Proposed Methods

- Noise Robustness
  - Null Subspace
  - MINT

- Common Zeros
  - Room Impulse Responses
  - Extraneous zeros resulting in from the overestimation

- The Demand for the Entire RIR Compensation
  - $\hat{n}_h \geq n_h$
Limitations of the Proposed Methods

- Noise Robustness
  - Null Subspace
  - MINT
- Common Zeros
  - Room Impulse Responses
  - Extraneous zeros resulting in from the overestimation
- The Demand for the Entire RIR Compensation
  - $\hat{n}_h \geq n_h$
- Filter-bank Design
Limitations of the Proposed Methods

- Noise Robustness
  - Null Subspace
  - MINT

- Common Zeros
  - Room Impulse Responses
  - Extraneous zeros resulting in from the overestimation

- The Demand for the Entire RIR Compensation
  - $\hat{n}_h \geq n_h$

- Filter-bank Design
  - Band overlap
Limitations of the Proposed Methods

- **Noise Robustness**
  - Null Subspace
  - MINT

- **Common Zeros**
  - Room Impulse Responses
  - Extraneous zeros resulting in from the overestimation

- **The Demand for the Entire RIR Compensation**
  - \( \hat{n}_h \geq n_h \)

- **Filter-bank Design**
  - Band overlap
  - Band gaps
Limitations of the Proposed Methods

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  - Null Subspace
  - MINT
- Common Zeros
  - Room Impulse Responses
  - Extraneous zeros resulting in from the overestimation
- The Demand for the Entire RIR Compensation
  - $\hat{n}_h \geq n_h$
- Filter-bank Design
  - Band overlap
  - Band gaps
- Gain Ambiguity
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- Noise Robustness
  - Null Subspace
  - MINT
- Common Zeros
  - Room Impulse Responses
  - Extraneous zeros resulting in from the overestimation
- The Demand for the Entire RIR Compensation
  - $\hat{n}_h \geq n_h$
- Filter-bank Design
  - Band overlap
  - Band gaps
- Gain Ambiguity
  - Subband method
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