LONG MICROWAVE-PHOTONIC VARIABLE DELAY OF CHIRPED WAVEFORMS

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THIS RESEARCH THESIS WAS CARRIED OUT UNDER THE SUPERVISION OF DR. AVI ZADOK FROM THE FACULTY OF ENGINEERING AT BAR-ILAN UNIVERSITY
Two years ago, when I started working on my Master’s degree under the supervision of Dr. Avi Zadok, I didn’t really know what the future holds for me. I knew what it means to study for a test or how to wire an electrical circuit, but I didn’t really have a clue what it means to be a researcher.

The first few months of working in the lab posed me with many questions: how to work with the optical equipment, what is the difference between the black coated fiber and the green coated one, and what does that black thing with the red handle do (today I know that it is used to clean fibers...). There were also many frustrating moments, when nothing seemed to work as expected, although minutes earlier everything was working perfectly.

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Glossary

AA  Anti-Aliasing
AFG  Arbitrary Function Generator
AM  Amplitude Modulator
AOM  Acousto-Optic Modulator
AWG  Arbitrary Waveform Generator
AWGN  Additive White Gaussian Noise
CW  Continuous Wave
DC  Direct Current
DSP  Digital Signal Processing
EDFA  Erbium-Doped Fiber Amplifier
FBG  Fiber Bragg Grating
FM  Frequency Modulation
FWHM  Full-Width Half-Maximum
HPF  High-Pass Filter
IF  Intermediate Frequency
ISLR  Integrated Side-Lobe Ratio (also referenced as ISL)
LCFBG  Linearly-Chirped Fiber Bragg Grating
LFM  Linear Frequency Modulation
LiNbO3  Lithium Niobate
LO  Local Oscillator
<table>
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<th>Description</th>
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<tr>
<td>LPF</td>
<td>Low-Pass Filter</td>
</tr>
<tr>
<td>MWP</td>
<td>Microwave Photonics</td>
</tr>
<tr>
<td>MZI</td>
<td>Mach-Zehnder Interferometer</td>
</tr>
<tr>
<td>NLFM</td>
<td>Non-Linear Frequency Modulation</td>
</tr>
<tr>
<td>ODL</td>
<td>Optical Delay Line</td>
</tr>
<tr>
<td>PAA</td>
<td>Phased-Array Antennas</td>
</tr>
<tr>
<td>PM</td>
<td>Phase Modulator</td>
</tr>
<tr>
<td>PSLR</td>
<td>Peak-to-Side-Lobe Ratio (also referenced as PSL)</td>
</tr>
<tr>
<td>RF</td>
<td>Radio Frequency</td>
</tr>
<tr>
<td>SBS</td>
<td>Stimulated Brillouin Scattering</td>
</tr>
<tr>
<td>SC-SSB</td>
<td>Suppressed-Carrier Single Sideband</td>
</tr>
<tr>
<td>SLM</td>
<td>Spatial Light Modulator</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal-to-Noise Ratio</td>
</tr>
<tr>
<td>SOA</td>
<td>Semiconductor Optical Amplifier</td>
</tr>
<tr>
<td>SSB</td>
<td>Single Sideband</td>
</tr>
<tr>
<td>TTD</td>
<td>True Time Delay</td>
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Abstract

The field of *microwave photonics* (MWP) has been a prolific area of research for over thirty years. MWP processing provides several potential advantages over electrical domain techniques: low propagation loss in optical fibers; ultra-broad transmission bandwidth; immunity to electro-magnetic interference; availability of high-bandwidth electro-optic modulators and detectors; potential for light-weight, small-footprint modules etc.

One promising potential application of MWP, which is being investigated for over twenty years, is the optical implementation of *radio-frequency* (RF) delay lines. Such delay lines are critical components in beam steering within phased-array radar systems. When broadband radar signals are used, variable group delay elements, rather than phase delays, are necessary to avoid angular dispersion of the beam. Such *true time delay* (TTD) elements must accommodate broadband RF waveforms and comply with stringent distortion requirements. Large antenna arrays require delay variations of tens of ns or longer. The variable TTD of high-frequency waveforms using RF cables tends to be lossy and bulky, rendering potential MWP alternatives appealing. Numerous methods for the realization of MWP TTD elements have been proposed and demonstrated. Despite considerable progress, the realization of MWP TTDs that are continuously variable over tens of ns and accommodate broadband waveforms remains challenging.

In this work I examine the potential of a significant relaxation in the objectives of the delay element: rather than provide an 'honest', universal group delay of every
incoming signal, I focus on the processing of specific waveforms which are of particular interest to actual radar systems. While the proposed processing is not strictly a variable delay of the waveforms, I show through simulations and experiment that it is has a nearly equivalent effect on this family of signals. In compromising the universality of the delay approach, I was able to obtain delays that are up to 100 times longer than what was previously reported, while retaining sufficient quality of the processed waveform.

The signals I worked with are collectively referred to as chirped waveforms. These waveforms sweep across a broad spectral bandwidth through long intervals of several µs. Chirped waveforms are processed at the receiver via a matched filter, to provide a compressed form known as their impulse response. The impulse response is characterized by a narrow, ns-scale-long correlation peak with low sidelobes. Furthermore, chirped waveforms have a constant magnitude, which is a very important characteristic for radar systems, and they can be designed to fit certain specifications.

The simplest chirped waveform is the linear frequency modulated (LFM) signal, in which the instantaneous frequency is swept at a constant rate. While comparatively simple to generate and process, its impulse response is characterized by relatively strong sidelobes. All the other chirped waveforms may be regarded as nonlinear frequency modulated (NLFM) signals. While it is more complex to generate NLFM waveforms due to their unique phase profiles, they provide several advantages over LFM waveforms: they can be tailored to work with certain system specifications or existing components, it is possible to shape their spectrum, and they have better sidelobe suppression without the need for digital windowing.
The principle of delaying chirped waveforms is based on their underlying time-frequency (or time-phase) relation. The chirped waveform is first modulated onto an optical carrier. Then, one sideband and the carrier are filtered out, and a phase-correction term is then applied to the remaining sideband. Afterwards, the optical carrier is reintroduced, and the processed sideband interferes with the optical carrier to reconstruct a modified chirped waveform upon detection. The effect of the phase correction on the impulse response function of the modified chirped waveform is nearly equivalent to its delay.

Simulations suggest that the delay of the impulse response of chirped waveforms according to the principle outlined above is possible. In the experiment, I was able to delay the impulse response of LFM waveforms by up to ±50 ns with adequate sidelobe suppression. Furthermore, I was able to delay the impulse response of 4th-order NLFM waveforms by up to 50 ns, and 8th and 16th-order NLFM waveforms by up to 20 ns. To the best of my knowledge, MWP processing of NLFM waveforms was not reported to-date.

Future research will extend this work to support chirped waveforms whose amplitude is varying in time, and incorporate the experimental setup within a phased-array antenna.
1 Introduction

The processing of analog radio-frequency (RF) and microwave signals using photonic means, known as RF-photonics or microwave-photonics (MWP) [1], has been a prolific area of research for over thirty years [1] [2]. In MWP systems, the electrical signal of interest is used to modulate an optical carrier wave. The modulation side-band (or side-bands) is modified in transmission through optical fibers and devices, and a modified electrical signal is reconstructed through the beating between the side-band and carrier upon detection. The entire MWP process is therefore equivalent to a direct RF filtering. MWP processing provides several unique advantages [1]: low propagation loss in optical fibers, that is independent of the microwave frequency; an ultra-broad transmission bandwidth of several THz; an inherent immunity to electro-magnetic interference; the availability of high-bandwidth modulators and detectors developed for optical communication; the potential for parallel processing through the use of multiple optical carriers; potential for light-weight, small-footprint modules etc. Considering all these advantages, MWP can be used to either perform tasks that are too complex to be carried out in the RF domain, or provide an improved optical version of an already operational RF system.

Many potential applications of MWP appear in the research literature. One conceptually simple application, which is already deployed in analog links for the defense sector [1], is that of antenna remoting: the use of fiber-optic transmission for the physical separation between an end unit of a radar system, for example, and a central office. Optical fibers can replace bulky and lossy RF cables, and provide
Introduction

Optical Delay Lines

2 propagation distances of tens of km without amplification. Another commonly discussed application is that of MWP filters, implementing both finite-impulse-response and infinite-impulse-response RF transfer functions in the optical media \[3\]. Other applications include the photonic generation of arbitrary and ultra-wideband RF signals \[4,5\]; the photonic implementation of advanced RF modulation formats \[6\], and analog-to-digital conversion using photonic devices \[7\].

1.1 Optical Delay Lines

1.1.1 Motivation: Phased-Array Antennas

One of the most interesting and promising potential applications of MWP, which is being investigated for over twenty years \[3,8\], is the optical implementation of RF delay lines. Such delay lines are critical components in radar systems based on phased-array antennas (PAAs), in which the spatial beam steering relies on the proper interference between the radiation patterns from individual antenna elements. The principle of beam-steering in phased-array antennas is illustrated in Figure 1. Each individual antenna within the array radiates isotropically. When all the signals feeding the antennas are in-phase, the intensity peak of the interference pattern is centered at the middle of the transmitted beam. When the signal feeding each antenna element is slightly phase-shifted with respect to the neighboring one, the interference pattern is reconstructed in the direction of the incrementing phases, allowing for the effective tilt of the beam. Phased-array antennas are far superior to previous, mechanically-rotating ones, in terms of scanning rates and reliability \[9,10\]. Incremental phase delays are
sufficient for the steering of beams of a single frequency or a narrow bandwidth. When broadband radar signals are used, variable group delay elements, rather than phase delays, are necessary to avoid spatial dispersion of the beam (known as ‘beam squinting’ [11]). These elements, often referred to as true time delays (TTDs), must accommodate broadband RF waveforms and meet the stringent distortion requirements of analog processing [12].

![Diagram of phased-array antennas](image)

**Figure 1:** An example for phased-array antennas. The steering of the beam is achieved by differential phase shifts between the array elements.

### 1.1.2 Implementations of Optical Delay Lines

TTDs can be implemented, obviously, through electrical switching among different cable paths [13,14]. However, such elements are restricted to a discrete set of delays, tend to be bulky and lossy, and do not easily scale to high frequencies. One
alternative, which is being rapidly adapted as technology progresses, relies on high-rate digital signal processing (DSP) for generating delayed replicas of an input waveform. However, even today, state-of-the-art analog to digital converters and DSP elements struggle to reach tens-of-GHz rates. Therefore, optical delay lines (ODLs) based on MWP methods provide an attractive alternative.

The simplest form of an ODL can be constructed by a switching matrix between fiber sections of different lengths. In a series of works by Tur and coauthors [15,16,17], discrete ODLs were realized through wavelength-selective switching among different paths. The quality of the delayed waveforms, in terms of distortion etc., was excellent. However, the precise path lengths are difficult to control and delay values cannot be varied continuously. Therefore, several, more elaborate methods were proposed in order to provide a continuously variable TTD, some of which will be further discussed below.

1.1.2.1 FBG based Optical Delay Lines

One such method involves using a linearly-chirped fiber Bragg grating (LCFBG). In a LCFBG the periodicity of the refractive index perturbation is position-dependent, as opposed to a standard fiber Bragg grating (FBG) in which the perturbation is periodic. This way, the Bragg reflectivity conditions for different incoming wavelengths are met at different points along the LCFBG. Using this property, LCFBGs can be used to obtain TTD: an RF signal is modulated onto a continuously-tunable laser source, and then transmitted through the LCFBG. Since the distance the light travels before it bounces back depends on its wavelength, the signal undergoes a wavelength dependent delay,
which can be translated into TTD. The required laser tuning range can be very narrow if a high-dispersion chirped FBG is used. Back in 1997, this concept was demonstrated by Cruz et al. [18], as they used it to achieve time delay variations up to 556 ps.

Regular, periodic FBGs may also be used to obtain TTD. To that end, a tunable laser source modulated by an RF signal is divided into a group of $N$ single-mode fibers, each of which includes a spatially distributed array of FBGs. Similar to the LCFBG method, different wavelengths are being reflected from the FBGs, such that the reflected light is delayed in accordance with the channel addressed [19]. Time delays of up to 145 ps were demonstrated by Molony et al. [20] using that principle.

1.1.2.2 Fiber Prisms

In many cases an ODL cannot provide the range of delay that is necessary to support an entire array. In the work of Vidal et al. [21], several discrete ODLs were used to provide delay for a sub-array, whereas a spatial light modulator (SLM) provided finer delay to individual elements. The maximum delay achieved using this method was 100 ps.

Another approach combines a single wavelength tunable laser with an array of fixed length fibers, comprising both highly-dispersive and non-dispersive media. The first fiber is composed only from non-dispersive material and the last one is composed only from a highly-dispersive material, while each fiber contains a slightly longer segment of the high-dispersion medium, with respect to the one above it. The entire construction forms a fiber optical prism. At the central wavelength, the main antenna beam is directed broadside. The shorter the wavelength, the shorter the time delay each fiber
adds to the signal, and the more steered the beam is towards the first, non-dispersive fiber side. In initial experiments, a relative tuning delay of 520 ps was obtained [22].

1.1.2.3 Slow Light

Slow light, the phenomena in which the group velocity of light pulses within optical media becomes slower than its expected value subject to standard propagation conditions, can also be used to provide TTD. In several works, slow light was generated using stimulated Brillouin scattering (SBS). This technique can achieve delays in the order of tens of nano-seconds [23]; however, its usable bandwidth is restricted to only 30 MHz, the SBS bandwidth [23,24]. Several studies demonstrated SBS bandwidth broadening up to 10 GHz [25,26,27], though the product of attainable delay times bandwidth remains limited to the order of unity. Another drawback of this approach lies in the SBS polarization dependence, which might cause pulse distortion [23,28].

TTD was also demonstrated using semiconductor optical amplifier (SOA) in several works over the last few years [29,30]. The propagation of dual-sideband modulated waveforms in an SOA led to partial fading, resulting in a modest slow-light-induced phase shifts of only 20°. Blocking one sideband produces a phase shift of 150° over a large RF bandwidth, reaching up to 40 GHz. This phase shift corresponded, subject to the conditions of the experiment, to a TTD of several hundred pico-seconds.

1.1.2.4 Hybrid Analog-Digital Optical Delay Lines

Another approach towards the realization of a variable fiber-optic delay line (VFODL) was proposed by Nabeel A. Riza et al. in [31]. This approach is unique for its use of both analog and digital delay lines (Hybrid VFODL). This hybrid combination solves
earlier resolution-range dilemmas as discrete fiber paths are used for long-time-delays, while LCFBGs provide near-continuous high-resolution delays, down to 0.5 ps, between the discrete delays of the switched paths. Using hybrid VFODL, TTDs of up to 25.6 ns were achieved.

1.2 Optical Delay Lines for Frequency Modulated Waveforms

As illustrated by the above examples, long variable time delays of broadband signals are still difficult to achieve. Moreover, most of the systems discussed so far are complicated to implement. In this work I explore the potential benefits of a significant relaxation in the objective of MWP delay elements. Rather than target the universal TTD of any waveform, I focus on the processing of a particular group of signals, that is prevalent in many radar systems: the linear frequency modulated (LFM) and non-linear frequency modulated (NLFM) waveforms. As their names suggest, the instantaneous frequency of these signals is time dependent (linear or non-linear dependence for the LFM and NLFM signals, respectively), spanning a broad bandwidth over relatively long durations. These categories of signals are often referred to collectively as chirped waveforms. Chirped waveforms are of constant magnitude, and they circumvent the transmission of short, high-peak power pulses, whose generation is more difficult, for practical considerations [32].

LFM signals are simple to generate and process, and with proper post-detection processing they can be compressed into an impulse response function which is characterized by a high spatial resolution and low side-lobes. To further reduce the impulse response side-lobes, NLFM signals were introduced. These signals are
characterized by a changing rate of frequency sweep, rather than a constant one, as in LFM signals. The freedom to determine the frequency sweep provides a way to shape the impulse response and get lower side-lobes than those achieved with LFM signals. Both types of waveforms will be addressed in much detail in the following chapters.

Few previous works addressed the microwave-photonic true-time-delay of LFM waveforms. For example, LFM waveforms were used in the aforementioned examples of discrete optical delay lines [15,16,17]. The continuously-variable true time delay of LFM waveforms was reported based on stimulated Brillouin scattering slow light [33], however the obtained delay was restricted to 230 ps. To the best of my knowledge, the microwave photonic processing of NLFM waveforms is not reported to-date. In sacrificing the universality of the delay approach, I was able to extend the range of chirped waveform ODL by more than a hundred-fold.

In the next chapters I will explain the underlying theory behind the long delay of chirped waveforms; I will then show and discuss simulation of the proposed method and proceed to the experimental setup and results. Finally, I will bring up some ideas for improvement in the results, and some suggestions for possible future research directions.

The method proposed in this work relies on phase and frequency modulation of an optical sideband. Before proceeding to the delay principle, in the next two sections I will briefly introduce the devices that were used for the implementation of such modulation: the LiNbO₃ electro-optic phase modulator and the acousto-optic frequency shifter.
1.3 LiNbO3 Phase and Amplitude Modulators

LiNbO3 is a uniaxial material, meaning that its refractive index is the same for two directions of the optical-frequency electric field, and different for the third. In most modulators, a waveguide is made within the LiNbO3 crystal. Light is allowed to travel along the x-axis of the crystal, with the optical-frequency electric field pointing in the z direction. A RF-domain electrical field is also applied along the z-axis. The applied electrical voltage \( V \) changes the refractive index along the z-axis as seen by the optical wave. Denoting the refractive index along the z-axis by \( n_e \) (the so-called extra-ordinary index), than the change in the refractive index is given by [34]:

\[
\Delta n = -\frac{1}{2} n_e^3 r_{33} \frac{V}{d}
\]

Here \( d \) is the distance between the electrodes surrounding the crystal is the z direction, and \( r_{33} \) is an element of the electro-optic tensor of LiNbO3 which equals \( r_{33} = 30.9 \cdot 10^{-12} \text{[m/V]} \). The change in refractive index modifies the phase of the propagating optical wave. The change of the phase in a waveguide of length \( l \) is given by:

\[
\Delta \varphi = \frac{2\pi}{\lambda} \Delta n \cdot l = -\pi \frac{V}{V_\pi}
\]

where \( V_\pi \) is the electrical voltage required to introduce a phase shift of \( \pi \) to the optical wave, and it is defined as:
\[ V_\pi = \frac{\lambda d}{n^3 r_{33} l} \] (3)

Phase modulators (PM) are built by a single LiNbO\(_3\) waveguide. Many amplitude modulators are based on the Mach-Zehnder Interferometer (MZI) (see Figure 2), in which incoming light is split into two paths. Each path includes a LiNbO\(_3\) based phase modulator that is driven by a different voltage, meaning that the light in the two paths experience two different phase shifts. Later, the two paths are combined causing the waves from the two paths to interfere with each other.

![Figure 2: MZI LiNbO\(_3\) Intensity Modulator](image)

Let us denote the incoming optical field as \( E_{in} \). Then the field at the output of the amplitude modulator will be:

\[ E_{out} = \frac{1}{2} E_{in} (e^{j\Delta \phi_1} + e^{j\Delta \phi_2}) = \frac{1}{2} E_{in} e^{j\Delta \phi_+} (e^{j\Delta \phi_+} + e^{-j\Delta \phi_-}) \] (4)

Where \( \Delta \phi_1 \), \( \Delta \phi_2 \) are the phase added by the two arms of the MZI, \( \Delta \phi_+ = \frac{1}{2} (\Delta \phi_1 + \Delta \phi_2) \) and \( \Delta \phi_- = \frac{1}{2} (\Delta \phi_1 - \Delta \phi_2) \). Using (4), the output is:

\[ P_{out} = \frac{P_{in}}{2} \left[ 1 + \cos \left( \frac{V_1 - V_2}{V_\pi} \right) \right] \] (5)
The result shows that indeed the above mentioned architecture provides intensity modulation of the incoming light.

1.4 Acousto-Optic Modulators

Acoustic waves passing through a dielectric medium modify its dielectric constant based on the effect of elasto-optic [34]. The change in the dielectric constant forms an effective grating, whose spatial period is the acoustic wavelength. In an Acousto-Optic Modulator (AOM), as shown in Figure 3, a piezo-electric transducer is used to drive an ultra-sonic acoustic wave through an optical medium (RF Input). Part of the power of an incident optical beam is deflected by the acoustic wave, and collected into an output fiber port [34]. The deflected beam is offset in frequency by \( f_{\text{AOM}} \), the RF input frequency.

![Figure 3: Acousto-optic Modulation](image)

The AOM is designed to operate at a nominal drive frequency of \( f_{\text{AOM},0} \). Since the angle of deflection varies with the driving frequency [34], the usable range of frequency offsets \( \Delta f \) that is attainable in an AOM is limited to deviations of only a few
MHz from $f_{AOM,0}$. In the processing of LFM waveforms, we consider the timing of the impulse response that is obtained using $f_{AOM,0}$ as our zero delay reference. Relative delay or advancement of the impulse response are obtained through deviations of the driving frequency: $\Delta f = f_{AOM} - f_{AOM,0}$. The range of frequency supported by the AOM is sufficient to delay or advance the impulse response of LFM waveforms by tens of ns, as discussed later.
2 Theory

2.1 Overview

The MWP TTD method proposed here takes advantage of the one-to-one mapping between time $t$ and instantaneous frequency $f(t)$ that is underlying both LFM and NLFM waveforms. Consider for example LFM signals, for which $f(t)$ is a linear function of time. Due to this dependence, the application of a constant frequency shift, or in other words a correction to the instantaneous phase that is linearly varying with time, to an infinitely long LFM waveform is equivalent to delaying it. Even though in practice the waveform duration is limited, and the aforementioned equivalency is not absolute, I will later show that for sufficiently long durations of LFM pulses, a time delay and a frequency shift have nearly identical effects on the impulse response. Therefore, this property of LFM might be used to obtain TTD. The concept for LFM signals can be generalized to NLFM signals, using a phase correction term that is not a linear function of time.

As we approach implementing this method, the first step would be to modulate the chirped waveform onto an optical carrier. After modulation, the optical signal comprises of two sidebands around the carrier. Trying to apply a phase shift at this point would not result in a delayed signal. The reason lies within the nature of the detection. The detector uses the optical carrier as its quiescent point. If we shift the carrier’s frequency along with those of sidebands, the frequency of the photo-detected, RF-domain waveform would remain unchanged. Thus, before applying the frequency
offset, it is imperative that we filter out the optical carrier. Subsequently, after optical processing and prior to detection, an unprocessed carrier is reintroduced. In addition we must also filter out one of the sidebands, otherwise two replicas of the RF waveform would appear in the photo-detected signal, with frequency offsets in different directions (Figure 4).

![Figure 4: A modulated signal is offset without filtering and with filtering. Without filtering, two replicas of the RF waveform appear with frequency offsets in different directions.](image)

After we filter the optical signal, leaving only one sideband to work with, a phase offset is applied to the remaining sideband, using an electro-optic phase modulator or an acousto-optic modulator. Following detection, the recovered RF signal is practically indistinguishable from a delayed replica of the original chirped waveform, except for small fractions of its duration near both temporal edges. The principle is elaborated in the following section.
2.2 Principle of Operation

Before introducing the method for delaying a general NLFM waveform, I address the specific case of LFM signals first. The instantaneous phase of a LFM waveform can be expressed as:

\[ \varphi_{\text{LFM}}(t) = 2\pi f_0 t + \pi \frac{B}{T} t^2 \] (6)

Thus, an LFM waveform of duration \( T \), bandwidth \( B \) and central radio-frequency \( f_0 \) can be expressed as:

\[ A_{\text{LFM}}(t) = A_0 \cos \left( 2\pi f_0 t + \pi \frac{B}{T} t^2 \right) \text{rect} \left( \frac{t}{T} \right) \] (7)

where \( A_0 \) is a constant magnitude, \( t \) denotes time, and \( \text{rect}(x) = 1 \) for \( |x| \leq 0.5 \) and equals zero elsewhere. The instantaneous frequency \( f(t) \) of the waveform is:

\[ f(t) = \frac{1}{2\pi} \frac{d}{dt} \varphi(t) = \frac{1}{2\pi} \frac{d}{dt} \left( 2\pi f_0 t + \pi \frac{B}{T} t^2 \right) = f_0 + \frac{B}{T} t \] (8)

Since the signal is nonzero only for \( |t| \leq 0.5T \), \( f(t) \) is linearly sweeping between \( f_0 \pm 0.5B \) along the waveform duration \( T \).

Suppose next that a phase offset \( \Delta \varphi \) is introduced to the LFM waveform of (7), so that:

\[ A_{\text{LFM}}^{\text{offset}}(t) = A_0 \cos \left( \Delta \varphi + 2\pi f_0 t + \pi \frac{B}{T} t^2 \right) \text{rect} \left( \frac{t}{T} \right) \] (9)
The phase-offset waveform closely resembles a replica of (7) that is temporally delayed by \( \tau \), provided that:

\[
\Delta \varphi = \varphi(t + \tau) - \varphi(t) = 2\pi f_0(t + \tau) + \pi \frac{B}{T}(t + \tau)^2 - \left(2\pi f_0 t + \pi \frac{B}{T} t^2\right)
\]

\[
= 2\pi f_0(t + \tau) + \pi \frac{B}{T}(t^2 + 2t\tau + \tau^2) - \left(2\pi f_0 t + \pi \frac{B}{T} t^2\right)
\]

\[
= 2\pi f_0 \tau + \pi \frac{B}{T} \tau^2 + 2\pi \frac{B}{T} \tau \cdot t
\]

The necessary phase correction therefore consists of a bias term:

\[
\Delta \varphi_0 = 2\pi f_0 \tau + \pi \frac{B}{T} \tau^2
\]

and a linearly varying term that represents a frequency offset \( \Delta f \):

\[
2\pi \frac{B}{T} \tau \cdot t \equiv 2\pi \Delta f \cdot t
\]

\[
\downarrow
\]

\[
\Delta f \equiv \frac{B}{T} \tau \cdot t
\]

Substituting (10) into (9) yields:

\[
A_{LFM}^{\text{offset}}(t) = A_0 \cos\left(\Delta \varphi + 2\pi f_0 t + \pi \frac{B}{T} t^2\right) \text{rect}\left(\frac{t}{T}\right)
\]

\[
= A_0 \cos\left(2\pi f_0(t + \tau) + \pi \frac{B}{T}(t + \tau)^2\right) \text{rect}\left(\frac{t}{T}\right) = A_{LFM}(t + \tau)
\]

When (10) is met, differences between (9) and a delayed replica of (7) are confined to the edges of the rectangular temporal window, which are not delayed (further explanation will be given shortly). As long as \( \tau \ll T \left(\Delta f \ll B\right) \), the differences
between the impulse responses of the offset waveform and the 'truly delayed' one are expected to be negligible. Since $T$ is typically many µs long, substantial delays to the impulse response should be possible, with tolerable performance degradation. Careful adjustment of the bias phase term according to (10) is necessary to avoid spatial distortion of the broadband transmitted beam [35].

Although the time delay we can obtain using this principle is much longer compared to what prior methods had achieved, it is not without limits. The equivalency between time delay and frequency shift is based on the LFM signal’s nature, and is absolute only for theoretical, infinitely long signals. However, for real world, finite-duration LFM waveform this equivalency strictly depends on both the duration of the pulse and its bandwidth. If, for instance, the instantaneous frequency is sweeping between 1 GHz and 2 GHz, and the LFM signal undergoes a frequency shift of 500 MHz, the output-signal’s frequency would sweep between 1.5 GHz and 2.5 GHz. This means that the first half of the output signal is indeed a delayed replica of the input’s first half, but only this portion of the signal has been delayed, while the rest is distorted.

To demonstrate, let us examine the instantaneous frequency of an LFM waveform (Figure 5). Both the delayed and the frequency-shifted replicas are shown in figures 6 and 7, respectively. Comparing the two (Figure 8), it is obvious that even though the central portions of the waveform (noted in green) of the delayed and the frequency-shifted waveforms are in overlap, their edges differ, leading to the eventual degradation of the impulse response function of the frequency-offset waveform.
2.3 In-Depth Analysis of LFM Waveforms

Thus far, the discussion around LFM waves dealt with their general characteristics, namely their time-frequency ambiguity. The most beneficial property of the LFM signal lies in the connection between the duration of a pulse and its bandwidth. To illustrate, let us first discuss the superiority of LFM pulses over simple sine wave pulses.
For this purpose, a careful examination of a sine pulse is required [36]. The simplest signal a pulse radar can transmit is a sinusoidal pulse of amplitude $A$ and carrier frequency $f_0$, truncated by a rectangular function of width $T$:

$$s(t) = \begin{cases} A e^{2\pi f_0 t} & 0 \leq t \leq T \\ 0 & \text{o.w} \end{cases}$$

(14)

As this pulse is transmitted, the returned signal can be expressed as:

$$r(t) = \begin{cases} KA_i e^{2\pi f_0 (t-t_r)} + n(t) & t_r \leq t \leq t_r + T \\ n(t) & \text{o.w} \end{cases}$$

(15)

$r(t)$, the signal obtained by the receiver, is an attenuated ($K$) and time-shifted ($t_r$) replica of the original, transmitted signal. I assume that an additive white Gaussian noise (AWGN), denoted as $n(t)$, is received along with the returned waveform. The impulse response is calculated by cross-correlating the transmitted signal $s(t)$ with the returned signal $r(t)$ [37]. The cross-correlation is therefore of the form:

$$\langle s, r \rangle(t) = KA_i A \left( \frac{t-t_r}{T} \right) e^{2\pi f_0 (t-t_r)} + n'(t)$$

(16)

$n'(t)$ is a term of inter-correlation between the transmitted signal $s(t)$, and the noise $n(t)$, hence it is still an AWGN since the noise is uncorrelated with $s(t)$. $\Lambda$ denotes a triangle function centered at $t_r$ and with width that is defined by $T$. The important point here is that the correlation resolution is inversely proportional to the pulse duration $T$: long pulses provide poor resolution. Since the signal power intensifies as the pulse lengthens, the SNR improves for longer pulses, as opposed to the
resolution. Therefore, for simple sine pulses, good SNR comes at the expense of poor resolution, and vice versa.

In contrast, LFM pulses provide good SNR along with good resolution. For the purpose of this discussion, the following form of (7) is shown:

$$s_{LFM}(t) = \begin{cases} A_0 e^{2\alpha t \left( \frac{\delta}{2T} \right)^2} & -T/2 \leq t \leq T/2 \\ 0 & \text{o.w} \end{cases}$$  \hspace{1cm} (17)

The auto-correlation of $s_{LFM}(t)$ is equal to its cross-correlation with a delayed replica of itself (excluding the attenuator factor $K$). The auto-correlation is expressed as:

$$\langle s_{LFM}, s_{LFM} \rangle(t) = T \Lambda \left( \frac{t}{T} \right) \text{sinc} \left[ \pi B t \Lambda \left( \frac{t}{T} \right) \right] e^{2\pi \delta t}$$  \hspace{1cm} (18)

The peak of this auto-correlation is centered at $t = 0$. Around that point, the auto-correlation behaves like a sinc function. The -3dB temporal width $T'$, also known as full-width half-maximum (FWHM), is approximately:

$$T' = \frac{1}{B}$$  \hspace{1cm} (19)

The resolution obtained here is the same as would obtained for a simple sine pulse of duration $T'$. For most cases, $B$ is such that $T' \ll T$, so using wideband LFM wave provides better resolution than sine pulses. Moreover, the bandwidth of an LFM signal is independent of its duration. Therefore, the SNR can be improved by transmitting longer pulses, whereas the bandwidth remains unchanged, so that the resolution is not deteriorated.
2.4 Delay of Nonlinear Frequency Modulated Waveforms

The extension of the delay principle to NLFM waveforms is addressed next. Any NLFM waveform can be represented as:

\[
A_{\text{NLFM}}(t) = A_0 \cos[\varphi(t)] \text{rect}\left(\frac{t}{T}\right)
\]  

(20)

The corresponding pulse delayed by \( \Delta t \) is given by:

\[
A_{\text{NLFM}}(t - \Delta t) = A_0 \cos[\varphi(t - \Delta t)] \text{rect}\left(\frac{t-\Delta t}{T}\right)
\]  

(21)

Consider now the introduction of an instantaneous phase correction term \( \Delta \varphi(t) \):

\[
A_{\text{NLFM}}^{\text{shifted}}(t) = A_0 \cos[\varphi(t) + \Delta \varphi(t)] \text{rect}\left(\frac{t}{T}\right)
\]  

(22)

The phase-corrected waveform could well approximate the delayed one,\n
\[A_{\text{NLFM}}^{\text{shifted}}(t) \approx A_{\text{NLFM}}(t - \Delta t), \text{ provided that } \Delta t \ll T \text{ and}
\]

\[
\Delta \varphi(t) = \varphi(t - \Delta t) - \varphi(t)
\]  

(23)

Differences between the delayed signal and the phase-shifted one are confined to the edges of the rectangular temporal window function, which is of course not delayed by the phase-correction process. Since the extent of required delay is typically much shorter than the pulse duration, differences between (21) and (22) are expected to be negligible.
2.5 Design of NLFM Waveforms

The impulse response of LFM waveforms suffers from relatively high side-lobes [37]. The impulse response is the inverse Fourier transform of the power spectrum of the signal, and hence it can be modified by shaping its power spectrum. To that end there are two main methods.

The first relies on changing the instantaneous amplitude of the signal over time. Since for LFM signals, there is a linear relation between frequency and time, this means that the amplitude of the signal is also changing along the frequency axis. From a practical and radar systems point-of-view, changing the amplitude of the signal over time complicates the transmitter and the receiver and can cause SNR loss [37].

The second method for shaping the power spectrum of an LFM signal is by using variable rate of frequency change in time. Notice that by deviating from a constant frequency sweep in time, we no longer have a linear-FM signal, but a nonlinear-FM signal, or NLFM signal instead. The idea behind NLFM waveforms is to 'spend more time' at frequencies that need to be enhanced.

Assume that the NLFM signal’s complex envelope is given by:

\[ u(t) = u_0(t)e^{j\phi(t)} \]  \hspace{1cm} (24)

where \( u_0(t) \) is the amplitude and \( \phi(t) \) is the instantaneous temporal phase of the signal. Denote \( \theta(f) \) as the spectral phase, than the Fourier transform of \( u(t) \) is given by the following equation:
The design objective in constructing the NLFM waveform is to find \( \varphi(t) \) so that the power spectral density of the waveform \( |U_m(f)|^2 \) approximates a favorable target function. The inverse Fourier transform of this target power spectral density, would provide an auto-correlation trace with a narrow main lobe and low sidelobes.

The design procedure relies on an approximate evaluation of the integral (25). The approximation is based on the principal of the stationary-phase [38,39]: The integral of a rapidly oscillating function acquires little contribution, except in regions where the phase is stationary (or in other words where the derivative of the phase is zero.)

The complex envelope \( u(t) \) can be defined using its Fourier transform:

\[
u(t) = \mathcal{F}^{-1}\{U(f)\} = \int_{-\infty}^{\infty} U(f)e^{j2\pi ft} df = \int_{-\infty}^{\infty} U_m(f)e^{j\varphi(f)} e^{j2\pi ft} df = \int_{-\infty}^{\infty} U_m(f)e^{j[\varphi(f)+2\pi ft]} df\]

The expression in (26) is an integral of a rapidly oscillating function. Therefore, the principal of stationary phase can be invoked to approximate this integral. The phase of the integrand in (26) is stationary whenever

\[
\frac{df}{df}[\theta(f)+2\pi ft] = 0 \quad (27)
\]

Denote the value of \( f \) which satisfies (27) by \( \lambda \), then:
2\pi t = -\theta'(\lambda) \quad (28)

Following further derivation that is outside the scope of this discussion, the method of stationary phase leads to the following approximate expression for \( u(t) \) [39]:

\[
u(t) \approx \sqrt{2\pi} \frac{U_m(\lambda)}{\sqrt{\left|\theta''(\lambda)\right|}} \exp\left\{ j \left( 2\pi \lambda t + \theta(\lambda) \pm \frac{\pi}{4} \right) \right\}
\]

(29)

The absolute value of the second derivative of the spectral phase is therefore given by:

\[
|\theta''(\lambda)| \approx 2\pi \frac{U_m^2(\lambda)}{u_0^2(t)}
\]

(30)

Integrating (30) yields \( \theta'(\lambda) \). This quantity is related by definition to the group delay at frequency \( \lambda \):

\[
D(\lambda) = -\frac{1}{2\pi} \theta'(\lambda)
\]

(31)

On the other hand equation (28) states that the same \( \theta'(\lambda) \) governs the time in which the waveform goes through a particular instantaneous frequency \( f(t) \), so that [39]:

\[
f(t) \approx D^{-1}(t)
\]

(32)
We can now use $D(\lambda)$ to find $f(t)$ using (32). With knowledge of the instantaneous frequency, the instantaneous phase may be found using a second integration:

$$\varphi(t) = 2\pi \int_0^t f(x) \, dx$$

(33)

The procedure therefore determines $\varphi(t)$ that provides an approximation to the target power spectral density $U_m(f)$, subject to the constraint of a magnitude $u_0(t)$ that is fixed over a duration $T$. The NLFM waveform is now reconstructed.

As an illustration, let us apply the procedure above to a specific combination of target power spectral density $U_m(f)$ and instantaneous magnitude $u_0(t)$, that are both fixed within a window:

$$U_m(f) = \begin{cases} 
\frac{1}{\sqrt{B}} & f_0 < f < f_0 + B \\
0 & o.w
\end{cases}$$

(34)

$$u_0(t) = \begin{cases} 
\frac{1}{\sqrt{T}} & 0 < t < T \\
0 & o.w
\end{cases}$$

(35)

According to (30), $\theta^*(\lambda)$ can be written as:

$$\theta^*(\lambda) = -2\pi \frac{T}{B}$$

(36)

Integrating (36) once:
\[
\theta'(\lambda) = \int_{j_0}^\lambda \theta''(x) \, dx = \int_{j_0}^\lambda -2\pi \frac{T}{B} \, dx = -2\pi \frac{T}{B} \bigg|_{j_0}^\lambda = 2\pi \frac{T}{B} (-\lambda + f_0) \quad (37)
\]

Using the result of \(\theta'(\lambda)\) from (37) and equation (31), we can determine the expression for \(D(\lambda)\):

\[
D(\lambda) = -\frac{1}{2\pi} \theta'(\lambda) = \frac{T}{B} (\lambda - f_0) \quad (38)
\]

Insert \(D(\lambda)\) into (32) to get \(f(t)\):

\[
f(t) = \frac{B}{T} t + f_0 \quad (39)
\]

Finally, integrate (39) once as in (33) to get \(\varphi(t)\):

\[
\varphi(t) = 2\pi \int_0^t f(x) \, dx = 2\pi \bigg|_{0}^{t} \frac{B}{T} x + f_0 \, dx = \pi \frac{B}{T} x^2 + 2\pi f_0 x \bigg|_{0}^{t} = 2\pi f_0 t + \pi \frac{B}{T} t^2 \quad (40)
\]

We therefore obtained the instantaneous phase \(\varphi(t)\) of an LFM waveform as defined in (6). Indeed, the LFM waveform is characterized by uniform magnitude and power spectral density.

The target spectrum \(U_m(f)\) in more advanced waveforms is chosen so that the autocorrelation that is associated with it yields sidelobes that are strongly suppressed:

\[
R_c(\tau) \approx \mathcal{F}^{-1} \left[ |U_m(f)|^2 \right] \quad (41)
\]
In this work, we chose target functions that are the square root of raised cosines with power $n$:

$$U_m(f) = \begin{cases} 
 k + (1-k) \cos^n \left( \pi \frac{f}{B} \right) & -\frac{B}{2} \leq f \leq \frac{B}{2} \\ 
 0 & \text{o.w.} 
\end{cases}$$

Here $0 \leq k \leq 1$ is a design parameter, and $B$ is the bandwidth of the NLFM waveform. Figure 9 shows that for higher values of $n$, $U_m(f)$ becomes narrower, and Figure 10 shows that $U_m(f)$ becomes flatter as $k$ gets higher. Note that for $k=1$ and any choice of $n$, $U_m(f)$ reduces to the spectrum of the LFM waveform which was discussed earlier.

The decision to use $U_m(f)$ from (42) is not arbitrary. Going back to (41) and using $U_m(f)$ from (42), we find that the sidelobes in the NLFM signal’s auto-correlation are lowered as the order $n$ increases. However, as $n$ increases, the resolution of the
signal’s auto-correlation is degraded. In the next chapter the impulse responses of NLFM waveforms are simulated and discussed.
2.6 Simulation

Before discussing the experiment, it is worthwhile to understand the method used in the experiment by simulating it.

![Flowchart](image)

**Figure 11: Flowchart demonstrating the method of operation for the simulation**

The flowchart in Figure 11 shows the method of operation for the simulation demonstrating the suggested method for the delay of chirped waveforms. First, an NLFM waveform is generated and modulated onto an optical carrier. Afterwards, a filter in the optical frequency domain is applied to retain only one sideband. The ODL is then implemented through phase modulation of the remaining sideband, using either an ideal $\Delta \phi$ correction shape or a real-world one sampled from the function generator used in the experiment. Finally, the signal is detected and down-converted to the baseband.

The value of TTD produced in the circuit of Figure 11 can be found by measuring the distance between the main-lobe of the reference signal’s auto correlation, and that of the cross correlation between the reference and processed signals.

The following discussion uses three figures of merit to describe the quality of an impulse response function: PSLR, ISLR and resolution. All three are illustrated in Figure 12; the peak-to-side-lobe ratio (PSLR) is the ratio of main lobe peak to the highest side-lobe peak, and it quantifies the degradation due to a localized noise. The integrated
**Side-lobe ratio (ISLR)** is the ratio of the energy within the main lobe to the energy outside the main lobe (illustrated in Figure 12 as the ratio between the red painted area and the yellow painted area), and it quantifies the degradation due to distributed noise. **Resolution** is defined as the width of the main correlation lobe, also referred to as the full width at half maximum (FWHM). The intersection of the impulse response with the -3 dB line (dashed), defines the boundaries of the main lobe for all the above. The first two are measured in dB, and the latter is measured in nano-seconds.

The correlation sidebands can be suppressed using a spectral windowing. The simulation uses Hann’s window [40], however, the choice of a window is arbitrary, as every window which reduces the correlation side-lobes is suitable, for example: Hamming’s window [40]. It is important to mention that windowing should only be applied to LFM waveforms and not to NLFM waveforms, since the spectrum of NLFM signals is already shaped to meet the low side-lobes requirement, at least in principle.
2.6.1 Simulation Using LFM Waveform

Figure 13 shows the simulated impulse responses of the LFM waveforms of (7)-(9), with $T=5\mu s$, $B=500\,\text{MHz}$, and $\Delta f=10\,\text{MHz}$ which corresponds to a delay of $\tau=100\,\text{ns}$ according to (12). The impulse response of the frequency offset waveform is effectively delayed by $\tau$. The PSLR of the offset and ideal waveforms are 37.5 dB and 39.5 dB, respectively, and the corresponding ISLR values are 32.9 dB and 32.1 dB, respectively. The PSLR values are governed by highest sidelobes, which are immediately adjacent to the main peak. The FWHM of the impulse response main lobe, signifying resolution, is 2.5 ns for both waveforms. Both the PSLR and the ISLR remain above 20 dB for delays up to $\tau=0.2\cdot T=1\mu s$.

This maximum obtainable delay is proportional to the LFM pulse duration, meaning that for $T=1\mu s$, a maximum delay of $\tau=200\,\text{ns}$ can be achieved with the above PSLR values. Furthermore, as explained above, the correlation resolution is the
inverse of the LFM bandwidth as can be seen in equation (19). Therefore, wider bandwidth will result in higher resolution. This can be seen in Figure 14, where the FWHM of the LFM pulse with \( B = 500 \text{MHz} \) is twice the FWHM of the LFM pulse with \( B = 1 \text{GHz} \).

![Simulated correlations of LFM waveforms showing that the wider the bandwidth is, the higher the resolution is](image)

Numerical simulations show that the impulse responses of manipulated waveforms are effectively delayed with only marginal degradations in their figures of merit: resolution, PSLR and ISLR. By compromising the universality of the TTD element, we are able to scale the effective delay of LFM waveforms towards the requirements of large phased-array antennas. Furthermore, the proposed method is very simple and requires only few elements.

### 2.6.2 Simulation Using NLFM Waveform

Figure 15 shows the simulation results for NLFM 4th-order \((n = 4, k = 0.015)\) with pulse duration of \( T_{\text{NLFM}} = 5 \mu\text{s} \), bandwidth of \( B = 500 \text{MHz} \), and a phase correction term that is designed to delay the impulse response by \( \tau = 100 \text{ns} \). It can be seen that
the impulse response is indeed effectively delayed by $\tau$. The PSLR values of the non-delayed and delayed impulse responses are 42.7 dB and 35 dB, respectively, and the ISLR values are 37.9 dB and 31.2 dB, respectively. The resolution of NLFM 4th-order is 3 ns, for 8th-order is 5 ns, and for 16th-order is 6.8 ns. As the NLFM order $n$ increases, the sidelobes suppression improves at the expanse of resolution.

As said in previous chapters, NLFM waveform’s figures of merit are supposed to be better than those of an LFM waveform. Indeed, Figure 16 and Figure 17 show a clear difference between the PSLR and ISLR of a non-delayed LFM waveform and those of a non-delayed 4th-order NLFM waveform, in favor of the NLFM waveform.
However, as both waveforms are effectively delayed based on phase corrections, we see that the ISLR and PSLR values of the NLFM waveform degrade more severely than those of the LFM waveform, until they eventually go below them. One explanation for the quicker degradation in the figures of merit of NLFM waveforms lays in their design. NLFM waveforms are designed from a specific spectrum (denoted as $U_m(f)$ above), and that is what gives them better PSLR and ISLR values. Figure 18 and Figure 19 show that delaying an NLFM waveforms by phase shifting cause their spectrum to...
distort. Hence, NLFM waveforms lose their advantage over LFM waveforms for long delays.

### 2.7 Single Sideband MWP Processing

In MWP implementations, the RF NLFM waveform is used to modulate an optical carrier, and is later recovered through the beating of the optical carrier and the modulation sidebands on a photo-detector. The delay of single-sideband analog waveforms benefits from piece-wise treatment [35]: since $f_{RF} \gg B$, it is sufficient for a MWP setup to provide an appropriate spectral phase across the window:

$$\left[ f_{opt} + f_{RF} - \frac{1}{2} B, \ f_{opt} + f_{RF} + \frac{1}{2} B \right]$$

(43)

as well as for the carrier frequency $f_{opt}$ itself. The spectral phases at all other optical frequencies need not be specified. On the other hand, if dual sideband modulation is used, then the spectral phase acquired by the signal needs be specified over a far broader spectral range of twice $f_{RF}$. The technique proposed in this work for the TTD of chirped waveforms may be regarded as a specific case of single sideband MWP processing, which takes advantage of the particular attributes of the waveform.
The photonic realization of our proposed TTD method therefore requires that relative $\Delta \varphi$ and $\Delta f$ offsets are generated between carrier and sidebands. To that end, the optical carrier of frequency $f_{opt}$ is split in two paths. Light in one path is modulated in suppressed-carrier single sideband (SC-SSB) manner, to retain only one modulation sideband which carries the NLFM waveform and is subsequently offset in phase and frequency. The original, unmodulated carrier is retained in the other path, and the two are recombined prior to detection.

The processing of a single optical sideband was previously employed in several MWP TTD demonstrations [33,35,41,42]. In the works of Chin et al. [41] and Sancho et al. [42], SSB treatment was used to simplify the optical processing needed to provide a MWP TTD element and dynamic MWP filter, respectively. SSB modulation was also used in the work of Zadok et al. [33], where a tunable delay of LFM signals was demonstrated using SBS.
3 Experimental Setup

The laboratory experiment was divided into three parts. In the first two, a TTD element for LFM waveforms was implemented, first using a PM and then using an AOM. In the third one, the PM-based TTD setup was generalized to accommodate NLFM signals as well. The following sections present the experimental setup for each one of the above parts.

Figure 20 shows the main building blocks of the experimental setup for the variable delay of the impulse response of LFM and NLFM waveforms. The differences in implementation between the three parts of the experiment are all in the optical processing block. Hence, the RF Up-Conversion and RF Down-Conversion blocks will be discussed in general, and the discussion over the Optical Processing block will be specific for each part of the experiment.

3.1 RF Up-Conversion Circuit

The main purpose of this circuit is, as its name suggests, to take a low frequency chirped waveform and to convert it to a high frequency waveform. Up-conversion is needed to generate a large separation between the frequencies of the optical carrier and the modulation sidebands, to be used in later stages of the
processing flow. Sufficient separation is required to filter out the optical carrier while retaining one modulation sideband.

Figure 21 shows the RF up-conversion circuit in detail. A chirped waveform is first generated using an arbitrary waveform generator (AWG). The waveform is defined by pulse duration of $T = 5 \mu s$, center frequency of $f_0 = 1$ GHz and bandwidth of $B = 500$ MHz. The AWG used for this setup has a maximal sampling rate of 5 GSamples/s. Without an anti-aliasing (AA) filter, a parasitic aliasing signal appears at the output of the AWG (see Figure 22). That signal is centered at a frequency of 4 GHz with its lowest frequency component reaching 3.75 GHz. In order to eliminate this parasitic spectral replica of the signal, a low-pass filter (LPF) with cut-off frequency of
$f_c = 2.25\text{GHz}$ is used at the AWG output. The signal at the output of the LPF is shown in Figure 23.

![Figure 22: Output of the AWG without an AA filter (NLFM 4th-order waveform $B=500\text{ MHz}$ wide, $T=5\text{ µs}$ pulse duration and center frequency of $f_0 = 1\text{ GHz}$)](image1)

![Figure 23: Output of the AWG after an AA filter. Only the desired signal is preserved.](image2)
After the chirped waveform is filtered, it passes through a chain of attenuators and one amplifier. The purpose of this chain is to amplify the waveform before it enters the RF mixer, without reaching the amplifier’s saturation. To prevent edge clipping, a specific order of components is required. For example, an attenuator is placed before the amplifier to avoid distortion. The overall amplification obtained here is 11 dB. In principal, a single RF amplifier could have replaced the whole chain; however, we did not have such RF amplifier at our disposal.

The next step would be to up-convert the intermediate frequency (IF) chirped signal so that it reaches a center frequency of $f_0 = 7.5\,\text{GHz}$, using an RF mixer. High center frequency is vital, as the optical filtering requires the ability to tell the optical carrier and the modulated side-lobe apart. A sine-wave local oscillator (LO) at a frequency of $f_s = 6.5\,\text{GHz}$ is used, generated by a high frequency microwave signal
generator. The frequency $f_r$ was chosen based on the limits imposed by both the bandwidth of the RF amplifier placed at the output end of the mixer, and the bandwidth of the amplitude modulator (AM) that is a part of the optical processing block.

![Figure 25: NLFM 4th-order waveform up-converted to a center frequency of $f_0 = 7.5$ GHz](image)

Since the frequency mixer produces copies of the chirped waveform not only around the desired frequency, but also around other frequencies, as can be seen in Figure 24, additional filtering was necessary. Thus, two high-pass filters were connected to the output end of the amplifier, prior to the optical modulation. A cutoff frequency of 7.15 GHz met our needs, since the desired replica of the up-converted signal lies between 7.25 GHz and 7.75 GHz. Within this range, the desired signal suffers a 3 dB loss, whereas the residues of the LO carrier and the undesired spectral replicas of the waveform are confined to the stopband of the filters (DC – 6000 MHz), where the attenuation is at least 20 dB. Figure 25 shows the residual LO carrier and undesired chirped signal replica, attenuated by 25 dB, while the desired up-converted waveform
was almost unchanged. Higher frequency replicas do not represent a problem, since the modulator’s cutoff frequency is around 12 GHz.

3.2 Optical Processing Circuit

Figure 26 shows the optical processing circuit in detail. The output of a continuous wave (CW) laser diode is split in two paths. Light in the upper path is modulated by an electro-optic AM, which is driven by an up-converted chirped waveform coming from the output end of the up-conversion RF circuit (see Figure 27).

Figure 26: Optical processing circuit

Figure 27: Optical carrier (middle peak) modulated by an AM which is driven by an LFM waveform (two peaks around the carrier).
A narrow-band FBG with bandwidth of 0.05 nm, along with a magneto-optical fiber circulator, retains one of the modulation side-bands and rejects the optical carrier and the other side-band (Figure 28). Then, fine adjustment of the tunable laser’s emission wavelength is carried out, in order to optimize the single-sideband selectivity through the FBG. The remaining side-band undergoes an optical processing by either a PM or an AOM, as will be explained in the next sub-sections.

The processed side-band is mixed with the original optical carrier (Figure 29), which is retained in the lower path (see Figure 26). A modified electrical NLFM waveform is reconstructed through the beating of the carrier and side-band on a broadband detector.

In order to improve the signal-to-noise ratio (SNR) of the detected signal, an Erbium-doped fiber amplifier (EDFA) with 17 dBm average output power was added to the upper branch, at the output end of the laser. We use the EDFA to amplify the signal,
since it is significantly attenuated as it passes through the optical processing (mainly because of the FBG and the PM). The signal power at the detector input was kept below -3 dBm to avoid compression.

As mentioned earlier, the experiment is divided into three parts that will be explained in the following sub-sections.

### 3.2.1 Delaying LFM Waveforms Using a Phase Modulator

In the first part, an LFM waveform was delayed using a phase modulator [43]. As explained in the Principle of Operation section above, delaying LFM waveforms can be accomplished by introducing the phase shift of (10) to the LFM signal. This phase shift is linearly varying in time, and it is equivalent to a frequency shift of

\[
\Delta f = \frac{B}{T} \tau
\]  

\[ (44) \]
As discussed earlier, the above frequency shift introduces an effective delay of the reconstructed RF waveform, provided that a constant bias phase of

$$\varphi_0 = 2\pi f_o \tau + \pi B \frac{\tau^2}{T}$$

is added as well. Without adequate control of individual bias phases, each antenna within a PAA would delay its input with respect to a different wavefront, and the beam will be spatially distorted. Bias phase can be controlled using piezoelectric transducer. Feedback information for the transducer can be provided by the transmission of an auxiliary signal at a different wavelength.

Going back to the experiment, right after the FBG and the circulator only one side-band is left. The remaining side-band undergoes a frequency offset, introduced by an electro-optic phase modulator which is driven by a ramp waveform of magnitude $V$ and period $1/\Delta f$:

$$\varphi(t) = \pi \frac{V}{\Delta f} \sum_{n=-\infty}^{\infty} \left[ t \cdot \Delta f - n \right] \text{rect}(t \cdot \Delta f - n)$$

When the ramp waveform magnitude is precisely adjusted to $V = 2V_\pi$, the coefficient preceding the sum becomes $2\pi$, and the modulation range extends between $[-\pi, \pi]$. The discontinuities of the phase modulation are then removed, leading to an effective frequency offset of $\Delta f$. 
The ramp waveform is generated by an arbitrary function generator (AFG). Given the choice of pulse duration $T = 5\mu s$ and bandwidth $B = 500\text{MHz}$, a frequency offset $\Delta f$ of 100 kHz corresponds to $1\text{ns}$ of TTD.

### 3.2.2 Delaying LFM Waveforms Using an Acousto-Optic Modulator

This part of the experiment still uses LFM waveforms, however, the frequency offset that was introduced earlier by phase shifting the signal using a phase modulator, is now introduced by a direct frequency shift using an acousto-optic modulator [44]. The AOM is driven by a sine wave of frequency $f_{AOM}$. The circuit is shown is Figure 30.

### 3.2.3 Delaying NLFM Waveforms Using a Phase Modulator

For the last part of the experiment, a phase modulator was used to delay NLFM waveforms using the setup in Figure 26. This part is basically a generalization of the first part of the experiment where a specific type of chirped waveforms, the LFM, was delayed. Note that for delaying NLFM waveforms, a general phase correction term that does not vary linearly with time is needed, according to equation (23). In order to generate the necessary $\Delta \phi$, a low-rate arbitrary function generator (AFG) was...
employed. Since $\Delta \varphi$ is usually not a periodic signal, synchronization between the phase correction term and the NLFM waveform is required. Otherwise, if the two are not synchronized, the phase offset will distort the NLFM signal.

![Phase correction term for LFM waveform](image1)

*Figure 31: Phase correction term for LFM waveform of period 5\(\mu\)s*

![Phase correction term for 4th-order NLFM waveform](image2)

*Figure 32: Phase correction term for 4th-order NLFM waveform of period 5\(\mu\)s*
A specific case of $\Delta \varphi$ is the ramp wave used for frequency shifting LFM waveforms from the first part of the experiment (Figure 31). In this case, the phase correction term is periodic throughout the pulse duration and hence it can be repeated continuously without the need for synchronization between $\Delta \varphi$ and the LFM waveform (except for bias phase adjustments in multiple-element arrays).

However, the $\Delta \varphi$ required for delaying NLFM 4th-order waveforms, for example, is not periodic during the pulse duration, as can be seen from Figure 32, and hence it needs to be synchronized with the NLFM pulse.
Figure 33: RF down-conversion circuit
3.3 RF Down-Conversion Circuit

Figure 33 shows the RF down-conversion circuit in detail. The detected waveform is down-converted to an intermediate center frequency of 1 GHz, using the...
same method used for the aforementioned up-conversion, mixing the detected signal with a copy of the 6.5 GHz LO used for the up-conversion (Figure 34). An RF amplifier is added prior to this mixing, to meet the mixer’s minimum gain limitations. Two low-pass filters connected to the output end of the mixer filter out all the mixer artifacts (Figure 35).

The final step is to sample the down-converted signal by a real-time oscilloscope of 6 GHz bandwidth. The impulse response of the processed LFM waveform is calculated by cross-correlating the sampled signal with a reference waveform, which was sampled at the AWG output by a second oscilloscope channel. All the measurements are synchronized using an external clock originating from the AWG.

The complete experimental setup comprises all the modules discussed in this chapter, and it is shown in Figure 36.
Figure 36: Complete experimental setup
4 Experimental Results

4.1 Delaying LFM Waveforms Using a Phase Modulator

Figure 37 shows the experimental impulse responses of LFM waveforms that were offset in frequency by several values of $\Delta f$, in the range of $0 \text{ MHz} – 25 \text{ MHz}$. Since the setup used an LFM waveform defined by pulse duration of $T = 5\mu s$ and bandwidth of $B = 500\text{MHz}$, as mentioned earlier, the corresponding group delays were in the range of $0 \text{ ns} – 250 \text{ ns}$. Figure 38 shows the corresponding simulation results, for $0 \text{ ns}$, $10 \text{ ns}$, $20 \text{ ns}$, $30 \text{ ns}$, $40 \text{ ns}$, $50 \text{ ns}$, $100 \text{ ns}$, $150 \text{ ns}$, $200 \text{ ns}$, $250 \text{ ns}$, from right to left. A variable delay in the impulse response is observed, in complete agreement with (12). The maximum delay obtained was $250 \text{ ns}$, restricted only by the bandwidth of the available ramp waveform generator.

The PSL and ISL ratios of the non-delayed impulse response were $35.8 \text{ dB}$ and $21.3 \text{ dB}$, respectively. The PSL and ISL ratios of frequency-offset waveforms were degraded by multiple pronounced side-lobes separated by $\tau$, as can be seen in figures...
39 through 56. The pronounced discrete side-lobes stem from distortion due the non-ideal ramp waveform used in the phase modulation.

Figure 39: Experimental correlation of TTD = 10 ns

Figure 40: Simulated correlation of TTD = 10 ns

Figure 41: Experimental correlation of TTD = 20 ns

Figure 42: Simulated correlation of TTD = 20 ns

Figure 43: Experimental correlation of TTD = 30 ns

Figure 44: Simulated correlation of TTD = 30 ns
Experimental Results

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<th>Figure</th>
<th>Description</th>
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<tr>
<td>45</td>
<td>Experimental correlation of ( TTD = 40 ) ns</td>
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<tr>
<td>46</td>
<td>Simulated correlation of ( TTD = 40 ) ns</td>
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<tr>
<td>47</td>
<td>Experimental correlation of ( TTD = 50 ) ns</td>
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<tr>
<td>48</td>
<td>Simulated correlation of ( TTD = 50 ) ns</td>
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<tr>
<td>49</td>
<td>Experimental correlation of ( TTD = 100 ) ns</td>
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<tr>
<td>50</td>
<td>Simulated correlation of ( TTD = 100 ) ns</td>
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</table>
Experimental Results

Delaying LFM Waveforms Using a Phase Modulator

Figure 51: Experimental correlation of TTD = 150 ns

Figure 52: Simulated correlation of TTD = 150 ns

Figure 53: Experimental correlation of TTD = 200 ns

Figure 54: Simulated correlation of TTD = 200 ns

Figure 55: Experimental correlation of TTD = 250 ns

Figure 56: Simulated correlation of TTD = 250 ns
In order to demonstrate that the correlation side-lobes are caused by the non-ideal ramp waveform, samples of the real ramp waveform were taken and incorporated into the simulation in place of the ideal ramp waveform. Figure 57 shows the simulation results with the ideal (left) and lab-sampled (right) ramp waveforms for $\tau = 100\text{ns}$. It can be seen from Figure 57 that the real ramp waveform is indeed the cause for the pronounced side-lobes.

Deviations from a perfect ramp wave worsen with $\Delta f$: the PSLR for $\tau \leq 100\text{ns}$ was higher than 19 dB, however the corresponding ratio for $\tau = 200\text{ns}$ has dropped to 14.5 dB. The ISLR values degraded from 14.6 dB for $\tau = 20\text{ns}$ to 7.2 dB and only 4.2 dB for $\tau = 100\text{ns}$ and $\tau = 200\text{ns}$, respectively. Figure 58 shows the PSLR and ISLR results for both the simulation and the experiment.

As can be seen from the graphs above, the simulated PSLR and ISLR values degrade, in a near monotonic way, as the generated delay lengthens. It also stands out that even for long simulated TTDs, the performance suggested by simulation remains satisfactory, with both figures of merit above 30 dB. In the experimental results, however, both the PSLR and ISLR values plunge way underneath the 20 dB line as the...
delay prolongs. This difference supports the claim that the non-ideal sawtooth wave used in the phase modulation is majorly responsible for the experimental performance degradation, since it gets more and more distorted as its frequency increases. If it wasn't for this, we might have encountered more subtle performance degradation as a function of delay, like the one obtained through simulation. The sidelobe suppression metrics of the processed waveform degrade due to periodic discontinuities at the $2\pi\tau$ ramp-phase correction term, which manifest in strong parasitic sidelobes, equally spaced by $\tau$. 

As to the third figure of merit, resolution, we can see that the FWHM of the main correlation peaks of both delayed and non-delayed impulse responses were 2 ns. Note that the bias phase $\Delta\phi_0$ from (11) was not monitored during this preliminary
experiment. Control over $\Delta \varphi_0$ would be mandatory when the application of the proposed method is scaled to actual beam-forming using multiple antenna elements.

4.2 Delaying LFM Waveforms Using an Acousto-Optic Modulator

Figure 59 shows the experimental impulse response functions, obtained for five values of $f_{\text{AOM}}$ in the range of 35 - 45 MHz. A relative delay of the impulse response by as much as $\pm 50$ ns is evident, in agreement with (12). As was discussed in the Theory chapter, the delay obtained using an AOM is restricted by the reduced efficiency of the AOM at drive frequencies that are far detuned from its pre-designed value of $f_{\text{AOM,0}} = 40$ MHz.

Figure 59: Measured, normalized impulse responses of 500 MHz-wide, 5 μs-long LFM waveforms. The frequency offsets $f_{\text{AOM}}$ (in MHz) are: 35 (blue), 38 (red), 40 (green), 42 (black), and 45 (magenta). A magnified view of the primary correlation peaks is shown in the inset.
For all values of $\Delta f$, the resolution of the impulse response main lobe was 2 ns.

Figures 60 and 61 show the PSLR and ISLR values for the various frequency offsets. The highest PSLR value is at the $f_{AOM,0}$ frequency, as anticipated, and it decreases as $\Delta f$ increases. The lowest PSLR value is 17.5 dB. The ISLR for all impulse responses is better than 20 dB.
The previous section discussed the experimental results for delaying LFM waveforms using ramp wave. Although in that approach longer delay variations of ±125 ns with equal resolution and PSLR can be reached, the ISLR values for delay variations larger than 20 ns were unacceptably low, between 7-9 dB. The application of an AOM eliminated the multiple periodic correlation sidelobes separated by $\tau$ that hindered the performance of the earlier setup. The remaining sidelobes stem from a residual incident beam that is not shifted, and from anti-Stokes and higher-order scattering in the AOM (figures 62 to 65).

![Figure 62: Experimental correlation of TTD = -50 ns](image1)
![Figure 63: Experimental correlation of TTD = -20 ns](image2)

![Figure 64: Experimental correlation of TTD = +20 ns](image3)
![Figure 65: Experimental correlation of TTD = +50 ns](image4)
The drive voltage $V$ was adjusted to minimize the sidelobes. The range of frequency offset values $\Delta f$ may be increased, in principle, with the construction of double-pass AOM in which the angle of beam deflection remains fixed. Here too, the bias phase $\Delta \phi_0$ (11) was not controlled in the experiment.

4.3 Delaying NLFM Waveforms Using a Phase Modulator

This section discusses the experimental results of the last part of the experiment [45]. This part focuses on delaying the impulse response of NLFM waveforms. Although NLFM is a generic name for a wide variety of waveforms, I will focus on a subset of NLFM waveforms that has a target Fourier transform $U_m(f)$ from (42).

![Figure 66: Experimental correlation of NLFM 4th-order waveforms for TTDs of 0 ns, 10 ns, 20 ns, 50 ns from right to left (k=0.015)](image)

![Figure 67: Simulated correlation of NLFM 4th-order waveforms for TTDs of 0 ns, 10 ns, 20 ns, 50 ns from right to left (k=0.015)](image)

The experimental impulse responses for delayed NLFM 4th-order waveforms are shown in Figure 66. The free parameter is $k = 0.015$, the signal’s bandwidth is $B = 500\text{MHz}$ and its duration is $T = 5\mu\text{s}$. The impulse responses were delayed by 0 ns (blue), 10 ns (red), 20 ns (purple) and 50 ns (pink). Figure 67 shows the corresponding
simulation results for TTDs of 0 ns, 10 ns, 20 ns and 50 ns, from right to left. Both, experiment and simulation, shows a variable delay can be achieved according to (23).

Figures 68 to 73 shows the various impulse responses in more detail. For higher TTDs the experimental impulse responses degrade considerably comparing to the ones from the simulation. The degradation is mainly due to the laboratory equipment that struggles to generate the fast changing phase correction term for high TTDs.
The PSLR values of the NLFM 4th-order impulse responses for the various TTDs are plotted in Figure 74, and their ISLR values in Figure 75. The PSLR values for delays of 10 ns and 20 ns are above 26 dB, and the ISLR values for those delays are 18.1 dB and 16.06 dB, respectively. The resolution for all impulse responses is 4 ns. For comparison, in the first part of the experiment, an LFM waveform was delayed using PM. The ISLR values obtained there were below 16 dB for all delays.
Figures 76 and 77 present, respectively, the experimental and simulated impulse responses for NLFM 8th-order waveform with $k = 0.015$, $B = 500\text{MHz}$, $T = 5\mu\text{s}$ and TTD of 20 ns. Figures 78 and 79 show the impulse responses for NLFM 16th-order waveforms with the same parameters. The PSLR values are 16.7 dB and 18 dB for NLFM 8th-order and 16th-order, respectively, and the ISLR values are 24.2 dB and 27 dB, respectively. The simulated impulse response of 16th-order NLFM delayed by 20 ns, for comparison, has PSLR and ISLR values of 34.08 dB and 29.2 dB, respectively. The resolution for NLFM 8th-order is 5 ns, and for NLFM 16th-order it is 6.8 ns. The experimental results support the theoretical predictions: as the NLFM order $n$ increases, the sidelobes suppression improves at the expense of resolution.

The longer delay of higher-order NLFM waveforms would be limited by the relatively narrow bandwidth of the AFG used in the generation of the phase correction term. It can clearly be seen by the difference between the above figures, for example, the simulation results from Figure 79, which doesn’t have bandwidth limitations, are far
better comparing to the experimental results from Figure 78, both in terms of PSLR and ISLR.

In order to identify the limiting factors for sidelobe suppression, the impulse responses of NLFM waveforms captured at different points along the experimental setup were calculated. Figure 80 shows the impulse response of the 4th-order NLFM waveform that was sampled directly at the AWG output. Figure 81 shows the impulse response of a 4th-order NLFM waveform that was processed only with the RF up-conversion and down-conversion parts, bypassing the optical processing block.
altogether. Figure 82 shows the impulse response of a 4th-order NLFM waveform that was passing through the entire setup but without delaying it.

The PSLR and ISLR values for all the above NLFM waveforms are shown in Table 1. The results of table one indicate that the RF circuitry degrades the PSLR of the waveform from 51 dB to 38 dB, and their ISLR from 32 dB to 26 dB. The degradation is primarily due to a single pronounced sidelobe. The RF-induces sidelobes set an upper limit on the compression performance that could be achieved. At this time, we do not have the know-how or the necessary equipment to improve the RF circuitry at our disposal.

The sidelobes suppression is further degraded by the optical processing. The PSLR and ISLR of non-delayed NLFM waveforms were 32 dB and 23 dB, respectively. The degradation due to the optical processing is smaller than the one caused by the RF circuitry. Nevertheless, the optical processing module is currently restricting the quality of the waveform. Potential performance-degrading mechanisms include noise from optical amplifiers, nonlinearities of the electro-optic modulator and detector, thermal noise at the detector and uneven spectral reflection of the optical sideband form the fiber Bragg grating.

<table>
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<th>From AWG</th>
<th>RF Only</th>
<th>Non-delayed</th>
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<tr>
<td><strong>PSLR [dB]</strong></td>
<td>51</td>
<td>38</td>
<td>32</td>
</tr>
<tr>
<td><strong>ISLR [dB]</strong></td>
<td>32</td>
<td>26</td>
<td>23</td>
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*Table 1: PSLR and ISLR values of the impulse response of 4th-order NLFM waveform from AWG vs. RF only vs. non-delayed*
Experimental Results | Delaying NLFM Waveforms Using a Phase Modulator

Figure 80: Impulse response of 4th-order NLFM waveform measured from the AWG to the scope

Figure 81: Impulse response of 4th-order NLFM waveform processed only with the RF circuitry

Figure 82: Impulse response of 4th-order NLFM waveform without delay
5 Discussion and Conclusions

The long, continuously-variable microwave-photonic delay of chirped waveforms, based on their time-frequency relations, was proposed and demonstrated in this work. These waveforms are prevalent in many radar systems as they provide narrow impulse response functions with low sidelobes despite their long duration. Strictly speaking, the microwave-photonic processing described in this work does not truly delay the waveforms. Nevertheless, I had shown through analysis, simulations and experiments that the method has a nearly equivalent effect on their impulse response functions. The method is applicable, at least in principle, to chirped waveforms of an arbitrary frequency sweep profile, duration, bandwidth, or central radio frequency. The method provides an improvement of up to 2 orders of magnitude in the delay-bandwidth product compared to previously reported approaches. It can provide the long delays that are necessary for beam forming in large phased-array antennas.

The method relies on the application of a low-bandwidth correction term to the instantaneous phase of a single modulation sideband. Thus, delaying high-frequency chirped waveforms can be accomplished using simpler and cheaper equipment. In addition, it allows a greater flexibility in the design of chirped waveform, since waveforms with complex sweep rates can be delayed by the same equipment used to delay simpler waveforms, such as the LFM waveform.

The results obtained in this work were limited by several factors. First of all, more than 5 dB in the ISLR were lost in the RF up-conversion and down-conversion...
parts. Better tuning of the RF circuitry will result in better sidelobe suppression. At this time, we do not have the know-how or the necessary equipment to improve the RF circuitry at our disposal. Further work in the delay of high-order NLFM waveforms would have to address that part of experimental setup. With the RF circuitry improved, the further sidelobe degradation which takes place within the optical module would have to be addressed as well. To that end, the effects of detector and optical amplifier noise, spectral reflectivity of the grating and modulation and detection nonlinearities should be examined closely. In addition, the limited bandwidth of the AFG used for providing the phase-correction term restricted the longer delay of higher-order waveforms. Nevertheless, LFM waveforms were successfully delayed by as much as ±50 ns, and 16th-order NLFM waveforms were delayed by as much as 20 ns, with adequate PSLR and ISLR values. The method can be extended further to the TTD of chirped pulses whose amplitudes are temporally-varying as well, with the application of an electro-optic intensity modulator in series. This will add another freedom in the application of the method to radar systems.

An interesting future experiment would be to feed the output of the basic setup introduced above to an RF-transmitting antenna. The antenna would launch the group-delayed chirped waveform to free space, while another antenna, positioned nearby, would receive the echoes of the signal, as they bounce back from potential targets. Assuming only one significant object is in the vicinity of the transmitting antenna, correlating the transmitted signal with the received one should result in a single peak, whose location corresponds with the distance between the transmitter and the object,
while other, more distanced objects would result in negligible correlation peaks, since the collected power is inversely proportional to the distance cubed. Such an experiment would validate the proposed true-time-delay principle, as the application of the phase-correction term to the chirped waveform would appear to 'push the object away'.

Lastly, it would be worthwhile to incorporate several replicas of the setup described in this work within multiple elements of a PAA, and demonstrate the actual steering of a radar beam. Each element would be fed with a laser carrier and the input chirped waveform. In order to achieve beam steering, however, one must address the absolute phases of the signals passing through each element. So far, the goal was merely to have a single chirped waveform undergo a group-delay, while constant phase-shifts were inconsequential. Without adequate control of individual bias phases, each antenna within the array would delay its input with respect to a different wavefront, and the beam will be spatially distorted. With proper closed-loop phase control, the novel method introduced above could be applicable to beam steering in large phased-array antennas.
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תקציר

עיבוד אותות אלקטרו-מגנטיים בדד. רדיי ומיקרו-רדיו, או בפונולוגיה, יכול להותוך לשתיERVED
פורת מהתווך-50 שנה. לעיבוד אותות אלו בפונולוגיה רדיי-פונולוגיה,חלק מהתווך פונולוגיה
בשהואות לעיבוד אותות אלקטרו-מגנטיים, לרבות אלקטרו-магנטים בוצק, גרידוד של מכונת-מדבק
רובה סרטי בדד, ולסיטונת בפונולוגיה אלקטור-מגנטית. ממון של מודולים

זבליהם-אלקטור-אופטיים, וצביעה לממדים קוליםữa. עותב

אותה המשותפת של מידע זה ليست במילים

 policemen, אך נתון כר במלת-20 שנה, הוא

ישום של קיום השיחה לא藍ות בדד, רדיי-במפתחים אופטיים. עותב שאותה "במחבטית" שיר

לשמחונך ברבי השיחה בתווך משנתה, להבידת התווך המשקיע השיחה פאזה בחלב, על

メント למחנה עותוי מרחב של אלאולה המשוות. רכיבי אלו דרסים להופך רדיי רגב סרטי

ולעומד כתיבות יומיות מתמורות.ሥר, עד לנטון התווך של אלえた המשודרות. רכיבים אלו דרסים להופך רדיי רגב סרטי

כנל- TableColumn.

ישום בבלים חשמליים לש意義 השיחה בתווך משנתה, ושופר של מחובר

ברב. מסמרים, ושלחון רדיי-פונולוגיה למ裨וי השיחה בתווך משנתה, מועדים עיניין. ו

מסתרות מופיעה משטר שים-שם למידה רכיב השיחה בתווך משנתה, באפשרויות פונולוגיה-למורות.

התקדמת התווך בתווך, השניה אואכה של אוזות-רחב, בראש סדרה של יוזמות מתמשכות.

 mz ב_rnnדר nutrita, או אוזות מחשבה של התווך המשותפת דופטרו-דרישות של רכיב

השיחה בתווך. במקומ תŦיע רכיב אוניברסלי, אשר יפה לשלוחות כל אוזות-שוח, או מתמדד

בועד לשמשק החיפש של אוזות-שוח ממציא בשמיש התווך במערכת מ"יכול, ושל מחובר

אותה איויג מובית השיחה בתווך משנתה, ראשית בראש ובראשך שיוול ל Bộתיות לשל השיחה בתווך. במערכת לוייתו

הששות על משקח האוזות האמורות זה ממון לה(Callisto" לשל השיחה בתווך. במערכת לוייתו

על כלכלות של רכיב השיחה בתווך, הצלחת לדוגו להנהיג האוזות עד פי 100 מצלדות

בעבר, והי כי שтвержден על אוזות משקחת של אוזות המשובה.

אותה בצימ במקסיק קוריאני כלכלות אוזות סורק-תדר. כי משedido שמם, אוותות אלו

سورקיט צוות רהב של רדיאר-רבעים על פי מוסר שיוור אורח, מסדר גולדה של כמה מ"ملك.

שערון. נ oko, אוותות אלה בeği משโช את, ממקימי שיוור עבורה מערות מכ"מל. על אַף

maktadır.
המשר האורק של אורות אלו, יינת ל红枣 את האגרוביות שלهما לתוםZO כדי מתור גודל של קונ.

שיגור בודד, עם אוניות-רנמות. הבאת שיווש מגננות מתائك את במקל התערוכות.

אות סורק ההדר העסוק בזיר הוא בוד אינפין תדר תליין (LFM), שב הסרירה של
הختارטר המ牝_between בקבר קובר. את זה הילוי ערבד, לעוף תגרבות התדר שיל מתאינו
באנונת זך גבורה יתסיט. ית האותות סורקית ההדר ניקרין בכלל גאולהות בולמי אינפין תדר-א
לייזרי (NLFM). עקיפ פורפי הופשואત הייתordan, אוחזות אל-כיסות וול לייזרי, אולמות, יש להם começa
יתרון עד פי אותות. יינת להשק חום לעברות עם דרישת מערערים או רכיבים משמיליס, יש
אפרשות לעבר את הסprzedות שלמה, יש לה אחות אל דינוטד ובלא השימשemento סינו
디יגיטליות.

הגייקורי הטאצואונות אוצ לאבעב השתייה של אורות סורקית תדר מתבוסס על הקperPage המובנה ביז
החות טלדר הרוגה (או בז עמד לפקד ראית) אشرط עמד-ברסימ וארישות, האות מוא?><ו על-גב תדר
פוש אופטיק. לאחר מק, מתבצער סינו הפסיט את התדר הרוגה האופטיקית את חת מאיונת צד האופטיקית.
בשלב הבה, מתווספת איבר תיקון-פסטאלאז baz את תדר הרגה. לעסר, ההדר הנושא האופטי משולב
בחזרה, על-ידי התאבהות של ל-סለ אוף תדר התמשכות מתkelig לאות היגלי את סורק תדר מתווק
בתדר הרודי המ꼬וד, ההשפעה של איבר תיקון-פסטאלאז על האות סורק התרדה, בموتאת המסנטן
המתאימות, זה оста מתולחלוול השתייה שלב.

סימוליצית מפרק נאות שיאית לэтажתות ביעסיק שמיוגר 그런 שליול כדי להשתת(core, תדר.

במחלק הניסי, הצלחת הלאותות אורות LFM למסך 450 נוג-שויוט עם אוניות-רנמות דיי.
בנוספ, אורות LFM מסדר 8 נוג-שויוט, במרכז LFM מסדר 8 נוג-שויוט 16 נוג-שויוט
ב-20 נוג-שויוט. לפוס -דייט, יעיבוד רדי-פופטי lässt אורות enjoying לא Żyżn deport.

מקראות עדיני, יניח את העייקורי עם לאורות בולי, משענות ומתחון בתמק, וישלב את מרוץ
הגייקורי באנטונט ומ"מ מרותב אלמנטים.
המתחת לתזה בוצע תחת הנהייתו של ד"ר אבי צדוק
מתקולטה לתנודה באוניברסיטת בר-אילן
השהייה ארוכה ומתחוננת של אותות רדיי סורקין-תדר במאפשרים פוטוניים

אופיר קלינגר

עבורה זו מוגשת חלק המדענות של שם קבלת תואר מוסמך

בפקולטה להנדסת של אוניברסיטת בר-אילן

רמת גן, ישראל

אלול תשע"ב