THE EFFECT OF DIRECTIONAL ANTENNAS ON SLOTTED CSMA AD HOC NETWORKS

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ABSTRACT

The performance of wireless ad hoc network are mostly limited by the self interference caused by the network members. Smart antennas and carrier sensing multiple access (CSMA) have been considered separately as efficient methods for self-interference reduction. In this paper we analyze the effect of directional antennas on the transmission capacity of slotted CSMA networks. The analysis is based on a Poisson point process shot noise model for the aggregate interference. The effect of directional antennas is shown to be equivalent to network density scaling. This scaling is expressed through a single parameter, which depends only on the antenna and the channel exponential decay factor. The paper also presents a useful bound on rate ratio of a pair of slotted CSMA networks. This bound is used to demonstrate the transmission capacity gain of slotted CSMA over slotted ALOHA networks. In particular it is shown that if the antennas have very narrow beams then the advantage of CSMA over ALOHA is negligible.

1. INTRODUCTION

Wireless ad hoc networks (WANET) had attracted a lot of attention in recent years. These networks do not depend on infrastructure and therefore are suitable for a variety of applications. The achievable data rates in WANET are mostly limited by the self interference caused by the network members (e.g., [1]). Several works had shown performance improvement by applying smart antennas or smart access mechanisms.

Carrier sensing multiple access (CSMA) is one of the most popular smart access mechanism. Its performance was studied extensively (e.g., [2] and refs therein), and many variants were considered. In this paper we focus on the slotted CSMA variant, [3], where all users transmission are aligned to predetermined time slots.

Hasan and Andrews [4] studied the effect of slotted CSMA on the transmission capacity (TC) of a WANET applying spread spectrum techniques. They suggested a homogenous Poisson approximation to the interfering users distribution. This approximation was validated by comprehensive simulations and tests. However, in that work, the aggregate interference was modeled as Gaussian random variable which was shown to be realistic only for large spreading factors. Motivated by the above, in this work we use the Poisson Point Process (PPP) to model the position of the interfering users and use precise Poisson shot noise model for the aggregate interference.

The use of directional antennas in slotted ALOHA WANET reduces the interference between users and improves the network capacity (e.g., [5, 6] and refs therein).

However, the effect of directional antennas on the performance of slotted CSMA WANET had not been studied yet.

In this work we extend the result derived in [6] for the TC gain of directional antennas in ALOHA networks to slotted CSMA networks. We also present a bound on the rate ratio of two slotted CSMA networks, and use it to shed some light on the gain of slotted CSMA over slotted ALOHA networks.

The rest of the paper is organized as follows: Section 2 describes the system and interference models; Section 3 gives the performance analysis of the slotted CSMA network with directional antennas; Section 4 gives numerical and simulation results and conclusions are given in section 5.

2. SYSTEM MODEL

We assume a decentralized wireless ad-hoc network utilizing a slotted CSMA media access control (MAC) protocol (e.g., [3]). At each slot, part of the nodes try to transmit data (based on availability of data in the buffer, and on a probabilistic access mechanism). These nodes are termed active transmitters. The distribution of the active transmitters in the plane is modeled by a two-dimensional PPP with density λ_p ¹. Assuming high mobility and/or a large number of nodes in the network, we assume that the active transmitters in one slot are completely independent of the active transmitters in other slots. Each transmission has a specific destination node. Considering the operation of a reasonable routing protocol the distance between each two communicating nodes is approximated as constant, d . The relative angle between transmitter-receiver pairs is distributed uniformly. The power received at receiver j from the transmission of user i is:

$$W_{i,j} = \rho X_{i,j}^{-\alpha} V_{i,j} G_{i,j} \quad (1)$$

where ρ is the node transmission power; $X_{i,j}$ and $G_{i,j}$ are the distance and the antenna gain between the i -th transmitter and the j -th receiver, respectively, and $\alpha > 2$ is the exponential decay path loss; $V_{i,j}$ is an addition channel attenuation between the i -th transmitter and the j -th receiver (e.g., due to fading). The random variable (RV) $V_{i,j}$ is statistically independent of any other channel attenuation variable ($V_{k,\ell}$ for $k \neq i$ or $\ell \neq j$) and of all distance and antenna gain variables

¹Note that λ_p can be tuned to optimize the network performance by adjusting the users transmission probability

($X_{k,\ell}$ and $G_{k,\ell}$ for any k, ℓ). All results presented herein hold for any channel fading (described by the distribution of V).

We assume that all nodes are equipped with identical directional antennas and each communicating transmitter-receiver pair points their antennas toward each other. The directional antenna gain as function of the relative angle θ can be written as (see also [6]):

$$G(\theta) = \begin{cases} 1 & |\theta| \leq \frac{\pi}{M}, \\ \frac{1}{M} & |\theta| > \frac{\pi}{M}. \end{cases} \quad (2)$$

where M is the antenna spatial selectivity, which determines the part of space covered by the antenna main lobe (i.e., the main lobe width is $2\pi/M$). The overall antenna gain between a transmitter and a receiver is the product of the two antennas gain. Note that for all communicating pairs we have a gain $G_{i,i} = 1$. For any other transmitter and receiver, the RV $G_{i,j}$ (with $i \neq j$) is statistically independent of any other gain ($G_{k,\ell}$ for $k \neq i$ or $\ell \neq j$) and from all distance and channel variables ($X_{k,\ell}$ and $V_{k,\ell}$ for any k, ℓ).

The slotted CSMA protocol is based on periodic slots, where each slot consists of an access sub-slot (sometimes termed CAP - contention access period) and data sub-slot (sometimes termed CFP - contention free period). During the access sub-slot each active transmitter waits a random time and then tries to transmit a request-to-send (RTS) message, and to receive a clear-to-send (CTS) message. The RTS-CTS procedure for the k -th pair is termed successful only if it guarantees that the tested pair and all previous successful pairs satisfy:

$$W_{ij} < \delta W_{jj}, \quad \forall i \neq j \in \{\mathcal{S}_k \cup k\} \quad (3)$$

where $\mathcal{S}_k$ is the set of all pairs that were termed successful at the time that transmitter k started to transmit and δ is the allowed interference ratio threshold (note that the special case of $\delta \rightarrow \infty$ represents the slotted ALOHA protocol). For simplicity we assume that the access sub-slot is significantly larger than the duration of the RTS-CTS process, and hence we neglect the effects of RTS-CTS collisions). We also assume that the data sub-slot is significantly larger than the access sub-slot, and hence the access overhead is negligible. At the end of the access sub-slot, all successful transmitters ($\mathcal{S} = \cup \mathcal{S}_k$) start to transmit data all through the data sub-slot (and (3) is satisfied for all $i \neq j \in \mathcal{S}$).

Fig.1 illustrates a sample realization of small area of the CSMA network. In the figure triangle and squares depict the locations of transmitter and receiver respectively, and the curve around each node represents its antenna gain pattern. In the realization illustrated in this figure all pairs were successful except for pair number 1 which was disabled due to its high interference to receiver 0.

In the following we focus on analyzing the achievable data rates in the data sub-slots. We use the shift invariant property of the system, and Slivnyaks theorem [7], to analyze the performance of the network using a probe receiver. Without loss of generality, we assume that the probe receiver is part of the successful set, and for notation simplicity we drop the probe

Fig. 1. Illustration of several types of interferers referring to a probe receiver. Squares mark receivers while the transmitters are numbered and marked by triangles.

receiver index throughout the paper. We also define the set $\bar{\mathcal{S}}$ as the set $\mathcal{S}$ excluding the probe receiver.

As WANETs typically works at the interference limited regime, in the following we neglect the contribution of the thermal noise. The interference to signal ratio (ISR) measured at the probe receiver can be written as:

$$I = \frac{\sum_{i \in \bar{\mathcal{S}}} W_i}{\rho d^{-\alpha} V_0} = \rho^{-1} d^\alpha V_0^{-1} \sum_{i \in \bar{\mathcal{S}}} X_i^{-\alpha} G_i V_i. \quad (4)$$

Without loss of generality, in the following we set $d = 1$ and $\rho = 1$. Denoting the single user rate by R_0 and assuming an efficient communication system, each user will require a SIR of $\beta = e^{R_0} - 1$, and the outage probability is $\Pr(\frac{1}{I} < \beta)$. A useful measure of the network performance is the mean of the sum rate of all successful transmissions defined as:

$$R^M(\lambda_c, \delta, \beta) \triangleq \lambda_c \cdot \Pr\left(I \leq \frac{1}{\beta}\right) \cdot \log(1 + \beta) \quad (5)$$

where λ_c denote the density of the successful transmitters. The maximum sum rate is termed the transmission capacity:

$$T_c^M \triangleq \max_{\beta, (\lambda_c, \delta) \in \mathcal{C}^M} R^M(\lambda_c, \delta, \beta) \quad (6)$$

Where $\mathcal{C}^M$ is the set of all achievable pairs of λ_c and δ given the density of the original Poisson point process, λ_p . The resulting distribution of the successful transmitters is quite complex. In the simplest case of omnidirectional antenna and $d \rightarrow 0$ this distribution is termed Matern type III [8]. This distribution was studied extensively in many contexts, and yet the results are not sufficient to allow a system analysis. Recently, several works have suggested to approximate the distribution of the successful transmitters as a PPP with density λ_c outside of the guard-zone of the probe receiver [4, 9]. In this work, we adopt this approximation, and use it to evaluate

the TC. In the following we will denote the interference received by the probe user in the approximating PPP by $\tilde{I}$, the resulting sum rate by:

$$\tilde{R}^M(\lambda_c, \delta, \beta) \triangleq \lambda_c \cdot \Pr\left(\tilde{I} \leq \frac{1}{\beta}\right) \cdot \log(1 + \beta) \quad (7)$$

and the transmission capacity by:

$$\tilde{T}_c^M \triangleq \max_{\beta, (\lambda_c, \delta) \in \mathcal{C}^M} \tilde{R}^M(\lambda_c, \delta, \beta) \quad (8)$$

In addition to the approximation accuracy demonstrated previously in [4], we also demonstrate this accuracy in the numerical evaluation section below.

2.1. ISR distribution for omnidirectional antenna

The characteristic function of the ISR, (4), for $M = 1$ and $V_i = 1$ was presented in [9]. We give here an alternative formulation, based on the presentation of the ISR as a sample of a two dimensional shot-noise process (see appendix A):

$$\Phi_I(s) = \exp\left(-\pi\lambda_c \int_{\delta^{-2/\alpha}}^{\infty} (1 - e^{-st^{-\alpha/2}})dt\right) \quad (9)$$

3. PERFORMANCE ANALYSIS OF SLOTTED CSMA WITH DIRECTIONAL ANTENNAS

3.1. The effect of Directional Antennas

In this subsection we analyze the effect of directional antennas on the achievable data rates in a WANET utilizing a slotted CSMA protocol. This effect was studied for slotted ALOHA networks [6] and was shown to depend mostly on the antenna factor:

$$\Psi_M \triangleq \frac{M^2}{((M-1)M^{-2/\alpha} + 1)^2}. \quad (10)$$

Notice that $\Psi_1 = 1$ and for very narrow main-lobe-width $\Psi_M \approx M^{\frac{4}{\alpha}}$. In this work we extend this result to the slotted CSMA case:

Theorem 3.1. *The sum rate gain of a slotted CSMA WANET with a directional antenna is:*

$$\tilde{R}^M(\lambda_c, \delta, \beta) = \Psi_M \cdot \tilde{R}^1\left(\frac{\lambda_c}{\Psi_M}, \delta, \beta\right) \quad (11)$$

In addition, a trivial corollary of theorem 3.1 is that if $(\lambda_c \Psi_M, \delta) \in \mathcal{C}^M$ and $(\lambda_c, \delta) \in \mathcal{C}^M$ then the ratio of the TC of the two networks is:

$$\frac{\tilde{T}_c^M}{\tilde{T}_c^1} = \Psi_M \quad (12)$$

Note however that the CSMA density cannot increase without limit. This is because any practical system has a finite number of users, and hence a maximal density. We'll denote hereafter this maximal density by $\lambda_{\max}$. Once this maximal density is reached the TC gain due to the use of directional antennas will be smaller than Ψ_M .

Proof of Theorem 3.1. The proof of the theorem has some resemblance to the proof in [6], and the major difference is the need to address the guard zones created by the CSMA protocol.

The RV of the antenna gain, G_i , can have one of 4 values g_k with probability p_k . The different values are function of the side of the antennas at the transmitter and the receiver that are directed toward each other (ML-main lobe or SL-side lobe). The values and their probabilities are detailed in Table 1. This types of interferers are also demonstrated in Fig. 1, where transmitters 2,3 and 4 are interferers to receiver 0 of types ML-ML, SL-SL and ML-SL respectively.

Table 1. Interferers properties

Index (k)	Type	p_k	g_k
1	SL-SL	$(1 - \frac{1}{M})^2$	$\frac{1}{M^2}$
2	SL-ML	$(\frac{1}{M} - \frac{1}{M^2})$	$\frac{1}{M}$
3	ML-SL	$(\frac{1}{M} - \frac{1}{M^2})$	$\frac{1}{M}$
4	ML-ML	$\frac{1}{M^2}$	1

In the CSMA process a receiver disable transmitters that satisfy $X_i^{-\alpha} Y_i \geq \delta$, where $Y_i \triangleq G_i V_i V_0^{-1}$. Thus, for all successful pairs, for each active transmitter $X_i \geq (\frac{Y_i}{\delta})^{1/\alpha}$ and by adjusting (9) to the random start point, the characteristic function of the aggregate interference is:

$$\begin{aligned} \Phi_I(s) &= \exp\left(-\lambda_c E_Y \int_{(\frac{Y}{\delta})^{1/\alpha}}^{\infty} (1 - e^{-sYt^{-\alpha}}) 2\pi t dt\right) \\ &= \exp\left(-\lambda_c E_Y Y^{2/\alpha} \int_{\delta^{-1/\alpha}}^{\infty} (1 - e^{-sr^{-\alpha}}) 2\pi r dr\right) \\ &= \exp\left(-\frac{\lambda_c}{\Psi_M} \Phi \int_{\delta^{-1/\alpha}}^{\infty} (1 - e^{-sr^{-\alpha}}) 2\pi r dr\right) \end{aligned} \quad (13)$$

where the second line results from the change of variables $r = tY^{-1/\alpha}$, and the last line by substituting $\Phi \triangleq E[V^{2/\alpha}]E[V^{-2/\alpha}]$ and $\Psi_M^{-1} = E[G^{2/\alpha}] = \sum_{k=1}^4 p_k g_k^{2/\alpha} = ((M-1)M^{-2/\alpha} + 1)^2 M^{-2}$. From (13) we observe that a network with directional antennas can achieve the same outage probability, $\Pr(I \leq \frac{1}{\beta})$, as with omnidirectional antenna if the transmitters density is multiplied by Ψ_M . Using this conclusion and using (5) completes the proof of the theorem. $\square$

3.2. Slotted CSMA Sum-Rate Upper Bound

In this section we present a bound on the ratio of the sum-rates of two Slotted CSMA systems with identical SIR threshold operating over the same channel. Denote the parameters of the first networks by $M_I, \lambda_{c,I}, \delta_I, \beta$ and the parameters of the second network by $M_{II}, \lambda_{c,II}, \delta_{II}, \beta$, the bound applies to networks that satisfy a specific ratio of the interferes density and is given by:

Theorem 3.2. For two networks that satisfy $\delta_I \geq \delta_{II} \geq 0$ the sum rate satisfies:

$$\frac{\tilde{R}^{M_{II}}(\lambda\Psi_{M_{II}}, \delta_{II}, \beta)}{\tilde{R}^{M_I}(\lambda\Psi_{M_I}, \delta_I, \beta)} \leq \frac{\Psi_{M_{II}}}{\Psi_{M_I}} e^{\pi\lambda\Phi(\delta_{II}^{-\frac{2}{\alpha}} - \delta_I^{-\frac{2}{\alpha}})}, \quad (14)$$

with equality whenever $\beta \geq \frac{1}{\delta_{II}}$. Note that $\Phi \triangleq E[V^{2/\alpha}]E[V^{-2/\alpha}]$ as defined in subsection 3.1.

In particular, theorem 3.2 can be used to upper bound the sum-rate gain of slotted CSMA over slotted ALOHA. Taking the limit $\delta_I \rightarrow \infty$ we set the first network to be slotted ALOHA, and we have:

$$\frac{\tilde{R}^{M_{II}}(\lambda\Psi_{M_{II}}, \delta_{II}, \beta)}{\tilde{R}^{M_I}(\lambda\Psi_{M_I}, \infty, \beta)} \leq \frac{\Psi_{M_{II}}}{\Psi_{M_I}} e^{\pi\lambda\Phi\delta_{II}^{-\frac{2}{\alpha}}}. \quad (15)$$

Further more, if we consider a slotted CSMA network and a slotted ALOHA network with identical antennas, and take into account the maximal slotted CSMA density we will have:

$$\frac{\tilde{R}^M(\lambda_c, \delta, \beta)}{\tilde{R}^M(\lambda_c, \infty, \beta)} \leq e^{\pi(\lambda_c/\Psi_M)\Phi\delta_{II}^{-\frac{2}{\alpha}}} \leq e^{\pi(\lambda_{\max}/\Psi_M)\Phi\delta^{-\frac{2}{\alpha}}}. \quad (16)$$

Thus we deduce that for very high antenna selectivity ($M \rightarrow \infty$, $\Psi_M \rightarrow \infty$) the advantage of slotted CSMA over slotted ALOHA becomes negligible. Taking into account the significant CSMA overhead we conclude that networks with very selective antennas can achieve higher TC by employing slotted ALOHA instead of slotted CSMA.

Proof of Theorem 3.2. We first apply Theorem 3.1 and we have:

$$\frac{\tilde{R}^{M_{II}}(\lambda\Psi_{M_{II}}, \delta_{II}, \beta)}{\tilde{R}^{M_I}(\lambda\Psi_{M_I}, \delta_I, \beta)} = \frac{\Psi_{M_{II}}}{\Psi_{M_I}} \frac{\tilde{R}^1(\lambda, \delta_{II}, \beta)}{\tilde{R}^1(\lambda, \delta_I, \beta)}. \quad (17)$$

Therefore, for the rest of the proof we consider two networks with omnidirectional antennas and identical densities. Furthermore, from (13) we can see that the effect of the fading can be compensated by dividing the density with the constant Φ . Denote by I_I and I_{II} the interference received by the probe receiver in two equivalent networks with density λ operating over a channel without fading. Using (5) and (11) we can write:

$$\frac{\tilde{R}^1(\lambda, \delta_{II}, \beta)}{\tilde{R}^1(\lambda, \delta_I, \beta)} = \frac{\Pr(I_{II} \leq \frac{1}{\beta})}{\Pr(I_I \leq \frac{1}{\beta})} \quad (18)$$

For a one-dimensional Poisson shot-noise with density λ and a guard threshold c , the PDF is described at [10],[11] by the following integral equation:

$$f_I(\zeta) = \frac{\lambda}{\alpha\zeta} \int_0^{\min(\zeta, c)} f_I(\zeta - u) u^{-1/\alpha} du \quad (19)$$

By using the equivalency, (24), we can deduce the following equation for the two-dimensional omnidirectional Poisson process and guard threshold of δ :

$$f_I(\zeta) = \frac{2\pi\lambda}{\alpha\zeta} \int_0^{\min(\zeta, \delta)} f_I(\zeta - u) u^{-2/\alpha} du \quad (20)$$

Note that this equation evaluates the pdf at ζ_I only as a function of the PDF at $0 \leq \zeta < \zeta_I$

We next divide the proof into two cases. We first discuss the case in which Equation (14) is achieved equality, and then extend the result to an inequality.

I. The equality case: $\beta \geq \frac{1}{\delta_{II}} (\geq \frac{1}{\delta_I})$

In this case, for both systems the pdf at the range $0 \leq \zeta \leq \beta$ does not depend on the value of δ . Therefore, at this region, the two PDFs will be identical up to a multiplication constant (scaling factor). We calculate this scaling factor at the point $\zeta = 0$ using the Laplace initial value theorem, $f(0^+) = \lim_{s \rightarrow \infty} sF(s)$, and by noting that the characteristic function is the Laplace transform of the PDF, which is the derivative of the CDF, on which we focus here:

$$\begin{aligned} \frac{\Pr(I_{II} = 0)}{\Pr(I_I = 0)} &= \frac{\lim_{s \rightarrow \infty} \Phi_{I_{II}}(s)}{\lim_{s \rightarrow \infty} \Phi_{I_I}(s)} \\ &= \frac{\lim_{s \rightarrow \infty} \exp\left(-\lambda \int_{\delta_{II}^{-1/\alpha}}^{\infty} (1 - e^{-sr^{-\alpha}}) 2\pi r dr\right)}{\lim_{s \rightarrow \infty} \exp\left(-\lambda \int_{\delta_I^{-1/\alpha}}^{\infty} (1 - e^{-sr^{-\alpha}}) 2\pi r dr\right)} \\ &= \lim_{s \rightarrow \infty} \exp\left(-\lambda \int_{\delta_{II}^{-1/\alpha}}^{\delta_I^{-1/\alpha}} (1 - e^{-sr^{-\alpha}}) 2\pi r dr\right) \\ &= \exp\left(-\lambda \int_{\delta_{II}^{-1/\alpha}}^{\delta_I^{-1/\alpha}} 2\pi r dr\right) \\ &= e^{\pi\lambda(\delta_{II}^{-2/\alpha} - \delta_I^{-2/\alpha})} \end{aligned}$$

where the second line used (23).

II. The inequality case: $\beta \leq \frac{1}{\delta_{II}}$

We can split the integral in (20) for network I into two intervals:

$$\begin{aligned} f_{II}(\zeta) &= \frac{2\pi\lambda}{\alpha\zeta} \int_0^{\min(\zeta, \delta_I)} f_{II}(\zeta - u) u^{-2/\alpha} du \\ &= \frac{2\pi\lambda}{\alpha\zeta} \int_0^{\delta_{II}} f_{II}(\zeta - u) u^{-2/\alpha} du \\ &\quad + \frac{2\pi\lambda}{\alpha\zeta} \int_{\delta_{II}}^{\min(\zeta, \delta_I)} f_{II}(\zeta - u) u^{-2/\alpha} du \\ &= f_{II}(\zeta) e^{\pi\lambda(\delta_{II}^{-\frac{2}{\alpha}} - \delta_I^{-\frac{2}{\alpha}})} \\ &\quad + \frac{2\pi\lambda}{\alpha\zeta} \int_{\delta_{II}}^{\min(\zeta, \delta_I)} f_{II}(\zeta - u) u^{-2/\alpha} du \quad (21) \end{aligned}$$

The expression in the last line is always positive and therefore the ratio $\frac{\Pr(I_{II} \leq \frac{1}{\beta})}{\Pr(I_I \leq \frac{1}{\beta})}$ decreases for $I > \delta_{II}$. $\square$

4. NUMERICAL RESULTS

In this section we demonstrate the accuracy of the PPP approximation for the interference of slotted CSMA WANET,

and the results obtained in subsection 3.1 using simulations. The slotted CSMA simulations were performed for different antennas main-lobe-width. On the other hand, the PPP simulation was performed only for the omnidirectional antenna case and the sum-rates for $M > 1$ were calculated by multiplication of the sum rate with the appropriate Ψ_M . Fig. 2 illustrates the sum-rate of the network as function of the single user rate for the case $V_i = 1$, $\alpha = 4$, $\delta = 0.2$ and various antenna width. The final density in this simulation was $\lambda_c = 0.08$. The actual density of the original Poisson point process was $\lambda_p = 0.13, 0.35, 0.75, 1.65$ for $M = 1, 4, 8, 16$ respectively. It can be noticed that PPP approximation of the network sum-rates for the slotted CSMA are very accurate and that the antenna factor Ψ_M can give a good insight on the directional antenna gain in slotted CSMA WANET.

Fig. 2. Semilogarithmic graph of the sum rate as function of single user rate for slotted CSMA and PPP approximation with parameters $V_i = 1$, $\alpha = 4$, $\delta = 0.2$, $\lambda_c = 0.08$

5. CONCLUSIONS

We studied the transmission capacity of a slotted CSMA WANET with directional antennas. The analysis was performed by using a PPP approximation of the interferers which was demonstrated to be very accurate.

The directional antennas effect was shown to be equivalent to a scaling factor of the network density. This scaling was expressed through a single parameter, which depends only on the antenna and the channel exponential decay factor. We also presented a bound on the ratio of the transmission capacity of slotted CSMA networks with different guard-zones. One application of this bound gives an upper bound on the advantage of slotted CSMA over slotted ALOHA networks, and shows that the CSMA gain becomes negligible as the antennas spatial selectivity goes to infinity (i.e., when the antennas have very narrow beams).

Appendix A

The characteristic function of a one-dimensional shot noise generated by a decaying power-law impulse response func-

tion with a limited range $t \in [c, \infty]$ is given by [11]:

$$\Phi_I(s) = \exp \left(-\lambda' \int_c^\infty (1 - e^{-st^{-\alpha'}}) dt \right). \quad (22)$$

Taking into account the radial symmetry, the characteristic function for the two dimensional case is:

$$\Phi_I(s) = \exp \left(-\lambda_c \int_a^\infty (1 - e^{-sr^{-\alpha}}) 2\pi r dr \right) \quad (23)$$

Using a variable substitution $t = r^2$ results in (9). By comparing (22) and (9) we can observe the following variable replacement between the one and two dimensional shot noises:

$$c \leftrightarrow a^2, \pi\lambda_c \leftrightarrow \lambda', \frac{\alpha}{2} \leftrightarrow \alpha' \quad (24)$$

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