Non-coherent pulse compression — aperiodic and periodic waveforms

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Abstract: The growing interest in adopting pulse compression waveforms to non-coherent radar and radar-like systems (e.g. lidar) invites this update and review. The authors present different approaches of designing on–off (1, 0) coded envelopes of transmitted waveforms whose returns can be envelope detected and non-coherently processed. Two approaches are discussed for the aperiodic case: (a) Manchester encoding and (b) mismatched reference. For the periodic case, on–off sequences are described, which produce perfect periodic cross-correlation when cross-correlated with one or more integer number of periods of a two-valued reference sequence (1, −1). This study provides comprehensive rules for designing periodic on–off waveforms and their references. The periodic waveform’s high-average duty cycle (over 50%) makes it a ‘quasi continuous wave (CW) non-coherent waveform’, which avoids the pulse–train conflict between average power and unambiguous range. Good experimental results with a laser range finder are presented. Reports on other uses are quoted.

1 Introduction

Direct-detection lidar, non-coherent (magneton) radar, sonar, ultrasound, ground penetrating radar, optical masks and optical time domain reflectometer (OTDR) are examples of non-coherent radar or radar-like systems that are likely to use unmodulated pulses and detect the intensity (envelope) of their reflection. Aperiodic on–off pulse sequences for such systems were discussed under the topic of non-coherent pulse compression (NCPC) [1]. Complementary pairs [2] and periodic sequences [3, 4] were also discussed. The basic approach of creating suitable waveforms for the aperiodic case is to create a unipolar sequence by Manchester encoding a binary sequence known to be useful in coherent pulse compression; or by Manchester encoding a binary complementary pair. The reference sequence with which the received signal is cross-correlated could be either a binary bipolar version of the Manchester-encoded sequence, or a straightforward mismatched filter (MMF) designed for the unipolar sequence. In its first part, the present paper reviews and compares the aperiodic waveforms and their different processing approaches. The second part presents new results for the periodic case. The importance of periodic on–off waveforms stems from the fact that with proper periodic reference sequences, periodic unipolar sequences can yield perfect periodic cross-correlation (PPCC) whose sidelobes are identically zero. Periodic sequences also exhibit high duty cycle which may be attractive to some applications and a hindrance to others. Moreover, described are unique periodic coherent waveforms that can be processed coherently or incoherently and in both cases produce PPCC.

2 Aperiodic on–off waveforms

The discussion and comparison of on–off aperiodic waveforms and their processing approaches will start with a simple example based on the well-known coherent pulse compression sequence Barker 13

\[
S_1 = +1 +1 +1 +1 +1 -1 -1 +1 +1 -1 +1
\]

Moreover, described are unique periodic coherent waveforms that can be processed coherently or incoherently and in both cases produce PPCC.

There are at least two ways in which Barker 13 can be transformed to an on–off transmitted signal: (a) Manchester encoding (+1, 1, 0, −1, 0) and (b) transmitting only the positive elements (+1 → 1, −1 → 0). We will now discuss both approaches.

2.1 Transmitting Manchester-encoded barker 13

Manchester encoding of the sequence \(S_1\) will create the sequence \(S_2\) that can be easily transmitted by pulses from a non-coherent source

\[
S_2 = 1010101010011010011001100110100110011001101100110011011001100110 (2)
\]

Ignoring attenuation, noise and multiple targets, envelope detection of point target reflected \(S_2\) will equal \(S_2\). Aperiodic cross-correlation of \(S_2\) with a reference binary sequence \(S_{ref} = +1, −1, +1, −1\) will produce the output shown in Fig. 1. The positive values resemble the compressed response of a coherent Barker 13. However, the negative deeps cannot be ignored. A weak target whose delay coincides with any negative deep in Fig. 1 (created by the response of a strong target) may not be detected.

A major drawback of the Manchester encoding approach is the doubling of the signal duration, compared with the duration of the coherent signal (assuming equal code element duration). The delay resolution is the width of the sub-pulse, which can be equal or smaller than the duration of the code element.

2.2 Transmitting unipolar Barker 13

An approach that is free of the Manchester encoding drawbacks is to transmit the unipolar version of \(S_1\), namely, \(S_1 = (S_1 + 1)/2\)

\[
S_1 = 11111001110101 (3)
\]

Following non-coherent envelope detection, the receiver cross-correlates the detected envelope with an MMF to \(S_1\), designed to minimise the integrated cross-correlation sidelobes. A twist to the design is to give more weight to the near sidelobes, as shown in [5]. If the extent of the zone defined as near sidelobes is about half the length of the entire sidelobes span, then there is
only negligible increase in the height of the remaining sidelobes. The middle subplot in Fig. 2 shows an example of a minimum integrated sidelobes filter of length 65 (=5 × 13) in which reduction of the cross-correlation near sidelobes was emphasised (bottom subplot).

The unipolar sequence will maintain the correlation property even if the sequences (transmitted and reference) are spread by inserting any fixed number of ‘zeros’ between sequence elements, which we will refer to as ‘zero padding’. Fig. 3 displays an example in which four zeros were inserted after each ‘1’ or ‘0’ in the transmitted signal (and also after each element of the MMF). The range resolution is now determined by the sub-pulse width rather than by the element width.

2.3 Transmitting complementary unipolar pair

Manchester encoding allows sidelobe reduction of on–off coding through the complementary pair concept [2]. We present an example based on the 26 element complementary kernel (Table 1).

Fig. 4 displays the delay response obtained by cross-correlation of the pair of transmitted pulses with the pair of reference pulses. The mainlobe reaches a value equal to the sum of ‘1’ elements in the pair (=52). Here again we see the two immediate large negative sidelobes with half the mainlobe height. Contrary to coherent complementary pair, the near sidelobes do not disappear completely but stay between −1 and 1 values, independent of the length of the sequences. Around a delay equal to the pulse repetition interval (PRI) we observe the expected recurrent lobes resulted from the cross-correlation between pulse #1 and reference #2, with a symmetrical lobe when pulse #2 is correlated with reference #1.

As pointed out and demonstrated in [2] non-coherent processing is sensitive to close targets, whose return may partially coincide. On the other hand, while usage of complementary pairs in coherent systems is limited because of strong sensitivity to Doppler shift, in non-coherent processing that problem has little or no impact.

It should be noted that the use of unipolar complementary pairs was reported with regard to direct-detection OTDR [6]. The approach there did not employ two Manchester-encoded unipolar pulses but four unipolar pulses: (a) the unipolar first sequence, (b) its 1’s complement, (c) the unipolar second sequence and (d) its 1’s complement. The detected returns of (a) and (b) were subtracted, and the detected returns of (c) and (d) were subtracted,
to result in a bipolar estimated versions of the two sequences. Those were then correlated with the two original bipolar sequences.

3 Periodic on–off waveforms

Barker 13 can serve also as a good example to start the discussion on periodic waveforms. Note that the unipolar version, $S_3$, of Barker 13 and the two-valued reference sequence

$$S_4 = \begin{bmatrix} 1 & 1 & 1 & 1 & b & 1 & 1 & b & 1, b = -2 \end{bmatrix}$$

will produce PPCC with a peak of 9 and all off-peak sidelobes identically zero (Fig. 5). As a matter of fact unipolar versions of all Barker sequences will produce PPCC with their respective two-valued reference sequence. The required values of $b$ are: $b = -2$ for Barker 4 and 13, $b = -3$ for Barker 5, and $b = -1$ for Barkers 3, 7 and 11.

In the periodic cases too, the unipolar sequence will maintain the PPCC property even if the sequences (transmitted and reference) are spread by zero padding (inserting any fixed number of ’zeros’ between sequence elements).

When using periodic sequences, we would like to get both good delay resolution (=element or sub-pulse width), large unambiguous delay (=element duration times the number of elements) and high average power (dense elements). This calls for long sequences, much longer than the 13 elements of the longest Barker. In the following sections, we will discuss three families of codes that can provide very long periodic sequences: M-sequences (maximum length shift register sequences), Legendre sequences and Ipatov sequences.

3.1 M-sequences (shift register sequences, maximal-length sequences)

The original binary $\{\pm 1\}$ version of a shift register sequence [7] produces a two-valued ideal periodic auto-correlation, with peak of $N$ (the code length) and uniform off-peak sidelobe level of $-1$. It turns out that the cross-correlation between the unipolar version of the code $\{1, 0\}$ and the binary version $\{+1, -1\}$ is perfect with a peak equal to the number of ’1’s in the code and uniform off-peak sidelobe level which is identically zero. To allow a readable drawing, the example for shift register sequence will be of length 15 (Fig. 6).

The auto-correlation of the coherent bipolar version is insensitive to reversed polarity. The situation is different for the unipolar version. Note that in a unipolar $m$-sequence the number of ’1’ elements is either larger or smaller than the number of ’0’ elements, by one element. For the bipolar $\{\pm 1\}$ reference to produce PPCC, the unipolar signal should be the one in which the number of ’1’s is larger than the number of ’0’s. M-sequences are found in lengths of $N = 2^n - 1$, with $n = 2, 3, \ldots$, which are rather sparse. Between lengths of 1000 and 10,000, there are only four available lengths (1023, 2047, 4095 and 8191). On the other hand, at each length several different codes can be found.

3.2 Legendre sequences (quadratic residue sequences)

A much more dense availability is obtained from Legendre sequences [8]. They are available for lengths $N = 4k - 1$, with $N$ a
prime and \( k \) any integer. Between lengths of 1000 and 10,000 there are 519 available Legendre sequence lengths. However, at each length there is only one sequence (excluding trivial transformations). Here too a binary \( \{+1, -1\} \) reference will result in PPCC as long as the unipolar signal has more ‘1’s than ‘0’s.

### 3.3 Ipatov binary sequences

Ipatov sequences \([9, 10]\) are binary sequences \( \{+1, -1\} \) that originally produce PPCC when the reference signal is \( \{+1, -b\} \), or \( \{+1, -b, -c\} \). Ipatov signals can also be used as unipolar...
signals with a two-valued (or three-valued) reference. A table of available Ipatov signals appears on p. 139 of [10]. For the unipolar signal lengths 5, 13, 21, 40, 121 and 1093, the respective values of b are: 3, 2, 3, 2, 2 and 2. Ipatov 624 is an example when a three-valued reference is required: \( b = 1 \) and \( c = 2 \). The unipolar Ipatov codes have significantly more ‘1’s than ‘0’s. For example, Ipatov 40 has 27 ‘1’s, Ipatov 121 has 81 ‘1’s, Ipatov 171 has 98 ‘1’s, Ipatov 624 has 498 ‘1’s and Ipatov 1093 has 728 ‘1’s.

### 4 Calculating the reference values

As mentioned earlier, the transformation of a binary signal \( s_b \) to a unipolar signal \( \tilde{s}_u \) is linear

\[
s_u = (s_b + 1)/2
\]

If the binary signal \( s_b \) and its reference signal \( s_{ref} \) have a two-level periodic cross-correlation, a PPCC can be obtained between the unipolar signal \( s_u \) and a reference signal \( s_{ref} \) which is a linear transformation of the reference for the binary signal

\[
\tilde{s}_{ref} = \alpha s_{ref} + \beta
\]

By definition, the two-level periodic cross-correlation function of the binary signal and its reference is

\[
C(n) = \sum_{k=0}^{N-1} s_b(k) \cdot s_{ref}^\ast((n + k) \mod N)
\]

\[
= \begin{cases} E, & n = 0 \mod N \\ F, & n \neq 0 \mod N \end{cases}
\]

where \( E = \sum_{k=0}^{N-1} s_b(k) \cdot s_{ref}^\ast(k) \) is the correlation’s mainlobe level (energy for auto-correlation) and \( F \) is the off-centre correlation level. The conjugate symbol \( ^\ast \) appears in (7) and the following equations in order to adhere to the general definition of correlation. However, in our case the signals, envelopes and sequences are all real.

Demanding PPCC for the unipolar and reference signals implies

\[
\tilde{C}(n) = \sum_{k=0}^{N-1} s_u(k) \cdot \tilde{s}_{ref}^\ast((n + k) \mod N) = \begin{cases} K, & n = 0 \mod N \\ 0, & n \neq 0 \mod N \end{cases}
\]

where the peak value \( K \) is arbitrarily defined as the number of 1’s in one period of \( s_{ref} \).

Combining (5)-(8) yields the required values for \( \alpha \) and \( \beta \) for PPCC

\[
\beta = -\frac{F + N \cdot \tilde{s}_{ref}}{N + N \cdot s_u}, \quad \alpha = \frac{2K}{E + N \cdot \tilde{s}_{ref} + 2\beta K}
\]

where the notation \( \tilde{a} \) denotes the average of the sequence \( a \) over one period. As the values of a binary signal \( s_b \) can only be ±1, the average over one period is

\[
\tilde{s}_b = \frac{(+1) \cdot K + (-1) \cdot (N - K)}{N} = \frac{2K - N}{N}
\]

Therefore, (9) can be simplified to

\[
\beta = -\frac{F + N \cdot \tilde{s}_{ref}}{2K}, \quad \alpha = \frac{2K}{E - F}
\]

### 4.1 Barker codes

For Barker codes \( E = N \), and the reference signal \( s_{ref} \) is identical to the binary signal \( s_b \). By applying the linear transformation (6) on the binary reference using the values in (11) reveals that the alphabet values of reference signal \( \tilde{s}_{ref} \) are \{ \( \alpha (\beta + 1), \alpha (\beta - 1) \} = \{1, -(4K(N-F)+1)\} \). Table 2 summarises the parameters for Barker codes.

### 4.2 M-sequences and Legendre sequences

M-sequences and Legendre sequences (as well as other types of binary sequences that correspond to cyclic Hadamard difference
intensity and 7 (or 13) times the PRI. Namely, the signal will behave like a conventional
The effective response extends the unambiguous delay to 7 (or 13) times the PRI. Namely, the signal will behave like a conventional

The number of 1s in a (q, v, r)-Ipatov sequence of order \(n\) is \(K = q^{n-1} - (q-r)\) (for a detailed treatment of Ipatov binary sequences see [9]). Table 3 summarises the parameters for several Ipatov codes.

Table 3 Parameters for several Ipatov codes

<table>
<thead>
<tr>
<th>(q, v, r)</th>
<th>n</th>
<th>K</th>
<th>F</th>
<th>α</th>
<th>β</th>
<th>Reference alphabet</th>
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<tr>
<td>3, 1, 1</td>
<td>2</td>
<td>4</td>
<td>3</td>
<td>0</td>
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<td>–1/3</td>
</tr>
<tr>
<td>3, 1, 2</td>
<td>3</td>
<td>13</td>
<td>9</td>
<td>0</td>
<td>3/3</td>
<td>–1/3</td>
</tr>
<tr>
<td>4, 1, 1</td>
<td>2</td>
<td>5</td>
<td>4</td>
<td>0</td>
<td>4/3</td>
<td>–1/4</td>
</tr>
<tr>
<td>5, 1, 1</td>
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<td>121</td>
<td>81</td>
<td>0</td>
<td>3/4</td>
<td>–1/3</td>
</tr>
<tr>
<td>4, 1, 2</td>
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<td>5</td>
<td>4</td>
<td>0</td>
<td>4/3</td>
<td>–1/4</td>
</tr>
<tr>
<td>5, 2, 1</td>
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<td>24</td>
<td>15</td>
<td>0</td>
<td>1/6</td>
<td>–1</td>
</tr>
<tr>
<td>7, 3, 2</td>
<td>2</td>
<td>24</td>
<td>14</td>
<td>0</td>
<td>1/4</td>
<td>–1</td>
</tr>
<tr>
<td>3</td>
<td>171</td>
<td>98</td>
<td>0</td>
<td>1/25</td>
<td>–1</td>
<td></td>
</tr>
</tbody>
</table>

5 Periodic waveforms that allow both coherent and non-coherent processing

The finding that unipolar Barker codes can produce PPCC with the corresponding binary Barker codes prompted an interesting example of combining coherent and non-coherent periodic processing. The example utilises Ipatov’s periodic ternary codes [11, 12] of length 7 \(\{1, 1, \ldots, 0, 1, 0, 1\}\) and 13 \(\{1, 1, \ldots, 1, 1, 0, 0, 1, \ldots, 0, -1, 0, 0, 1\}\). Each code can be transmitted as a train of pulses with inter-pulse coding of both amplitude and phase. When periodically transmitted, synchronously detected and coherently processed, such a periodic pulse–train yields perfect periodic auto-correlation. The effective response extends the unambiguous delay to 7 (or 13) times the PRI. Namely, the signal will behave like a conventional unencoded coherent pulse–train with 4 (or 9) times the pulse intensity and 7 (or 13) times the PRI.

However, when envelope detected and incoherently processed, the pulse–train becomes a unipolar Barker 7 (or Barker 13), which can also produce perfect correlation, albeit when cross-correlated with its periodic reference. For Ipatov 7 the signals involved in the two processing approaches are summarized in Table 4. The signals for the Ipatov 13 case can be easily deduced, but the negative values in the reference will be –2. Note that at length 13 there is another essentially different ternary code \(\{1, -1, -1, 1, 1, 0, 0, -1, 0, -1, 0, -1\}\). The identical (for both processing concepts) periodic correlations, seen on the last row of Table 4, are somewhat deceiving. This is a noise-free correlation result. The fact that the sum of squares of the reference elements in the non-coherent processor is 7/4 (or 25/9) times the sum in the coherent case, implies considerably more sensitivity of the non-coherent processor to the presence of noise. There are additional advantages to the coherent processing but the option to switch to non-coherent processing may be useful if the scene’s coherency is lost.

6 Performances with noise (simulation)

The simulation was performed with periodic Ipatov 121 unipolar signal, a section of which appears in the lower subplot of Fig. 7. There were five samples per code element (‘bit’). Independent additive white Gaussian noise (AWGN) was added to each received sample. An example of the same signal section with additive noise (signal-to-noise ratio (SNR)=0 dB) is shown in the top subplot. The reference Ipatov 121 contained 3 periods. Recall that for unipolar Ipatov 121 the reference replaces each 0 with –2. A section (longer than one period) of the output of the periodic cross-correlation is shown in Fig. 8. The peak sidelobe ratio (PSLR) in Fig. 8 is 19 dB, which promises both high probability of detection and good delay measurement accuracy.

7 Experimental results with a laser range finder and other applications

Pulse–train laser range finders suffer from the conflict between unambiguous range and average signal power [13, 14]. Large unambiguous range requires large PRI; hence, the target is illuminated by low-average power. In our periodic waveform, the average power is a constant and the unambiguous range can be increased by increasing the code length. To demonstrate this advantage NCPC of a periodic sequence was employed in a laser range-finding experiment. Light from a laser diode was repeatedly modulated by a 4003 bit-long unipolar Legendre sequence. The bit duration was 1 ns, corresponding to range resolution \(\Delta R\approx 15\) cm [15]. The modulated waveform was amplified to a transmission optical power of 22.5 dBm and launched toward a target at 100 m distance. The target was a sheet of white paper whose relative power reflectivity is \(\rho = 0.07\). Reflected echoes were collected by a lens of 10 cm diameter into an avalanche photo-receiver, which is characterised by noise-equivalent optical power of \(-43\) dBm. The optical power of the collected echoes was \(-58\) dBm, corresponding to an SNR of the electrical signal at the receiver output of \(-30\) dB. The received signal was cross-correlated with 45 periods of the reference, implying an integration time of 180 µs. The resulted range response is shown in Fig. 9 (zoom on ±30 m off the 100 m nominal range). Without zoom we would see: 0...0\(\times 4003+0.15\) m. Range resolution of 15 cm and a PSLR of over 27 dB are obtained.
Fig. 7  Section of unipolar Ipatov 121 signal (bottom) and received with AWGN (top). SNR = 0 dB

Fig. 8  Section (longer than one period) of the output of the periodic cross-correlation. SNR = 0 dB

Fig. 9  Measured delay response of incoherent, continuously coded lidar echo, collected from a Lambertian reflector target (white paper) at 100 m distance. The measurement electronic SNR was −30 dB
An earlier version of a laser range finder was reported in [16]. It used an aperiodic unipolar coded pulse based on an 1112 element binary sequence with low-correlation sidelobes. The same signal was also used in high-resolution, distributed fibre sensors of local temperature and mechanical strain. In this particular application, hundreds of weak reflection events were simultaneously interrogated [17]. The use of an on–off periodic code in an optical mask, in order to improve image resolution, was recently reported [18]. Aperiodic optical masks are used in many other fields. An example from cytometry is described in [19].

8 Eclipsing and interference

When the returns from one target coincide with the returns from a second target, an interference problem may result. Such interference is more pronounced in non-coherent processing than in coherent processing. When an aperiodic waveform is used, the interference effect declines as the delay difference between the two targets increases [3]. The interference disappears when the delay difference is longer than twice the signal duration. When a periodic signal is used, the interference from a second target may occur independent of the delay difference. This is why a periodic waveform suits a lidar application. The extremely narrow beam illumination makes it unlikely that a second target, especially at much closer range than the main target, will be also illuminated. A laser range finder is a practical and good example for our technique because there is negligible or no direct reception of the transmitted signal, and there is usually only one target within the illumination beam.

The near 50% duty cycle of transmitted periodic signals based on m-sequences. Legendre sequences and some Ipatov sequences, increases the eclipsing probability. Good isolation (e.g. in a bistatic configuration) could be one solution. The effect of eclipsing can be reduced by reducing the duty cycle, as shown by the zero padding described earlier. Another approach is choosing sequences with inherent low-average duty cycle. One example is a unipolar Ipatov sequence of length 133, whose average duty cycle is about 10%.

Integration of many coded pulses, or periods, improves SNR and also contributes to reduced interference from neighbouring targets. Non-coherent sub-pulses are likely to have random initial phases and possibly also some variability in carrier frequency. Sub-pulses returning from two targets are therefore likely to have their phase difference different for each coinciding sub-pulses. At the receiving antenna destructive interferences vary and are averaged, thanks to the large number of sub-pulses in an aperiodic pulse or in a periodic sequence. Furthermore, additional averaging is due to the large number of pulses or periods integrated during the relatively long processing interval. We therefore expect acceptable performances even in multi-target situation.

An experimental setup is shown in Fig. 10, where two closely spaced targets were positioned at a range of 273 m, in a way that will cause simultaneous illumination by the laser beam. The laser details are as outlined in Section 7. To accommodate the additional range and the halving of each target area, the processing included averaging of 1000 measurements. The resulted response (around a range of 273 m) is given in Fig. 11. Note the third peak, with relative intensity of −15 dB and additional delay equal to the delay between the two targets. We attribute the third peak to the triple bounce.

9 Conclusions

This paper reviewed NCPC and presented progress in both aperiodic and periodic on–off waveforms and in their non-coherent processing. A prominent new result in aperiodic waveforms is the transmission of their original unipolar version and correspondingly using an MMF in the receiver. For periodic waveforms, the use of unipolar versions of Shift register, Legendre and Ipatov sequences was described, which, when cross-correlated with their appropriate two-valued (or three-valued) bipolar reference, produce sidelobe-free periodic response. Good experimental results were
shown, as obtained from a low-power lidar range finder that uses a 4003 element on–off periodic code based on a Legendre sequence. The experimental results also showed good tolerance to a two-target illumination scene. Periodic on–off waveforms are especially suited to lidar range measurements because (a) lasers lend themselves to very fast on–off keying and their non-coherent detection is relatively simple to implement, (b) periodic waveforms avoid the pulse–train conflict between average power and unambiguous range and (c) due to the extremely narrow laser illumination, it is unlikely that clutter or close targets will be illuminated and eclipse the periodic waveform returning from the desired distant target.

10 References

12. Ipatov, V.P.: ‘Spread spectrum and CDMA: principles and applications’ (Wiley, Chichester, 2005), Sec. 6.11.3