Distributed Energy Efficient Channel Allocation

Oshri Naparstek, Member, IEEE, S. M. Zafaruddin, Member, IEEE, Amir Leshem, Senior Member, IEEE, and Eduard Jorswieck, Senior Member, IEEE

Abstract—Design of energy efficient protocols for modern wireless systems has become an important area of research. In this paper, we propose a distributed optimization algorithm for the channel assignment problem for multiple interfering transceiver pairs that cannot communicate with each other. We first modify the auction algorithm for maximal energy efficiency and show that the problem can be solved without explicit message passing using the carrier sense multiple access (CSMA) protocol. We then develop a novel scheme by converting the channel assignment problem into perfect matchings on bipartite graphs. The proposed scheme improves the energy efficiency and does not require any explicit message passing or a shared memory between the users. We derive bounds on the convergence rate and show that the proposed algorithm converges faster than the distributed auction algorithm and achieves near-optimal performance under Rayleigh fading channels. We also present an asymptotic performance analysis of the fast matching algorithm for energy efficient resource allocation and prove the optimality for a large enough number of users and number of channels. Finally, we provide numerical assessments that confirm the faster convergence of the proposed algorithm compared to the distributed auction algorithm.

Index Terms—Auction algorithm, bipartite graph, channel assignment, energy efficiency (EE), linear programming, distributed protocol, multi-access channel, Rayleigh fading channel, wireless networks.

I. INTRODUCTION

Channel allocation is a long-term issue in communication systems that has been investigated extensively for various wireless networks [2]–[6]. In many centralized systems (such as, cellular networks, digital subscriber lines, and optical systems etc.), Orthogonal Frequency Division Multiplexing (OFDM) is a standard technique to meet the high demand of efficient spectrum utilization. The centralized approach can obtain the optimal solution for joint power and sub-carrier allocation in the downlink direction [7]–[10] and sub-carrier allocation in the uplink direction [11]–[13] due to the global view of the whole network. However, there are some disadvantages that limit the practicality of the centralized approach such as significant signaling overhead, increased implementation complexity, and higher latency in dealing with resource allocation problems. On the other hand, emerging wireless networking paradigms such as cognitive radio networks, ad-hoc networks, and device-to-device communications are inherently distributed. Hence, computationally efficient distributed protocols are needed as a practical solution for resource allocation problems in dense and heterogeneous wireless environments. In this paper, we focus on developing distributed protocols for dynamic spectrum access where $N$ users shares $K$ orthogonal channels in an open sharing cognitive radio model [14].

The simplest distributed approach is to use random channel allocation. Although the random allocation has a very fast convergence time and is very simple to implement through standard random access techniques, its performance is significantly worse than the best centralized strategy for a large number of users. Distributed resource allocation using non-cooperative game theory has been shown to be promising for various network scenarios [15]–[19]. However, the game theoretic approaches may require some form of knowledge on other players actions. Recently, authors in [20] proposed a game theoretic approach with limited feedback for channel allocation problem over a frequency-selective channel. It is noted that the Nash equilibrium requires one-sided stability notions such that equilibrium deviations are evaluated unilaterally per player which may not be practical for the assignment problems between distinct sets of players. Furthermore, the tractability of equilibrium in game-theoretic methods require some structure in the objective functions which may not always be satisfied for practical wireless scenarios.

Recently, matching theory has emerged as a promising technique for wireless resource allocation which can overcome some limitations of centralized optimization and game theory theoretic approach [21]–[26]. While the stable allocation based on the Gale Shapley algorithm [27] turns out to be a greedy assignment and almost optimal for Rayleigh fading channels [22], it is desirable to obtain the optimal allocation. In order to compute the optimal distributed solution, the auction technique developed by Bertsekas [28] was revised for sum-rate maximization using multichannel opportunistic carrier sense multiple access (CSMA) [23]. However, one of the disadvantages of the distributed auction algorithm is its convergence time which might be too long in practical scenarios. It was shown in [23] that the number of iterations needed until the convergence of the distributed auction algorithm for the sum-rate maximization is bounded by $O(N^3)$. Other approaches to the distributed channel allocation problem include game theoretic stable matching [27], bargaining solutions [29]–[31], and distributed allocation using multichannel ALOHA [32],

This research was supported by the German-Israel Foundation for Scientific Research and Development under Grant I-1243-406.10. A preliminary work for sum-rate optimization was presented at the IEEE 18th International Conference on Digital Signal Processing (DSP), Fira, Greece, July 1-3, 2013 [1].

Oshri Naparstek was with the Faculty of Engineering, Bar-Ilan university, Ramat-Gan, 52900, Israel. Currently, he is with the Rafael Advanced Defense Systems Ltd., Haifa 31021, Israel (email: oshri8@gmail.com).

S. M. Zafaruddin was with the Faculty of Engineering, Bar-Ilan university, Ramat-Gan, 52900, Israel. Currently, he is with the Dept. of Electrical and Electronics Engineering, BITS Pilani, Pilani-333031, India (email: syed.zafaruddin@pilani.bits-pilani.ac.in).

Amir Leshem is with the Faculty of Engineering, Bar-Ilan university, Ramat-Gan, 52900, Israel (email: leshema@biu.ac.il).

Eduard Jorswieck is with the Faculty of Electrical and Computer Engineering, Communications Laboratory, TU Dresden, Dresden 01062, Germany (e-mail: eduard.jorswieck@tu-dresden.de).
While for decades communication networks have been designed to optimize performance measures such as bit error-rate, latency, data-rate, etc., in the last few years the issue of energy-efficient network design has gained more importance [34], [35]. This is due to the fact that the diverse and ubiquitous high-data-rate multimedia wireless applications cause rapid energy consumption of battery-powered mobile devices and improvement in battery capacity is unable to keep pace with increasing power dissipation in the transceiver circuitry. This has sparked research interests to develop energy efficient transmission protocols with a focus on channel allocation [36]–[45]. Although the channel allocation problem is solved by mapping it to a maximum matching problem in recent publications such as [38], [45], the issue of convergence in a large distributed network has not been considered.

In this paper, we propose a distributed optimization algorithm for the channel allocation problem to increase the energy efficiency of multiple interfering transceiver pairs that cannot communicate with each other. The main contributions of the paper are listed as follows:

- We show that the optimal energy efficient allocation can be computed distributely without explicit message passing using a modified distributed auction algorithm and opportunistic carrier sensing.
- We develop a novel scheme by converting the channel assignment problem of the distributed auction algorithm into finding perfect matchings on bipartite graphs. The proposed fast matching algorithm reduces the convergence time of the distributed auction algorithm and achieves near-optimal energy efficiency.
- We analyze the performance of the fast matching algorithm for energy efficiency maximization in the Rayleigh fading channels and show that the expected energy efficiency index achieved by the matching algorithm approaches the optimal energy efficiency index for large enough number of users and number of resources.
- We prove that the expected number of iterations until convergence of the fast matching algorithm is \( O(N \log(N)) \), where \( N \) denotes the number of users.
- We provide numerical assessments for various wireless channels comparing the energy efficiency performance of the fast matching algorithm with the optimal centralized method.

A. Related Works

The channel allocation problem is a special case of a very well studied combinatorial optimization problem known as the assignment problem. The original formulation of the assignment problem is as follows: Given a matrix \( A \) find a matrix \( P \) that maximize the trace \( tr AP \). The first specialized algorithm to solve the assignment problem was the Hungarian method suggested by Kuhn in 1955 [46] which is centralized and required full knowledge of the utility matrix. In 1979 a distributed relaxation of the assignment problem called the auction algorithm was introduced by Bertsekas [28]. It is called the auction algorithm since it was inspired by auctions where the users bid for objects and raise their bids until the highest bidder wins the object. The auction algorithm did not require full knowledge of the utility matrix but did need some kind of explicit message passing mechanism or a shared memory. The auction algorithm has previously been suggested as a way to solve the channel assignment problem [47], [48]. However, these modified auction algorithms also required shared memory. Based only on the local prices, authors in [23] revised the auction technique of Bertsekas [28] and implemented an optimal algorithm using multichannel opportunistic carrier sense multiple access (CSMA). In [49] and [50] a single user channel is considered and the optimization is carried out through transmit power control. In [51] the channel assignment problem is formulated as a many-to-one matching game under the limitation that each primary channel can only be assigned to one secondary user. In contrast, in [52]–[60], multiuser interference channels are considered and a competitive scenario in which users selfishly aim at individual EE maximization is addressed. Centralized and decentralized resource allocation in multi-hop networks for energy-efficiency maximization is studied in [61], [62]. One of the disadvantages of the distributed auction algorithm is its convergence time which might be too long in practical scenarios. It was proven that the matching algorithm is asymptotically optimal for sum rate maximization [23].

Information and communication technologies (ICT) represent about 2% of the entire world's energy consumption, and the situation is likely to reach a point where ICT equipment in large cities will require more energy than is actually available [63]. For data networks, contrary to the intuition, more energy is consumed in access networks than in core networks [64]. This happens because the number of devices in access networks (i.e. mobile terminals, base stations, and data modems installed on customers' premises) is much larger than the number of communication devices (routers, multiplexers, etc.) in the core network. This has sparked research in the field of wireless networks with a focus on the problem of energy efficient channel allocation schemes [36]–[45]. A joint maximization of the network energy and spectrum efficiency for user association in a heterogeneous network was considered in [65]. It is emphasized that the convergence issue for energy efficient channel allocation in a fully distributed network has not been yet considered.

B. Notations and Organization

Important notations used in the paper have been listed in Table 1. The paper is organized as follows: Section II describes the system model and defines the energy efficiency of a network. In Section III, we formulate the maximal energy efficiency channel allocation problem. Section IV discusses the distributed auction algorithm. In Section V we present a fast converging algorithm for a relaxation to the maximal energy efficiency problem and show that the algorithm for the relaxed problem terminates within \( O(N \log(N)) \) iterations with high probability. The performance of the fast matching algorithm under Rayleigh fading channel is studied in Section VI. In Section VII we discuss simulated results for the proposed algorithm. Finally, Section VIII concludes the paper.
TABLE I
THE LIST OF MAIN NOTATIONS

| $N$, $n$ | Number of users, user index |
| $K$, $k$ | Number of channels, channel index |
| $U$, $U_k$ | $N \times K$ utility matrix |
| $\gamma_n$ | SINR of the $n$-th user at $k$-th channel |
| $C_n$ | Cost of the $n$-th user at $k$-th channel |
| $P_{n,k}$ | Transmit power of the $n$-th user at $k$-th channel |
| $\eta_n$ | Channel coefficient of the $n$-th user at $k$-th channel |
| $\rho_{n,k}$ | Circuit transmit power of the $n$-th user at $k$-th channel |
| $D_{n,k}$ | Transmission bandwidth |
| $P_{max}$ | Maximum transmit power |
| $H_n$ | Channel coefficient of the $n$-th channel |
| $\kappa_n$ | $k$-th channel assigned to $n$-user |
| $H_n^c$ | Data rate of the $n$-th user |
| $\omega_n^c$ | Maximum profit of the $n$-user |
| $\sigma_n^e$ | Noise variance of the $n$-th user |
| $\zeta_n$ | Second maximum profit of the $n$-user |
| $\log(.)$ | Logarithm of base 2 |
| $\epsilon$ | A small positive constant |

Fig. 1. System model for channel allocation ($f_1, f_2, f_3$, i.e., $K = 3$) among three Tx-Rx pairs ($N = 3$) before convergence. Here, two users (user 2 and user 3) transmit the signal over same channel $f_1$ which results into interference and remain unassigned. However, only user 1 transmits over $f_2$ and thus become assigned. At the convergence, each Tx-Rx pair should be assigned with a unique channel.

II. SYSTEM DESCRIPTION AND ENERGY EFFICIENCY
Before formulating the optimization problem for maximal energy efficiency, we describe the system model and present various definitions of energy efficiency for communication networks used in the literature.

A. System Model
We consider an ad hoc network setup in an open spectrum sharing model with $K$ channels and $N$ users. A user is defined as a transmitter-receiver pair with a direct link and channels refer to the spectrum bands that have a specific central frequency. Since the users are distributed, we assume that there is no central coordinator or dedicated control channel, and each user independently searches available channels and makes its own access decision. We assume that the users are in a close range, and they can interfere with each other when simultaneously transmitting over the same channel. A toy example for channel allocation problem with $K = N = 3$ is depicted in Fig. 1. Deregulation of spectrum usage (for example, communication over the unlicensed 2.4 GHz band) is another paradigm where users can sense the whole spectrum and transmit over various frequency bands. This open spectrum access model of cognitive radio [14] is different from traditional multi-channel resource allocation scenario where only a few common channels are available. This requires a fundamental shift from traditional centralized mechanisms towards self-optimizing approaches in a distributed fashion.

Due to resource and hardware limitations for sensing, we assume that each user can only choose one channel to sense and access in each time slot [66] i.e., $N = K$. In general, we are interested in the scenario where $K > N$. This scenario arises in broadband spectrum access where there are a large number of available channels for consideration\(^1\). This assumption can always be fulfilled: if $N > K$ then $N - K$ artificial channels with rate zero could be added in order to make $N = K$ [28]. However, in a typical WiFi scenario (where there are more users than channels $N > K$, and back-off periods are slotted), a joint frequency domain multiple access (FDMA) and time domain multiple access (TDMA) approach would be superior. Thus, when $N > K$, we can solve the channel allocation problem by extra TDMA over the frequencies.

B. Definitions of Energy Efficiency
The first and most widely used definition of EE is the ratio between the throughput and the transmit power (see [35], [58], [67], and references therein). Another proposed metric uses the goodput in place of the throughput [59]. In all of the above works, as far as the computation of the consumed energy is concerned, only the transmit power is considered, whereas the power that is dissipated in the electronic circuitry of each terminal in order to keep the terminal active is neglected. This assumption was relaxed in [68], by defining the consumed power as the sum of the transmit power plus a constant term, independent of the transmit power, which models the circuit power needed to operate the terminal. Following [68], in [49], [50], [60] the consumed power is also defined as the sum of the transmitted power and the circuit power. Moreover, in these papers the throughput is replaced by the achievable rate in the definition of the energy efficiency.

Essentially, each user $n$ is not only interested in maximizing performance in terms of achieved signal-to-interference-ratio (SINR) $\gamma_n$ (interference can arise from adjacent cells) at an assigned channel, but also in saving as much battery energy as possible. This trade-off is well modeled by defining the EE of a given user $n$, as the ratio between efficiency function

\(^1\)This scenario is particularly suitable for the cognitive radio network where the secondary users may not be restricted to a certain frequency band and can search among a large set of channels.
f(.) which measures the SINR-based performance of user \( n \) and the power consumed to attain this performance level [35], [67]:

\[
EE_n = \frac{f(\gamma_n)}{P_n + P_n^{\text{c}}},
\]

(1)

where \( P_n \) is transmit power of the \( n \)-th user at a channel, and \( P_n^{\text{c}} \) is the power required by the \( n \)-th transmitter electronic circuitry to operate the device, and is dissipated even during non-transmission periods. For further details on the circuitry power term, we refer the reader to [69] and references therein, where several power consumption models for wireless networks are developed. As for \( f(\gamma_n) \), in principle it can be a generic increasing function of the \( n \)-th user’s SINR, with \( f(0) = 0 \) and such that (1) tends to approach zero for growing transmit power \( P_n \). Two widely used efficiency functions are:

1) \( f(\gamma_n) = R_n(1 - e^{-\gamma_n}) \), where \( R_n \) is the communication rate of the \( n \)-th user and \( (1 - e^{-\gamma_n}) \) is an approximation of the probability of correct symbol reception. A similar approximation was used in [53], [54]. Thus, \( f(\cdot) \) is the number of bits that are correctly demodulated at a receiver per unit of time.

2) \( f(\gamma_n) = B\log(1 + \gamma_n) \), where \( B \) is the communication bandwidth. For strictly static channels \( f \) represents the \( n \)-th user’s achievable rate. For quasi-static channels, the use of \( f(\cdot) \) for resource allocation purposes is still well-motivated in view of the assumption that the channel coefficients remain constant for longer than the resource allocation phase.

Variations of option 1) are also available in the literature in the form of \( f(\gamma_n) = R_n(1 - e^{-\gamma_n})^S \) and \( f(\gamma_n) = R_n(1 - e^{-\gamma_n}/2)^S \), and in this case the function \( f(\cdot) \) is an approximation of the probability of error-free reception of a data packet of \( S \) symbols. An EE that considers both the case of \( S > 1 \) and the circuit power \( P_n^{\text{c}} \) was considered in [68] for a single-hop system. There, it was shown that an equilibrium for the power allocation algorithm exists, but the convergence could not be proved. However, in the following we choose to focus on the equally well-motivated case of \( S = 1 \). Thus, for any \( S \), the resulting EE in (1) is a measure of the number of bits that are correctly decoded at the receiver, per unit of time and per Joule of energy drained from the battery of the transmitter. Moreover, all the efficiency functions that we consider result in an EE which is measured in bits per Joule, thus representing a natural measure of the efficiency with which each Joule of energy drained from the battery is being used.

Two pertinent welfare performance metrics are the average EE and the system global EE (GEE), respectively defined by [35], [58], [67] as:

\[
EE = \frac{1}{N} \sum_{n=1}^{N} \frac{f(\gamma_n)}{P_n + P_n^{\text{c}}},
\]

(2)

and

\[
\text{GEE} = \frac{\sum_{n=1}^{N} f(\gamma_n)}{\sum_{n=1}^{N} (P_n + P_n^{\text{c}})},
\]

(3)

It is noted that the assumption \( K = N \) ensures that each user is assigned with a single channel with SINR \( \gamma_n \). Thus, definition of EE in (2) and (3) does not contain channel index \( k \). Customarily, the GEE in (3) is used to describe the energy efficiency of the overall system while EE in (2) focuses on energy efficiency of individual users.

III. PROBLEM FORMULATION

Let \( P \) be a matrix of transmission powers where each channel is used by a single user and \( P_n \) is defined as the minimal power needed by the \( n \)-th user to achieve a preassigned target rate \( R_n \) in the \( k \)-th channel. We assume all the users have continuous sensing over all channels [22], [70]. This is a reasonable assumption since the sensing power is only a small percentage of the total power in wireless networks [71]. We also assume that only one user can transmit on each channel in each time slot and consider alien interference from other networks as the additive noise. Each user experiences frequency selective channel caused by both channel statistics and out of cell interference (since out of cell interference affects different users in a different way).

We propose a fully distributed method to maximize the energy efficiency of the system using different utility functions as described in the following:

1) Average EE under rate constraint: Under the expected rate constraints, each entry \( P_{n,k} \) of the matrix \( P \) is chosen to be the solution to the following ergodic rate equation

\[
R_n = \mathbb{E} \left[ \log_2 \left( 1 + \frac{|H_{n,k}|^2 P_{n,k}}{\sigma_n^2} \right) \right],
\]

(4)

where \( H_{n,k} \) is the channel coefficient for the \( n \)-th user at the \( k \)-th frequency band and \( \sigma_n^2 \) is the noise variance at the receiver of the \( n \)-th user. By using Jensen’s inequality, we get

\[
R_n \leq \log_2 \left( 1 + \frac{\mathbb{E} \left[ (|H_{n,k}|^2 P_{n,k}) \right]}{\sigma_n^2} \right).
\]

(5)

The solution to the above equation gives us to minimize the energy

\[
P_{n,k} \geq \frac{(2R_n - 1) \sigma_n^2}{\mathbb{E} \left[ (|H_{n,k}|^2) \right]}.
\]

(6)

Using equations (6) and (2), we define the utility matrix \( U^\text{av} \) such that \( U^\text{av}_{n,k} \) is chosen to be the optimal individual EE for the \( n \)-th user in the \( k \)-th channel under rate constraint \( R_n \)

\[
U^\text{av}_{n,k} = \frac{R_n}{\sigma_n^2 + |H_{n,k}|^2 + P_{n,k}}.
\]

(7)

where \( \phi = (2R_n - 1) \) is a function that makes the power \( P_{n,k} \) used on the \( k \)-th channel by the \( n \)-th user to satisfy a QoS requirement.

2) Average EE under goodput constraint: Another problem we solve is the energy efficient channel assignment under a goodput requirement. The achievable goodput is defined as the rate of the successfully transmitted symbols. In this case \( P_{n,k} \) is chosen to fulfill a goodput requirement for a given symbol error rate (SER):

\[
T_n = R_n (1 - \text{SER}) = R_n \left( 1 - e^{-\frac{|H_{n,k}|^2 P_{n,k}}{\sigma_n^2}} \right),
\]

(8)
and for a fixed $R_n > 0$ and $T_n$. The solution for $P_{n,k}$ is given by

$$P_{n,k} = \frac{\log \left( 1 - \frac{R_n}{\sigma_n^2} \right)}{|H_{n,k}|^2},$$ \hspace{1cm} (9)

and goodput utility with $\phi = \log \left( 1 - \frac{R_n}{\sigma_n^2} \right)$ is given by

$$U_{n,k}^{\text{good}} = \frac{R_n}{(\phi + \sigma_n^2) + P_{n,k}(\epsilon)}.$$ \hspace{1cm} (10)

3) Global EE under rate constraint: The third problem is the energy efficiency maximization with respect to the GEE criterion. Assuming preassigned target rates for the users, the problem simplifies into a power minimization problem under rate constraints. We assume that the instantaneous transmission power is limited per user by $P_{\text{max}}$. For simplicity, we formulate the utility of the GEE as a maximization problem. We define the utility matrix $U_{n,k}^{\text{GEE}}$ for the GEE criterion as

$$U_{n,k}^{\text{GEE}} = \begin{cases} P_{\text{max}} - P_{n,k}, & P_{n,k} \leq P_{\text{max}} \\ 0, & P_{n,k} > P_{\text{max}}. \end{cases}$$ \hspace{1cm} (11)

Combining the utility functions in (7), (10), and (11), the maximum energy efficiency problem can be formulated as an integer programming problem in a general form as

$$\max_{\eta} \sum_{n=1}^{N} \sum_{k=1}^{K} U_{n,k} \eta_{n,k} \hspace{1cm} \text{s.t.,}$$

$$\sum_{k} \eta_{n,k} = 1, \quad \forall n = 1, 2, ..., N$$

$$\sum_{n} \eta_{n,k} = 1, \quad \forall k = 1, 2, ..., K$$

$$\eta_{n,k} \in \{0, 1\}, \quad \forall n,k,$$

where $U_{n,k}$ denotes either $U_{n,k}^{\text{GEE}}$ or $U_{n,k}^{\text{good}}$ and $\eta_{n,k}$ is a channel assignment variable. Here, $\eta_{n,k} \in \{0, 1\}$ equals 1 if the $k$-th channel is assigned to the $n$-th user otherwise $\eta_{n,k}$ becomes 0. The constraint matrix of the problem in (12) is totally unimodular. Thus, the solution to the relaxed problem where we replace the integer constraint by $0 \leq \eta_{n,k} \leq 1$ is also the solution to the original problem. The relaxed problem is a linear programming (LP) problem and can be solved efficiently in a centralized manner by LP solutions methods such as the Hungarian method [72]. Although the original problem in (12) is relaxed, we prove that after the relaxation we can still achieve asymptotically optimal results in much lower time complexity than solving the original problem.

In the next section, we describe a distributed auction algorithm for the channel assignment problem in (12) to achieve maximal energy efficiency of the system. Subsequently, we describe a fast matching algorithm to reduce the convergence time of the distributed auction algorithm.

IV. DISTRIBUTED AUCTION ALGORITHM

We propose the use of a fully distributed channel assignment algorithm for maximal energy efficiency that does not require any explicit message passing or a shared memory between the users. The algorithm relies on the auction algorithm [28] and the distributed algorithm suggested in [23] for sum-rate maximization. The distributed protocol consists of a bidding stage and an assignment stage. The description of the algorithm is as follows. The utility matrix $U$ is an $N \times K$ matrix of energy efficiency indexes and $C$ is an $N \times K$ cost matrix. The cost of a channel $C_{n,k}$ is a unitless number that represents how much user $n$ wants channel $k$ in comparison to the other users. The utility parameter $U_{n,k}$ defines the energy efficiency of user $n$ if the $k$-th channel is assigned. We define the profit of user $n$ from channel $k$ as the reward (i.e. energy efficiency index) minus the price of the channel $U_{n,k} - C_{n,k}$. In the initialization stage each user sets the cost for all of the channels to be 0; i.e., $C_{n,k} = 0, \forall n,k$, select $\epsilon > 0$ and sets his state to unassigned. $\tilde{k}_n$ is defined as the most profitable channel of the $n$-th user:

$$\tilde{k}_n = \arg \max_k (U_{n,k} - C_{n,k}).$$ \hspace{1cm} (13)

The distributed protocol proceeds in iterations. In each iteration two stages are sequentially performed, a bidding stage where users raise the price on their most profitable channel and an assignment stage where channels are assigned to the users who proposed the highest prices. In the bidding stage, each unassigned user $n$ find his most profitable channel $\tilde{k}_n$, profit from that channel $\zeta_n$, and second most profitable channel $\omega_n$

$$\tilde{k}_n = \arg \max_k (U_{n,k} - C_{n,k})$$

$$\zeta_n = U_{n,\tilde{k}_n} - C_{n,\tilde{k}_n}$$

$$\omega_n = \max_{k \neq \tilde{k}_n} (U_{n,k} - C_{n,k}).$$ \hspace{1cm} (14)

Each unassigned user raises the price on his most profitable channel by

$$C_{n,\tilde{k}_n} = C_{n,\tilde{k}_n} + \zeta_n - \omega_n + \epsilon,$$ \hspace{1cm} (15)

where $\epsilon$ is a predetermined positive constant that can be seen as the price of participating in the auction. After the unassigned users update their prices, all the users bid on their most profitable channels. If user $n$ gets assigned to channel $k$ he continues to bid on that channel without raising his bid. If a user $n$ is unassigned he bids on $\tilde{k}_n$ with the new bid $C_{n,\tilde{k}_n}$. In the assignment stage each channel is assigned to the highest bidding user. A channel without bids stays unassigned and users who were not assigned to channels become unassigned. The bidding and assignment stages proceed in iterations until all the users are assigned to channels. Once all of the users are assigned to channels, no one raises his bid and as a result the assignment becomes static. When all the users are assigned we say that the algorithm has converged. The distributed auction algorithm appears in Algorithm 1.

It was proven in [23] that the solution of the distributed algorithm is at most $N \epsilon$ smaller from the optimal one. Note that choosing $\epsilon$ sufficiently small, (for integer utilities, as is the typical case in communication with quantized data rates) we can achieve the optimal solution.

The distributed auction algorithm can be implemented using an opportunistic CSMA protocol without the use of explicit message passing among users. However, the only coordination requirement between users lies with an auctioneer to decide which user made the highest bid. The opportunistic CSMA can be used as an auctioneer. We can define the reward that each user $n$ gets from channel $k$ to be the energy efficiency of
Algorithm 1 Distributed Auction Algorithm [23]

1: Select $\epsilon > 0$, set all the users as unassigned and set $C_{n,k} = 0$, $\forall n,k$

Repeat

2: Each unassigned user $n$ calculates his own maximum profit:
   $\zeta_n = \max_k(U_{n,k} - C_{n,k})$.

3: Each unassigned user $n$ calculates his second maximum profit:
   $k_n = \arg \max_k(U_{n,k} - C_{n,k})$
   $\omega_n = \max_{k \neq k_n}(U_{n,k} - C_{n,k})$.

4: Each unassigned user $n$ updates the price of his best channel
   $k_n$ to $C_{n,k_n} = C_{n,k_n} + \zeta_n - \omega_n + \epsilon$.

5: All the users bid. The unassigned users bid on their new best channel with the updated bid. The assigned users bid on the last channel they bid on and with the same price.

6: Assign channel to the highest bidder (channels with no bids stay unassigned).

Until all users are assigned.

that channel $U_{n,k}$. Using the opportunistic CSMA scheme, each user $n$ tries to access his best profit channel as defined in (13) with a backoff time of $\tau_n = f(k_n)$ where $f(x)$ is a positive monotonically decreasing function. The price $C_{n,k}$ is determined and updated if necessary as described in Algorithm 1. The prices and their corresponding waiting times must converge in a finite number of iterations as in the distributed auction algorithm.

It was shown in [23] that the number of iterations needed until the convergence of the distributed auction algorithm is bounded by $O(N^3)$. In the next sections, we suggest a relaxation to the maximal energy efficiency problem. We show that the suggested relaxation is asymptotically optimal and can be solved with $O(N \log(N))$ expected number of iterations.

V. Fast Matching Algorithm for Channel Assignment

In this section, we describe the fast matching algorithm, its relation with the bipartite graph, and a distributed implementation using the CSMA algorithm.

A. Fast Matching Algorithm

In (12), we have formulated the maximum energy efficiency channel assignment problem for arbitrary values. In the previous section, we showed that this problem can be optimally solved in a fully distributed manner using the distributed auction algorithm. However, the expected convergence time for the near-optimal distributed auction algorithm might be too high for practical use. To speed up the convergence time, we develop a novel scheme by considering a relaxation to the channel assignment problem where each channel can either be "good" or "bad". Essentially, channel $k$ is a good channel (corresponding to the channel gain) for the $n$-th user with respect to a properly chosen threshold. We represent a good channel by 1 and a bad channel by 0. Hence, the main idea of the fast matching algorithm is to transform $U$ into a binary 0, 1 matrix $\tilde{U}$, and then apply the matching algorithm on $\tilde{U}$. The transformation from $U$ to $\tilde{U}$ is done by applying a judiciously chosen threshold $a_n^{\text{thresh}} \geq 0$ for each row:

$$\tilde{U}_{n,k} = \begin{cases} 1, & U_{n,k} \geq a_n^{\text{thresh}} \\ 0, & U_{n,k} < a_n^{\text{thresh}}. \end{cases}$$

(16)

To ensure asymptotically optimal solutions to the max-energy efficiency problem, $a_n^{\text{thresh}}$ must satisfy the following requirements:

1) Only the best channels (corresponding to the channel gain) of each user should be above $a_n^{\text{thresh}}$.

2) $\tilde{U}$ should contain a perfect matching with high probability.

The first condition ensures that the solution to $\tilde{U}$ can provide a good solution to the max-sum problem. The second condition ensures that with high probability all of the users will get assigned by the algorithm. We define a parameter $m$ such that $m \log(N)$ best channels of each user are above the threshold.

Assume that each row $n$ of $\tilde{U}$ consists of i.i.d random variables and assume that the cumulative distribution function (CDF) of each entry of $U_{n,k}$ is given by $F_n$. A proper choice of $a_n^{\text{thresh}}$ would be

$$a_n^{\text{thresh}} = F_n^{-1}\left(1 - \frac{m \log(N)}{N}\right),$$

(17)

where $m > 2$ [73]. The choice of the threshold satisfies the first condition since only $m \log(N)$ best channels of each user are above the threshold. Theorem by Erdős and Rényi in [74] ensures that with high probability $\tilde{U}$ contains a perfect matching if $a_n^{\text{thresh}} \geq 0$ for all $n = 1, 2, ..., N$. Hence, the proposed algorithm can converge faster with high probability when target rate of each user be chosen independently such that with high probability $a_n^{\text{thresh}} \geq 0$ for all $n = 1, 2, ..., N$.

Since every channel can be either good or bad, the utility matrix of the relaxed problem becomes $\{0,1\}^{N \times K}$, which can be represented using bipartite graphs. Thus, the channel assignment problem for energy efficient transmissions is reduced finding perfect matchings on bipartite graphs.

B. Maximum Cardinality Matching on Bipartite Graphs

To formulate the maximum energy efficiency assignment problem as a matching problem on bipartite graph and for theoretical analysis, we need various definitions and relevant results on bipartite graph as presented in [73]. Using these definitions, we define the maximum cardinality matching problem as follows: Let $G = (U, V, E)$ be bipartite graph with vertex sets $|U| = |V| = N$ and an edge set $E$. Find a matching $M$ such that $|M|$ is maximal. Here, $U$ and $V$ in the bipartite graph represent the users and the channels, respectively, and the edges represent the energy efficiency of each user in each channel.

The maximum cardinality matching (MCM) problem can also be formulated as the max-energy efficiency problem (12) where the reward matrix is a binary matrix with 0, 1 values. We now present an algorithm that finds a maximum cardinality matching on bipartite graphs which can be implemented in a fully distributed manner. This iterative algorithm assigns
Algorithm 2 Algorithm for maximal cardinality matching

1: Initialize $h_v = 0, \forall v \in V$, $U_{\text{free}} = \{1, 2, ..., N\}$ and set $M = 0$
2: while $|M| < N$ do
3:     Choose $u \in U_{\text{free}} \triangleright$ An unassigned user is chosen
4:     $j = \arg\min_{v \in E_u} h_v \triangleright$ Choose a channel with minimum access value
5:     $M = M \cup \{(u, j)\} \triangleright$ Assign user $u$ on channel $j$
6:     $u_{\text{old}} = \{u \in U : (u, j) \in M\} \triangleright$ User $u_{\text{old}}$ was assigned earlier on channel $j$
7:     $M = M \setminus (u_{\text{old}}, j) \triangleright$ Unassign user $u_{\text{old}}$ from channel $j$
8:     $U_{\text{free}} = U_{\text{free}} \cup u_{\text{old}} \setminus \{u\} \triangleright$ Update the set of free users
9:     $h_j = h_j + 1 \triangleright$ Increase the channel access value by one
10: end while

an unassigned user to a channel according to the following scheme: Each channel $k \in K$ is assigned a value $h_k$ that represents how many times the channel was reassigned to different users. Let $h^{(i)} = [h_1, h_2, ..., h_K]$ be the vector of the values of the channels on the $i$-th iteration. At the beginning of the algorithm all the values of the channels are initialized to 0; i.e.,

$$h^{(0)} = 0$$

(18)

Let $U_{\text{free}}$ be the set of all free users. In each iteration, an unassigned user $u \in U_{\text{free}}$ is chosen and assigned to the channel with a minimal value he can access and raises its value by 1. The MCM algorithm is summarized in Algorithm 2.

C. Expected Number of Iterations of Fast Matching Algorithm

In this section, we analyze the expected number of iterations until the algorithm converges for random bipartite graphs. $G = (U, V, E)$ is called a random bipartite graph if $G$ is a bipartite graph and the edges in $E$ are independently chosen with probability $p$, i.e.,

$$\Pr((u, v) \in E) = p, \ \forall u \in U, \ \forall v \in V.$$  

(19)

Denote the set of all random bipartite graphs with vertex sets $|U| = |V| = N$ and probability $p$ for an edge by $B(N, p)$. The following known result on perfect matching in random bipartite graphs was proven by Erdős and Rényi in [74] and Motwani in [75]:

Theorem (Erdős and Rényi [74]). Let $\epsilon > 0$, $p = \frac{(1+\epsilon)\log(N)}{N}$, and $G \in B(N, p)$ then

$$\lim_{N \to \infty} \Pr(G \text{ contains a perfect matching}) - e^{-2N^{-\epsilon}} = 0.$$  

(20)

Theorem (Motwani [75]). Let $G \in B(N, p)$, where $p \geq \frac{(1+\epsilon)\log(N)}{N}$ and $\epsilon > 0$ then for every $\gamma > 0$ there exists $N_\gamma$ such that for every $N \geq N_\gamma$

$$\Pr(G \in B(N, p)) \geq 1 - N^{-\gamma}.$$  

(21)

The next theorem proven in [73] shows that for random bipartite graphs with $p \geq \frac{(1+\epsilon)\log(N)}{N}$ the number of iterations until the convergence of the algorithm is less than $cN \log(N)$ with high probability, where $c > 0$ is a constant.

Theorem (Naparstek and Leshem [73]). Let $G = (U, V, E)$ be a random bipartite graph with $|U| = |V| = N$, $p \geq \frac{(1+\epsilon)\log(N)}{N}$ and $\epsilon > 0$. Let $T$ be the number of iterations until the algorithm converges then:

$$\lim_{N \to \infty} \Pr \left( T \leq \frac{cN \log(N)}{\log(Np)} \right) = 1.$$  

(22)

Above theorem ensures that the fast matching finds a perfect matching with a probability that approaches 1 in $O(N \log(N))$ iterations.

D. Distributed Implementation of Fast Matching Algorithm

We implement the fast matching algorithm without the use of explicit message passing using the opportunistic CSMA. The opportunistic CSMA [76] is a distributed transmission protocol suggested for wireless sensor networks. It is composed of carrier sensing and a waiting strategy. Since continuous sensing of all channels by all users is assumed, each user in the network calculates a fitness measure $\psi_n$ and maps it into a waiting time $\tau_n$ based on a predetermined common decreasing function $f(\psi_n)$. Here, each user waits until the waiting time ends and if no one transmitted on its most wanted channel then it is allowed to transmit. Hence, the user with the highest $\psi_n$ transmits in the channel. This can be seen as a distributed winner determination algorithm where the winner gets the channel. Note that since instantaneous sensing is assumed it implies that there are no collisions. This is because the probability of two users having the same random listening time is zero.

The fully distributed fast matching algorithm for maximal energy efficiency using prioritized CSMA proceeds as follows: At the beginning of each time slot each unassigned user finds the channel with the lowest value among the good channels (defined in (16) i.e., one of the best $m \log(N)$ channels). Next, each unassigned user waits a random amount of time and if no one transmitted on any of the channels before the waiting time then that user transmits a busy signal on the channel with lowest value and raises the value of the channels by 1. Each assigned user waits for the maximum time allowed $\tau_{\text{max}}$. If no user transmits on the assigned channel until $\tau_{\text{max}}$, then the assigned user transmits on the channel. When a user senses that a busy signal was transmitted on a channel before $\tau_{\text{max}}$, that user raises the value of that channel by 1. The schematic description of algorithm is depicted in Algorithm 3.

VI. PERFORMANCE OF FAST MATCHING ALGORITHM UNDER WIRELESS FADING CHANNEL

In this section, we analyze the expected number of iterations required by the fast matching algorithm in a Rayleigh fading channel, and show that the proposed algorithm is asymptotically optimal for the maximally energy-efficient channel assignment problem.
Algorithm 3 Fast Matching Algorithm Using Opportunistic CSMA

Each user \( n \) performs the following:

1: Initialize \( h^{(0)} = 0 \), set assigned = false, set \( b_n \) to be the indices of \( m \log(N) \) best channels of the \( n \)-th user, and set \( N_{\text{iter}} = 0 \).

2: Repeat

3: \( N_{\text{iter}} = N_{\text{iter}} + 1 \)

4: Find channel \( k \) to be assigned to a user \( n \) with a minimal value \( \hat{k}_n = \arg\min_{k \in b_n} h_k \) > Choose a channel with minimum access value.

5: Choose random backoff time \( \tau_n \).

6: if a busy tone was transmitted on channel \( k \) before \( \tau_n \) then

7: No transmission attempt by the \( n \)-th user in the current time slot.

8: \( h_k = h_k + 1 \) > Increase the channel access value by one.

9: else

10: Transmit a busy tone on the \( \hat{k}_n \)-th channel.

11: \( h_{\hat{k}_n} = h_{\hat{k}_n} + 1 \)

12: Set assigned=true

13: end if

14: else

15: if a busy tone was transmitted on channel \( \hat{k}_n \) before \( \tau_{\text{max}} \) (where \( \tau_{\text{max}} \) is the maximum listening time) then

16: set \( h_k = h_k + 1 \)

17: \( N_{\text{iter}} = N_{\text{iter}} + 1 \)

18: Set assigned = false

19: else

20: transmit a busy tone on the \( k \)-th channel.

21: Set assigned = true

22: end if

23: end if

24: Until all users are assigned or \( N_{\text{iter}} = (N-1)^2 \)

25: Run the distributed auction algorithm from [23].

26: end if

A. Target Rates for Rayleigh Fading Channels

We model the channels of each user as Rayleigh fading channels; i.e., the channel attenuation \( |H_{n,k}|^2 \) is an exponential random variable given by

\[
|H_{n,k}|^2 = G_{n,k} \cdot F_{n,k} \cdot \frac{1}{r_n^\alpha}, \tag{23}
\]

where \( G_{n,k} \) is a global normalizing factor, \( F_{n,k} \) is an exponentially distributed gain (due to the Rayleigh fading channel with a multi-path effect), \( r_n \) is the distance between the \( n \)-th transmitter from its receiver and \( \alpha \) is the path loss exponent. Hence, the PDF of \( |H_{n,k}|^2 \) is:

\[
f_{|H_{n,k}|^2}(x) = \lambda_{n,k} x^\alpha e^{-\lambda_{n,k} x}, \tag{24}
\]

where

\[
\lambda_{n,k} = \frac{r_n^\alpha}{G_{n,k}} = \frac{1}{\mathbb{E}(|H_{n,k}|^2)}. \tag{25}
\]

The instantaneous rate of user \( n \) in channel \( k \) is given by [77]:

\[
R_{n,k} = \log_2 \left( 1 + \frac{|H_{n,k}|^2 P_{n,k}}{\sigma_n^2} \right). \tag{26}
\]

We now present sufficient conditions on the rate of users such that the fast matching algorithm converges within \( O(N\log(N)) \) expected number of iterations.

Theorem 1. The fast matching algorithm converges within \( O(N\log(N)) \) expected number of iterations in a Rayleigh fading channel with SNR \( \bar{\gamma} = \frac{P_{\text{max}} \mathbb{E}(|H_{n,k}|^2)}{\sigma_n^2} \) if the following requirement is satisfied for all \( n \):

\[
R_n \leq \log_2 \left( 1 + \bar{\gamma} \log \left( \frac{N}{m \log(N)} \right) \right) \tag{27}
\]

for the rate requirement, and

\[
\frac{T_n}{R_n} \geq 1 - \left( 1 + \frac{N}{m \log(N)} \right)^{\bar{\gamma}} \tag{28}
\]

for the goodput requirement.

**Proof:** The proof is presented in Appendix A. \( \blacksquare \)

Theorem 1 provides conditions on the target rate of each user under which the proposed algorithm has a faster convergence rate. Examining (27) reveals that a target rate equal to the capacity of the channel satisfies the rate requirement for faster convergence. Note that users do not require external knowledge or message passing among users since the conditions can be verified by each user independently. Hence each user can choose a target rate and verify the convergence conditions without relying on other users.

B. Asymptotic Optimality

We now show that the fast channel assignment is asymptotically optimal for Rayleigh fading with properly chosen target rates.

Theorem 2. Let \( A_{\text{OPT}}^{\text{GEE}} \) be the optimal solution to the max-energy efficiency problem for the global energy efficiency defined in (3) and let \( A_{\text{FMA}}^{\text{GEE}} \) be the solution obtained by the fast matching algorithm. If the rates satisfy (16) and a perfect matching exists, then for Rayleigh fading channels:

\[
\lim_{N \to \infty} \frac{\mathbb{E}\{A_{\text{FMA}}^{\text{GEE}}\}}{\mathbb{E}\{A_{\text{OPT}}^{\text{GEE}}\}} = 1 \tag{29}
\]

**Proof:** The proof is presented in Appendix B. \( \blacksquare \)

Theorem 3. Let \( A_{\text{OPT}}^{\text{EE}} \) be the optimal solution to the max-energy efficiency problem for the individual energy efficiency defined in (2) and let \( A_{\text{FMA}}^{\text{EE}} \) be the solution obtained by the fast matching algorithm. If the rates satisfy (16) and a perfect matching exists, then for Rayleigh fading channels:

\[
\lim_{N \to \infty} \frac{\mathbb{E}\{A_{\text{FMA}}^{\text{EE}}\}}{\mathbb{E}\{A_{\text{OPT}}^{\text{EE}}\}} = 1 \tag{30}
\]

**Proof:** The proof is presented in Appendix C. \( \blacksquare \)

Using the asymptotic results of Theorem 2 and Theorem 3, performance of the fast matching algorithm approaches to the optimal solutions for large enough users and channels. Since analytical treatment for convergence analysis is intractable, we demonstrate the asymptotic performance through simulation results in the next section.
TABLE II
SIMULATION PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Tx-Rx Pairs or users</td>
<td>2 to 512</td>
</tr>
<tr>
<td>Antenna gain</td>
<td>0 dBi</td>
</tr>
<tr>
<td>Parameter, m</td>
<td>2 − s</td>
</tr>
<tr>
<td>Maximum spectral efficiency</td>
<td>&gt; 5 bits/sec/Hz</td>
</tr>
<tr>
<td>Thermal noise power, N₀</td>
<td>−1/4 dBm/Hz</td>
</tr>
<tr>
<td>Noise figure</td>
<td>&gt; 3 dB</td>
</tr>
<tr>
<td>Mobility of users</td>
<td>3 km/h</td>
</tr>
</tbody>
</table>
| Path Loss                        | (i) LTE, α = 3, \( \sigma_{\text{dB}} = 4 \) dB (distance range: 50m ≤ r ≤ 500m)  
(ii) 5G-Uma (NLOS), \( \sigma_{\text{dB}} = 6 \) dB (distance range: 45m ≤ r ≤ 1429m)  
(iii) 5G-Umi-SC (NLOS), \( \sigma_{\text{dB}} = 7.82 \) dB (distance range: 19m ≤ r ≤ 272m)  
(iv) 5G-Shopping mall (LOS), \( \sigma_{\text{dB}} = 2 \) dB (distance range: 5m ≤ r ≤ 150m) |
| Carrier frequency                | LTE: 2 GHz                     
5G: 6 GHz                        |
| Fading Channel                   | LTE: EPA (9 taps)              
5G: TDL-A (23 taps), delay span 100ns |
| Bandwidth                        | LTE: 1800 MHz                  
5G: 720 KHz                      |
| UMa: Urban macro, UMi: Urban micro, SC: Street Canyon, LOS: line of sight, NLOS: non-line of sight, \( \sigma_{\text{dB}} \): log-normal shadowing factor. |

VII. NUMERICAL AND SIMULATION ANALYSIS

In this section, we demonstrate the performance of the fast matching algorithm using computer simulations based on the MATLAB software. We compared the proposed algorithm with the greedy algorithm, distributed auction algorithm, and centralized Hungarian method under various channel and network scenarios. In the greedy algorithm, the users maximize their profits selfishly without considering reward for other users, and thus not optimal for a network of large users. The performance metric is the required transmission power to achieve a desired spectral efficiency. We also compared the global energy efficiency performance of the network.

We fixed the parameter \( m = 2.5 \) to get \( m \log(N) \) channels above the threshold. The target rates were chosen using (37) and the threshold was computed using \( \alpha_{\text{thresh}} = F^{-1}(1 - \frac{m \log(N)}{N}) \). We considered network of various size ranging from \( N = 2 \) to \( N = 512 \) users. These users are assumed to be distributed uniformly. The users were assumed to be moving at a speed of 3 km/h. We considered channel models from ETSI 3rd Generation Partnership Project (3GPP) and used long term evaluation (LTE) and 5G channels for our simulations [78], [79]. The center frequency of carrier was taken as 2 GHz for the LTE and 6 GHz for the 5G channels. The path loss for LTE channels was generated using Friis transmission equation with antenna gain 0 dBi and path loss coefficient \( \alpha = 3 \). However, path loss for 5G channels was generated based on different network configurations [78], [80], as depicted in Table II. The users also experience long-term shadowing and the short term fading. The log-normal spreading factor ranges from 2 dB to 7.8 dB. For the LTE, the channel of each user pair was generated by the extended pedestrian A model (EPA) with 9 random taps [78], and tapped delay line (TDL) A-model with 23 taps for the 5G channels with delay spread ranging from 10 ns to 1000 ns. The channel bandwidth was divided into \( N \) sub-channels each with 180 KHz for the LTE and 720 KHz for the 5G system [78]. The simulations were averaged over 5000 iterations. The simulation parameters are listed in Table II.

First, we investigated the number of iterations achieved by the fast matching algorithm compared to the distributed auction algorithm. In Fig. 2a, we considered a network of 50 users where any user can select randomly one of the LTE and 5G channels, as listed in Table II, and plotted the number of iterations for each channel realizations. We verify that the number of iterations required by the fast matching algorithm does not exceed \( N \log(N) \). The figure shows that the simulations support theoretical result on the number of iterations for convergence in channel allocation to achieve an spectral efficiency of 5 bits/sec/Hz for each user with minimum transmit power. We also compared the expected number of iterations of the fast matching algorithm with the distributed auction algorithm until convergence for various network scenarios. The comparison is shown in Fig. 2b. As predicted, the expected number of iterations needed by the fast matching algorithm is smaller than the expected number of iterations by the distributed auction algorithm. Moreover, the expected number of iterations is almost independent of network scenario.

Next, we demonstrated the average transmit power requirement to achieve a desired target rate (i.e. spectral efficiency taken 5 bits/sec/Hz) by various algorithms under various channel scenarios in Fig. 3. It can be seen that the fast matching algorithms performs better than the greedy method. However, the proposed algorithm requires higher transmit power.
power than the auction method but requires significantly less number of iterations in convergence, as shown in Fig. 2b. As expected, the centralized scheme using Hungarian requires the minimum average transmit power. Moreover, the optimal distributed scheme, the auction method, performs very close to the Hungarian method. Plots show that an increase in the number of users reduces the power requirement of each user significantly. This increases the energy efficiency of network. The simulations were further verified by considering three different delay spread for 5G UMa channels, as depicted in Fig. 4.

Next, we compared the global energy efficiency performance of various network using different algorithms as shown in Fig. 5 and Fig. 6. Although the distributed auction algorithm achieves the maximum energy efficiency for various network scenarios (as depicted in Fig. 5), the proposed fast matching algorithm provide greater gain per iteration (as depicted in Fig. 6) due to less number of iterations required for its convergence.

Finally, we demonstrate the asymptotic optimality of the proposed algorithm by simulating over large network of different scenarios and plotting the ratio of global energy efficiency of the fast matching algorithm and the greedy algorithm to the optimal distributed auction, as shown in Fig. 7. The figure shows that the fast matching algorithm approaches the optimal auction algorithm as network size increases as proved through analysis in Theorem 2. However, it requires much larger system sizes to achieve the optimal solution since the increase in the reward becomes slower as number of users increases (i.e., when \( N > 200 \)). It can also be seen that the greedy algorithm is not asymptotic optimal and performs poorly compared with the fast matching algorithm.

**VIII. CONCLUSION**

We presented a fully distributed protocol for resource allocation to optimize the energy efficiency of a wireless network. We converted the channel assignment problem into
the proposed algorithm performs better than the greedy method. Simulations on realistic LTE and 5G channel show that asymptotically optimal solutions to the energy efficiency problem can be found using carrier sensing protocol to compute the channel assignments. We also showed that under mild assumptions on the fading distribution, the fast matching algorithm produces assignments with high probability. The algorithm was based on a version of the auction algorithm which solves a matching problem. The inverse CDF of $F_{|H_{n,k}|^2}(x)$ is exponentially distributed with the CDF:

$$F_{|H_{n,k}|^2}(x) = 1 - e^{-\lambda_{n,k} x}.$$  \hspace{1cm} (31)

Hence, the CDF of $P_{n,k}$ is given by

$$F_{P_{n,k}}(x) = e^{-\lambda_{n,k} \sigma_n^2 (2R_n - 1)/\log(2)}.$$  \hspace{1cm} (32)

The expected number of iterations will be $\mathcal{O}(N \log(N))$ only if there exists a perfect matching in the graph with a probability of at least $1 - \frac{2}{N^{m-1}}$. From Theorem of Erdős and Rényi in [74], there exists a perfect matching with a probability of at least $1 - \frac{2}{N^{m-1}}$ only if the expected number of edges connected to each vertex is at least $m \log(N)$ for $m > 2$. Hence, to fulfill this requirement, the expected number of channels in which each user is able to transmit without violating his power constraint is at least $m \log(N)$. A user can transmit on a channel only if the power needed on the channel to achieve the target rate is less than $P_{\max}$. Thus,

$$F_{P_{n,k}}(P_{\max}) \geq \frac{m \log(N)}{N}.$$  \hspace{1cm} (33)

Hence, the target rates for each user must satisfy:

$$F_{P_{n,k}}^{-1}\left(\frac{m \log(N)}{N}\right) \leq P_{\max}.$$  \hspace{1cm} (34)

The inverse CDF of $F_{P_{n,k}}(x)$ is given by

$$F_{P_{n,k}}^{-1}(x) = \frac{\lambda_{n,k} \sigma_n^2 (2R_n - 1)}{\log(2)}.$$  \hspace{1cm} (35)

Using (34) and (35), the target rates must satisfy:

$$F_{P_{n,k}}^{-1}\left(\frac{m \log(N)}{N}\right) = \frac{\lambda_{n,k} \sigma_n^2 (2R_n - 1)}{\log\left(\frac{N}{m \log(N)}\right)} \leq P_{\max}.$$  \hspace{1cm} (36)

Simplifying (36), we get:

$$R_n \leq \log_2 \left(1 + \frac{P_{\max} \log\left(\frac{N}{m \log(N)}\right)}{\lambda_{n,k} \sigma_n^2}\right).$$  \hspace{1cm} (37)

Thus, the fast matching algorithm converges with an expected $\mathcal{O}(N \log(N))$ number of iterations.

Following the same steps for the goodput requirement, we get

$$\frac{T_n}{R_n} \geq 1 - \left(1 + \frac{N}{m \log(N)}\right)^{\frac{P_{\max}}{\lambda_{n,k} \sigma_n^2}}.$$  \hspace{1cm} (38)

Using $\lambda_{n,k} = \frac{1}{E(|H_{n,k}|^2)}$, $\gamma = \frac{P_{\max} E(|H_{n,k}|^2)}{\sigma_n^2}$ in (37) and (38), we get the Theorem 1.
APPENDIX B: PROOF OF THEOREM 2 (GEE ASYMPTOTIC OPTIMALITY)

First, we require some known results from order statistics [83] to accomplish the proof for asymptotic analysis, as presented below:

Definition 1. Let $A$ be a random variable with CDF $F_A(r)$ and let $A_{1:N} < A_{2:N} < \ldots < A_{N:N}$ be random variables obtained by taking $N$ samples from $A$ and ordering the samples in an increasing order. $A_{k:N}$ is called the $k$-th order statistics of $A$ with $N$ samples.

Definition 2. Let $k_N$ be a function of $N$ such that $k_N \to \infty$ as $N \to \infty$ and $\lim_{N \to \infty} \frac{k_N}{N} = 0$ then $A_{N-k_N+1:N}$ and $A_{k_N:N}$ are called intermediate order statistics.

Definition 3. Let $F(x)$ be a differentiable, absolutely continuous distribution function. If

$$
\lim_{x \to F^{-1}(1)} \frac{d}{dx} \left(\frac{1 - F(x)}{f(x)}\right) = 0 \quad (39)
$$

then the third Von Mises condition is satisfied.

Theorem (Falk [84]). Let $F$ be an absolutely continuous CDF satisfying one of the Von Mises conditions. Suppose $k_N \to \infty$ as $N \to \infty$ and $\lim_{N \to \infty} \frac{k_N}{N} = 0$. Then there exist norming constants $\alpha_N$ and $\beta_N > 0$ such that

$$
\frac{A_{N-k_N+1:N} - \alpha_N}{\beta_N} \to N(0, 1) \quad (40)
$$

where $\alpha_N = F^{-1}(1 - \frac{k_N}{N})$ and $\beta_N = \frac{\sqrt{N}}{\sqrt{\alpha_N}}$.

Next, we derive probability distribution function and quantile function of $U_{n,k}$ given that the power is lower than $P_{\text{max}}$:

$$
F_{U_{n,k}^{\text{GEE}}}(x) = 1 - e^{-\frac{a_{n,k}}{P_{\text{max}}}} e^{-\frac{\log(N)}{P_{\text{max}}^2}} \quad (41)
$$

where $a_{n,k} = \lambda_n \sigma_n^2 (2^{R_n} - 1)$ and

$$
F_{U_{n,k}^{\text{GEE}}}^{-1} (\rho) = P_{\text{max}} + \frac{a_{n,k}}{\log(1 - \rho) - \log(N) + \frac{a_{n,k}}{P_{\text{max}}}} \quad (42)
$$

We now observe that the probability distribution in (42) satisfies the third Von Mises condition (as given in Definition 3) resulting

$$
\lim_{N \to \infty} \mathbb{E} \left( U_{n,m \log(N)+1:N} \right) = F_{U_{n,k}^{\text{GEE}}}^{-1}(1 - \frac{m \log(N)}{N}) = P_{\text{max}} + \frac{a_{n,k}}{\log(N) - \log(N) + \frac{a_{n,k}}{P_{\text{max}}}} \quad (43)
$$

We now obtain simple bounds on $\mathbb{E} \left( A_{n:N} \right)$ and $\mathbb{E} \left( A_{n,k}^{\text{FMA}} \right)$:

$$
\mathbb{E} \left( A_{n:k}^{\text{OPT}} \right) \leq \sum_{n=1}^{N} \mathbb{E} \left( A_{n:N} \right) \quad (44)
$$

It is now easy to see that

$$
\frac{NP_{\text{max}} - \sum_{n=1}^{N} \frac{a_{n,k}}{P_{\text{max}}}}{NP_{\text{max}} - \sum_{n=1}^{N} \frac{a_{n,k}}{P_{\text{max}}} - \frac{\log(N) - \log(m \log(N))}{P_{\text{max}}}} \leq \mathbb{E} \left( A_{n:k}^{\text{OPT}} \right) \leq 1.
$$

It can be seen that

$$
\lim_{N \to \infty} \frac{NP_{\text{max}} - \sum_{n=1}^{N} \frac{a_{n,k}}{P_{\text{max}}}}{NP_{\text{max}} - \sum_{n=1}^{N} \frac{a_{n,k}}{P_{\text{max}}} - \frac{\log(N) - \log(m \log(N))}{P_{\text{max}}}} = 1.
$$

Finally, we use (46) and (47) to get (29) of Theorem 2.

APPENDIX C: PROOF OF THEOREM 3 (EE ASYMPTOTIC OPTIMALITY)

The proof is identical to the proof of Theorem 2. The only difference is that the CDF $F_{U_{n,k}^{\text{OPT}}}(x)$ is given by

$$
F_{U_{n,k}^{\text{OPT}}}(x) = 1 - e^{-\frac{a_{n,k}}{P_{\text{max}}} e^{-\frac{\log(N)}{P_{\text{max}}^2}}} \quad (48)
$$

It is easy to verify that all arguments applied to the CDF in Theorem 2 can also be readily applied to the CDF in (48).

REFERENCES


[79] ETSI TR 138 900 V14.2.0 (2017-06), “Study on channel model for frequency spectrum above 6 ghz (3gpp tr 38.90 0 version 14.3.1 release 14),” June 2017.


