Robust Spectrum Management with Incomplete Information over Fading Channels

Yair Noam\textsuperscript{a,}\textsuperscript{*}, Amir Leshem\textsuperscript{a}, Hagit Messer\textsuperscript{b}

\textsuperscript{a}Faculty of engineering Bar-Ilan University, Ramat-Gan, Israel
\textsuperscript{b}School of Electrical Engineering, Tel Aviv University Tel Aviv, Israel

Abstract

This paper studies a wireless interference game in fading channels with partial channel information. Using a Bayesian game formulation, we consider two selfish wireless communication systems (players) who share the same frequency band, where each player only knows its own channel gains up to some estimation error. In the case of fixed channels with perfect channel estimation, this game is known to have a spectrally efficient equilibrium point, in comparison with the equilibrium point of constant mutual interference. However, wireless channels are usually not fixed, and, in addition, are not estimated perfectly. This leads to payoff perturbations, which may cause the spectrally efficient equilibrium point to be unstable. This paper makes two contributions. First, we extend the Bayesian interference game to frequency-selective block-fading channels, and show that the same strategy

\textsuperscript{*}Corresponding author.
Some of the results in this paper were published \cite{1} in the IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP) 2014.

\textit{Email addresses: yair.noam@biu.ac.il (Yair Noam), leshema@biu.ac.il (Amir Leshem), messer@eng.tau.ac.il (Hagit Messer)}

\textit{URL: http://www.eng.biu.ac.il/noamya/ (Yair Noam), http://www.eng.biu.ac.il/leshema/ (Amir Leshem), http://www.eng.tau.ac.il/~messer/ (Hagit Messer)}
that improves the spectrum utilization in the game with fixed channels can
produce an efficient equilibrium point in the frequency selective fading chan-
el game as well. The second contribution is the incorporation of channel
estimation errors into the game. In this case, we show that the spectrally effi-
cient equilibrium point is robust to small estimation errors. We demonstrate,
via simulation, that the robustness and the improved spectral efficiency are
maintained even if the channels are estimated via a training sequence as short
as one.

**Keywords:** Game theory; spectrum management, interference-avoidance,
frequency division duplex (FDD), Bayesian games, games with
incomplete-information, robust Nash equilibrium.

1. Introduction

Radio interference is a major problem in wireless communications, lead-
ing to a poor spectrum utilization. For this reason, efficient spectrum man-
agement, including centralized, distributed and competitive approaches, is
a proliferate field of research [2–25]. In competitive spectrum management,
systems are seen as selfish entities that shape the spectrum to maximize their
individual utility; e.g., the information rate. This approach is particularly
applicable to wireless users who are not served by the same service provider.
Because each users’ actions affect the other users’ utilities and vice versa, a
natural tool to analyze such interactions is game theory. One of the most
useful notions is the Nash Equilibrium (NE) ([26] see e.g.) which describes
a stable operating point; that is, a strategy profile such that each player
(system) can only lose if it unilaterally deviates from it. Operating at such
a point is highly desirable for wireless protocols, since it makes the protocol stable. In a game where players share a flat-fading interference channel with complete information\(^1\), it is known [10] that the Full Spread (FS) strategy, where both players spread their powers equally over the entire band, is a NE point. It is also known [11, 27] that in some cases, the FS NE point is spectrally inefficient. For example, joint Frequency Division Multiplexing (FDM) can yield a higher payoff to both players than mutual FS. This point, however, is not stable, since if one player uses FDM, the other player will have a higher payoff if it chooses FS rather than FDM; a situation known as the prisoner’s dilemma [8, 27]. Interestingly, if players are unaware of their opponents’ preferences, the game may have a more efficient equilibrium point [28] in which players choose between FS and FDM based on their local channel state information and the statistical properties of the other player’s channels. The latter non-pure FS equilibrium point Pareto dominates the FS NE; i.e., not only does it increase the sum-payoff of both players, it also produces a higher payoff for each player. The point is obtained using a Bayesian game framework which is a very useful approach for analyzing wireless communications setups in which users don’t know the others’ channel gains [5], and for understanding the effect of incomplete information on the game’s outcome [14]. The non-pure FS equilibrium point, achieves higher rates for both users in the two-user Bayesian Interference Game (BIG) [28]. However, the result is limited to time invariant channels; i.e., channels which remain constant

\(^1\)By complete information, we mean that every user knows all the direct and cross channel gains of all users in the network as well as the total power and any other limitations imposed.
during the entire codeword. Furthermore, with respect to the local channel state information, it is assumed in [28] that there is no estimation error; an assumption which is only approximately correct. In general, estimation errors affect the spectrum management problem, both in the competitive and cooperative cases, since it might make the equilibrium point unstable; i.e., under estimation error one of the players might gain by deviating from the NE point. So far, the only aspect of this problem that has been studied is where players fail to meet some interference constraints due to uncertainty about the channels to unintended receivers, but have perfect knowledge of the channel gains to the intended receivers. In this case, the perturbed game (the game under estimation error) can be seen as a robust game in which players maximize their payoff under tighter constraints on the maximum interference level that could be inflicted on other players. This game was first formulated and analyzed for spectrum sharing between cognitive radios and a primary user [29], and further studied in [30–34]. A similar problem was also studied in DSL [35] for non-competitive spectrum management; i.e., the problem of optimizing a joint utility rather than a game where each player selfishly optimizes its own utility.

This paper makes two contributions. The first is the extension of the BIG to time varying and frequency-selective fading channels which affect the overall achievable rates and require a different definition of the payoff functions. We show that even though the payoff function is different, the same strategy profile that yields the non-pure FS equilibrium point in the constant-channel case [28] forms a similar equilibrium point in these cases. The second contribution of this paper is to take into account channel estimation errors. We
show that BIG’s equilibrium points, which are functions of the true channel gains, are robust to estimation errors when the players use the estimated gains. By robust, we mean that the point remains stable in the sense that no player can obtain a significant gain by unilaterally deviating from the equilibrium strategy. Finally, we show, via simulation, that the BIG is robust even if the channels are estimated by a short training sequence.

This paper is organized as follows. Section 2 presents the system model and reviews the BIG. Section 3 extends the BIG to block-fading time varying and frequency selective channels. Section 4 discusses the BIG and its properties under estimation errors. Sections 5 and 6 present simulations and conclusions, respectively.

2. The Bayesian Interference Game (BIG)

We now present the system model and describe the Bayesian game which will be further extended and analyzed in the subsequent sections. A Bayesian game is an appropriate formulation for wireless communication systems where each player knows its own channel gains, but does not know the other player’s. This is a more practical model, than the classic full-information game for users who do not share their local channel state information (CSI) because of the considerable overhead it induces, or for users in the wireless unlicensed band. To make this paper self-contained, we review some results dealing with the BIG [28]. Consider a two-user flat-fading interference channel (Fig. II) with an overall bandwidth $B$. During the channel coherence time, player $i$ receives the signal

$$W_i(t) = H_{ii}V_i(t) + H_{ij}V_j(t) + N_i(t), \quad (1)$$
Figure 1: A wireless interference channel where the two systems (players) interact without complete information. Each player knows the square magnitudes of its direct and impinging channel gains and the statistics of its opponent’s channel gains. For example, player 1 knows $|H_{11}|^2$ and $|H_{12}|^2$ but only knows the statistics of $|H_{22}|^2$ and $|H_{21}|^2$.

where $i, j \in \{1, 2\}$, $i \neq j$, and $V_i(t), V_j(t) \in \mathbb{N}$ are user $i$’s and $j$’s transmit signals, respectively; $N_i(t)$ is a white Gaussian noise with variance $\sigma_N^2$ and $H_{iq}$, $i, q \in \{1, 2\}$ are random fading-channel gains. Both players have a total power constraint of $\bar{p}$. Throughout this paper, the index $j$ is never equal to $i$. We denote user $i$’s Signal to Noise Ratio (SNR) and Interference to Noise Ratio (INR) as $X_i = \gamma |H_{ii}|^2$ and $Y_i = \gamma |H_{ij}|^2$, respectively, where $\gamma = \bar{p}/\sigma_N^2$.

The realizations (sample points) of $X_i, Y_i$ are denoted by $x_i, y_i$, respectively.

In the BIG, player $i$ shapes its spectrum based on the information $x_i, y_i$, in order to maximize its own payoff. It does not observe $Y_j$ and $X_j$ but only knows its distributions. The channel is divided into two equal parallel sub-channels, of bandwidth $B/2$ each. Thus, each player observes

$$W_i^{(q)}(t) = H_{ii}V_i^{(q)}(t) + H_{ij}V_j^{(q)}(t) + N_i^{(q)}(t), \; q = 1, 2$$

where $V_i^{(1)}(t), V_i^{(2)}(t)$ are independent information bearing signals transmit-
Table 1: User $i$’s utility function $u_i(s_i, s_j, SNR_i, INR_i)$

<table>
<thead>
<tr>
<th>Player Actions</th>
<th>$s_j = 1$</th>
<th>$s_j = 1/2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Player $i$ chooses FDM ($s_i = 1$)</td>
<td>$\frac{1}{2} \log_2 (1 + SNR_i)$</td>
<td>$\frac{1}{2} \log_2 \left(1 + \frac{SNR_i}{1 + INR_i/2}\right)$</td>
</tr>
<tr>
<td>Player $i$ chooses FS ($s_i = 1/2$)</td>
<td>$\frac{1}{2} \log_2 \left(1 + \frac{SNR_i/2}{1 + INR_i/2}\right)$</td>
<td>$\log_2 \left(1 + \frac{SNR_i/2}{1 + INR_i/2}\right)$</td>
</tr>
</tbody>
</table>

An underlying assumption in the BIG is that the interference inflicted by each user is treated by the other user as noise and that each user is constrained to a Gaussian codebook. It is further assumed that the spectrum is shaped by each user only once during the entire coherence interval; this happens at the beginning of the interaction. In more explicit terms, the action $s_i = 1$ corresponds to FDM while $s_i = 1/2$ to FS. This formalism implies that players coordinate in advance to use disjoint sub-bands in the case of FDM.

An underlying assumption in the BIG is that the interference inflicted by each user is treated by the other user as noise and that each user is constrained to a Gaussian codebook. It is further assumed that the spectrum is shaped by each user only once during the entire coherence interval; this happens at the beginning of the interaction. In more explicit terms, the action $s_i = 1$ corresponds to FDM while $s_i = 1/2$ to FS. This formalism implies that players coordinate in advance to use disjoint sub-bands in the case of FDM.

An underlying assumption in the BIG is that the interference inflicted by each user is treated by the other user as noise and that each user is constrained to a Gaussian codebook. It is further assumed that the spectrum is shaped by each user only once during the entire coherence interval; this happens at the beginning of the interaction. In more explicit terms, the action $s_i = 1$ corresponds to FDM while $s_i = 1/2$ to FS. This formalism implies that players coordinate in advance to use disjoint sub-bands in the case of FDM.

An underlying assumption in the BIG is that the interference inflicted by each user is treated by the other user as noise and that each user is constrained to a Gaussian codebook. It is further assumed that the spectrum is shaped by each user only once during the entire coherence interval; this happens at the beginning of the interaction. In more explicit terms, the action $s_i = 1$ corresponds to FDM while $s_i = 1/2$ to FS. This formalism implies that players coordinate in advance to use disjoint sub-bands in the case of FDM.
sub-band\(^2\) and feeds it back to the transmitter (Tx). Based on this estimate, the codebook is determined, and then, the information is transmitted. This procedure is illustrated in Fig. 3. The assumption that the Tx knows the SINR will later be relaxed.

Since each Tx, say Tx\(i\), knows the SINR in each sub-band, it can infer whether \(S_j = 1\) or not. Furthermore, because the actions \(S_1, S_2\) cannot be modified at this point, and since the interference is treated as noise, the

---

\(^2\)This process is a common practice. For example, in LTE/LTE advanced [36], after the first estimation phase, there is another phase in which each mobile terminal estimates the equivalent link which is in effect after the precoding.
maximum achievable rate by each user is

\[ u_i(s_i, s_j, x_i, y_i) = I(V_i; W_i | S_i = s_i, S_j = s_j, X_i = x_i, Y_i = y_i), \quad (4) \]

where \( V_i = \{ V_i^{(1)}, V_i^{(2)} \} \) and \( W_i = \{ W_i^{(1)}, W_i^{(2)} \} \); this is the utility function. Under the Gaussian signaling assumption, this utility \( u_i(s_i, s_j, SNR_i, INR_i) \) is given in Table 1. We are now ready to define the Bayesian interference game.

**Definition 1.** The Bayesian Interference Game (BIG) \([28]\) is defined by the following:

1. A set of players \( \{1, 2\} \).
2. An action set \( \Theta = \{1, 1/2\} \). Let \( s_i \in \Theta \) be the action chosen by player \( i \), then, according to (3), \( s_i = 1 \) corresponds to FDM and \( s_i = 1/2 \) corresponds to FS.
3. A set of positive and independent random variables \( X_1, Y_1, X_2, Y_2 \) whose distributions are common knowledge. At the time the spectrum is shaped, each player \( i \) observes the realized values of \( X_i, Y_i \) but does not observe \( X_j, Y_j \).
4. A utility function \( u_i(s_i, s_j, x_i, y_i) \) given in Table 1.
5. A set of pure strategies \( S = S_1 \times S_2 \) where every \( S_i \in S_i \) is a function that maps values of \( x_i, y_i \) to an action in \( \Theta_i \); i.e., \( S_i : \mathcal{X}_i \times \mathcal{Y}_i \rightarrow \Theta_i \), where \( \mathcal{X}_i = \text{Range}(X_i) \) and \( \mathcal{Y}_i = \text{Range}(Y_i) \).

Player \( i \)'s objective is to maximize his conditional expected payoff given his

---

\(^3\)See [10], for further discussion of the rationale behind choosing this utility.
private information \( x_i, y_i \); i.e.,

\[
\pi_i(s_i, S_j, x_i, y_i) \equiv \mathbb{E} \{ u_i(S_i, S_j, x_i, y_i) | S_i = s_i, X_i = x_i, Y_i = y_i \}, \tag{5}
\]

and by substituting the expressions of Table 1 into (5), we obtain

\[
\begin{align*}
\pi_i(1, S_j, x_i, y_i) &= \frac{1}{2} a_j \log(1 + x_i) + \frac{1}{2} (1 - a_j) \log \left(1 + \frac{x_i}{1 + y_i/2}\right) \\
\pi_i(1/2, S_j, x_i, y_i) &= a_j \left(\frac{1}{2} \log_2 \left(1 + \frac{x_i}{y_i}\right) + \frac{1}{2} \log_2 \left(1 + \frac{x_i/2}{1 + y_i/2}\right)\right) + (1 - a_j) \log_2(1 + \frac{x_i/2}{1 + y_i/2}), \tag{6}
\end{align*}
\]

where, \( a_j = P(S_j = 1) \).

**Definition 2.** A NE point of the BIG is a strategy profile \( S = (S_i, S_j) \) such that for every strategy profile \( \tilde{S} = (\tilde{S}_i, \tilde{S}_j) \) and for every \( i \in \{1, 2\} \),

\[
\pi_i(S_i, S_j, x_i, y_i) \geq \pi_i(\tilde{S}_i, S_j, x_i, y_i) \quad \forall \ x_i, y_i \in X_i \times Y_i. \tag{7}
\]

Since the action space is binary, a strategy \( S_i(x_i, y_i) \) in the BIG is equivalent to a decision region \( D_i \subseteq X_i \times Y_i \) such that \( S_i(x_i, y_i) = 1 \) (i.e. FDM) if \( x_i, y_i \in D_i \) and \( S_i(x_i, y_i) = 0.5 \) if \( x_i, y_i \in D_i^c \). A more detailed description of the BIG and its properties is given in [28]. For example, we explain why it is sufficient to consider only pure strategies (that is, deterministic strategies) and why this setup is preferable over other game setups.

**Definition 3.** Let \( S_j \) be player \( j \)'s strategy with \( a_j = P(S_j = 1) \). Player \( i \)'s best response to \( S_j \) is defined by:

\[
\tilde{S}_i(x_i, y_i, a_j) \equiv \begin{cases} 
  s_i = 1, & \text{if } e(x_i, y_i, a_j) > 0 \text{ and } y_i/x_i > 1/2 \\
  s_i = 1/2, & \text{otherwise}
\end{cases}, \tag{8}
\]

where

\[
e(x_i, y_i, a_j) = \pi_i(1, S_j, x_i, y_i) - \pi_i(0, S_j, x_i, y_i). \tag{9}
\]
To calculate NE points we need to extract $\hat{a}_1$ and $\hat{a}_2$ from the equations

$$\hat{a}_1 = P (\hat{S}_1(X_1, Y_1, \hat{a}_2) = 1)$$
$$\hat{a}_2 = P (\hat{S}_2(X_2, Y_2, \hat{a}_1) = 1).$$

(10)

One of the solutions is $\hat{a}_1 = 0, \hat{a}_2 = 0$ (pure-FS by both users), which is a NE point, regardless of the channel distribution, since FS is the best response of each player if his opponent uses FS. In this case, each player’s payoff is $u_i(1/2, 1/2, x_i, y_i)$. In some cases, the pure-FS NE point may be very poor for both users; e.g., in cases where the mutual FDM yields a higher payoff for both players, but they will not operate at that point since the unilateral deviation will produce a higher payoff for each user. This means that a mutual FDM is not stable point from a game theoretic point of view, even when it leads to better spectrum utilization. This fact motivates the derivation of more efficient NE points. However, because it is impossible to derive other NE points in closed-form, only $\epsilon$-NE points in which both players have a higher payoff than in the FS-NE point are known in closed-form [28, Theorem 4]; where $\epsilon$-NE is defined as follows:

**Definition 4.** For $\epsilon > 0$, an $\epsilon$-NE point is a strategy profile $(\hat{S}_1, \hat{S}_2)$ such that

$$\pi_i (\hat{S}_1, \hat{S}_2, x_i, y_i) \geq \sup_{\hat{S}_i \in \hat{S}_i} \pi_i (\hat{S}_i, \hat{S}_j, x_i, y_i) - \epsilon, \ \forall x_i, y_i$$

(11)

The idea behind an $\epsilon$-NE point is that if one of the players deviates from it, he can gain no more than $\epsilon$ additional payoff. From a practical point of view, $\epsilon$-NE points are as stable as ordinary NE points if $\epsilon$ is sufficiently small, such that it is considered negligible by the players. We now present a spectrally $\epsilon$-NE for BIG.
Theorem 1 ([28] Theorem 4). Assume the channel gains $|H_{iq}|^2$, $i,q \in \{1,2\}$ are continuous random variables and let $\bar{p}$ be the overall power constraint; then for every $\epsilon > 0$, there exists some $\bar{p}_0$ such that for every $\bar{p} > \bar{p}_0$ the following strategy profile is an $\epsilon$-near NE point:

$$\hat{S}_1 = \tilde{S}_1(x_1, y_1, \hat{a}_2)$$

$$\hat{S}_2 = \tilde{S}_2(x_2, y_2, \hat{a}_1),$$

where $\tilde{S}_i$ is the best response given in (8), and $(\hat{a}_1, \hat{a}_2)$ is a solution to the following equation system

$$a_1 = 1 - F_{Z_1}(q(a_2))$$

$$a_2 = 1 - F_{Z_2}(q(a_1))$$

where $Z_i = |H_{ji}^2|/|H_{ii}^2|$, $F_{Z_i}(z)$ is the distribution function of the of $Z_i$ and $q : (0,1] \rightarrow (0.5, \infty]$ is a continuous, monotonically decreasing function given in [28, Proposition 1].

In [28] it was shown that this $\epsilon$-NE point produces higher rates for both players than the pure-FS NE; furthermore, this point is unique [37].

3. The block-fading BIG under a maximum power mask

An underlying assumption in [37] is that the channels are fixed for the entire codeword. Thus, the utility in Table 1 is the maximum achievable rate for user $i$ if the corresponding actions $s_i, s_j$ are taken by both players; and each Txi and Rxi know $X_i, Y_i, s_i,$ and $s_j$. Typically with Bayesian games, the payoff $\pi_i(S_i, S_j, x_i, y_i)$, given in (5), is not the actual achievable rate.
Rather, it is the statistical expectation of that rate, with respect to the a-priori distribution of $X_j, Y_j$, given that $X_i = x_i, Y_i = y_i$ and that the strategy profile $S_i, S_j$ is used. The actual achievable rate $u_i (S_i(x_i, y_i), S_j(x_j, x_j), x_i, y_i)$ depends on the realized values $x_j, y_j$ which are unknown to player $i$ at the time it chooses its action.

In this section, we study the BIG in a block-fading channel model (see e.g., [38], Sec. 4.2.1) where the channels vary many times across a single codeword. In addition, we relax the requirement that each Tx must know the actual SINR when choosing the codeword. This setup fits two scenarios: frequency-selective or time-varying block-fading channels. In the former, the channel is time-invariant during the entire codeword transmission. However, the codeword is transmitted over $K \in \mathbb{N}$ flat-fading sub-channels. Each sub-channel has a bandwidth $B$, and its gain is independent identically distributed with respect to the other sub-channels. In the time-varying case, the channel is flat over the entire spectrum, and the codeword is transmitted over $K$ blocks of length $T$, dubbed the channel coherence time. The signal

Figure 4: The BIG protocol in block fading channels.
received by user $i$ in block $k$ is given by

$$W_{i,k}(t) = H_{i,i,k}V_{i,k}(t) + H_{i,j,k}V_{j,k}(t) + N_{i,k}(t), \quad k = 0, ..., K - 1,$$

(16)

where, $V_{i,k}(t), V_{j,k}(t)$ are signals transmitted by user $i \in \{1, 2\}$ and by user $j \in \{1, 2\} \setminus \{i\}$, respectively. Similarly to (2), each block is divided into two sub-channels where $W_{i,k}(t) = \{W_{i,k}^{(q)}(t)\}_{q=1}^{2}$, $V_{i,k}(t) = \{V_{i,k}^{(q)}(t)\}_{q=1}^{2}$, $N_{i,k}(t) = \{N_{i,k}^{(q)}(t)\}_{q=1}^{2}$.

In the frequency selective block-fading case, $V_{i,k}(t)$ is the discrete-time, base-band equivalent representation of the narrow-band signal occupying the $k$th frequency block, whereas in the time-varying case, it represents the signal at the $k$th coherence time; i.e., $t \in \{(k-1)T, ..., kT - 1\}$. In both cases we assume that each user has a maximum power constraint of $\bar{p}$; i.e., the overall power of $V_{i,k}^{(1)}$ and $V_{i,k}^{(2)}$ is $\bar{p}$. In the frequency selective case, this constraint is translated into a power mask and in the time-varying case it is translated into a short-term or an instantaneous power constraint. We begin by analyzing the game in the frequency selective block fading channel.

### 3.1. The BIG in frequency-selective block-fading channel

For every sub-band $k$, each player chooses between FS and FDM as described in (2). Denote $X_{i,k} \triangleq \gamma |H_{i,i,k}|^2, Y_{i,k} \triangleq \gamma |H_{i,j,k}|^2, \mathbf{X}_i \triangleq [X_{i,k}, ..., X_{i,K}]^\top$ and $\mathbf{Y}_i = [Y_{i,1}, ..., Y_{i,K}]^\top$. A strategy, or a spectrum allocation policy, is a function $S_i(\mathbf{X}_i, \mathbf{Y}_i)$ which maps $\mathbf{X}_i, \mathbf{Y}_i$ to an action $\{s_{i,k}\}_{k=1}^{K} \in \{1/2, 1\}^K$, where $s_{i,k} \in \{1/2, 1\}$ is the action taken in sub-band $k$, as defined in (3) for a single-band game. We further denote the $k$th entry of $S_i(\mathbf{X}_i, \mathbf{Y}_i)$ by $S_{i,k}(\mathbf{X}_i, \mathbf{Y}_i)$, which will sometimes be denoted for brevity by $S_{i,k}$.
The BIG is played as follows: at the beginning of the interaction, upon receiving its private information $X_i, Y_i$ from Rx$i$, Tx$i$ shapes its spectrum according to the strategy $S_i(X_i, Y_i)$, and chooses the codeword which will be transmitted. Then, there is another round of estimation in which the SINR in each sub-band is estimated by Rx$i$, but this time, it is not fed back to Tx. Finally, the codeword is transmitted over all blocks. The procedure is depicted in Fig. 4. Under this protocol, each user observes $K$ parallel point-to-point flat-fading AWGN channels, where each channel $k$ is composed of two sub-channels of bandwidth $B/2$; i.e.,

$$W_{i,k}^{(q)}(t) = \sqrt{G_{i,k}^{(q)}X_{i,k}^{(q)}(t)} + \sqrt{G_{j,k}^{(q)}Y_{j,k}^{(q)}(t)} + Z_{i,k}^{(q)}(t), \ q = 1, 2, \ (17)$$

where

$$G_{i,k}^{(q)} = \begin{cases} S_{i,k}(X_i, Y_i) & \text{if } i = q \\ 1 - S_{i,k}(X_i, Y_i) & \text{if } i \neq q \end{cases}, \ (18)$$

$V_{i,k}^{(q)}(t), V_{j,k}^{(q)}(t)$ are the input and interference signals, respectively, and $Z_{i,k}^{(q)}(t)$ is an additive noise. Both the signals and noise satisfy $Z_{i,k}^{(q)}(t), V_{q,k}^{(q)}(t) \sim N(0, 1)$.

Next, we derive user $i$’s maximum achievable rate, given that user $j$ applies a strategy with an identical decision rule for every $k$, defined below.

**Definition 5 (A memoryless frequency-invariant strategy).** A strategy $S_j$, $j \in \{1, 2\}$ is called memoryless frequency-invariant, if for every $k \in \{1, ..., K\}$, the decision rule is a function of only $X_{j,k}, Y_{j,k}$; i.e., the $k$th entry of $S_j(X_i, Y_i)$ satisfies

$$S_{j,k}(X_i, Y_i) = S_j(X_{j,k}, Y_{j,k}), \forall k \in \{1, ..., K\}. \ (19)$$
For simplicity, since a memoryless frequency-invariant strategy is determined by \( S_j \), we denote such strategies by \( S_j \).

Given that user \( j \) applies a memoryless frequency-invariant strategy, user \( i \) observes a point-to-point channel with a state \( \{ Q_{i,k} \}_{k=1}^K \), where \( Q_{i,k} = \{ G_{j,k}^{(q)} \}_{q=1}^2, X_{i,k}, Y_{i,k} \} \), CSI at the receiver (CSIR) \( \{ E_{i,k} \}_{k=1}^K \) where \( E_{i,k} = Q_{i,k} \) (perfect CSIR), and CSI at the transmitter (CSIT) \( \{ U_{i,k} \}_{k=1}^K \), where \( U_{i,k} = \{ X_{i,k}, Y_{i,k} \} \). Because the knowledge of \( \{ G_{j,k}^{(q)} \}_{q=1}^2 \) is equivalent to knowing whether \( S_j(X_{j,k}, Y_{j,k}) \) is equal to 1 or 1/2, we write \( Q_{i,k} = \{ S_{j,k}, X_{i,k}, Y_{i,k} \} \) and \( U_{i,k} = \{ X_{i,k}, Y_{i,k} \} \).

This channel falls under the category of Proposition 2 [39], in which the CSIT is a deterministic function of the CSIR, 

\[
P(E_{i,k}|\{U_{i,k}\}_{k=1}^K) = P(E_{i,k}|U_{i,k}),
\]

and \( \{ U_{i,k} \}_{k=1}^K \) and \( \{ Q_{i,k} \}_{k=1}^K \) are jointly stationary and ergodic, for each \( i \in \{1, 2\} \). The maximum achievable rate is

\[
R_i(s_i, S_j) = \int_{F_{V_i|X_i,Y_i}(v_i|x_i,y_i)} \max I(V_i;W_i|S_i = s_i, S_j, X_i = x_i, Y_i = y_i) dF_{X_i,Y_i}(x_i,y_i),
\]

(20)

where \( F_{V_i|X_i,Y_i}(v_i|x_i,y_i) \) is the marginal distribution of \( V_{i,k} \); a distribution that is independent of \( k \). The subscript \( k \) is also omitted in \( S_j, X_i, Y_i \) and \( W_i \) due to stationarity, and therefore, \( S_{i,k} \) is not a function of \( k \) and is written as \( S_i \). Next, by taking into account that user \( j \) applies Gaussian signaling, and that the PSD must be as in Fig. 4, it follows that

\[
I(V_i;W_i|S_i = s_i, S_j, X_i = x_i, Y_i = y_i) = \pi_i(s_i, S_j, x_i, y_j),
\]

(21)

where \( \pi_i(s_i, S_j, x_i, y_j) \) is given in (11). Note that \( R_i(s_i, S_j) \) is not a function of a particular realization of the channel or of \( S_j \). Thus, it can be maximized with
respect to \( S_i \) at the decision-making phase (phase 2 in Fig. 4). Therefore, given that player \( j \) employs a frequency-invariant strategy, \( S_j \), player \( i \)'s best response is also a frequency invariant strategy, which is given by

\[
\tilde{s}_i = \arg \max_{s_i} \int \pi_i(s_i, S_j, x_i, y_j) dF_{X_i, Y_i}(x_i, y_i),
\]

(22)

Next, because the integrand \( \pi_i(s_i, S_j, x_i, y_j) \) is positive, it is possible to interchange the order of the maximization and the integration; thus,

\[
\tilde{s}_i = \arg \max_{s_i} \pi_i(s_i, S_j, x_i, y_j).
\]

(23)

It turns out that this best response is identical to the best response of Definition 3, for the case of a fixed channel. The difference is that in the frequency-selective block-fading BIG, the decision is made for every \( k \), according to \( \tilde{s}_i(x_{i,k}, y_{i,k}, a_j) \). Thus, BIG’s \( \epsilon \)-NE point of Theorem 1 is also an \( \epsilon \)-NE point of the frequency-selective block-fading BIG, even though in the latter the Tx does not observe the other player’s action, and even though the actual achievable rate in each game is different.

### 3.2. Time-varying block fading

This game is very similar to the frequency selective game; however, there is an important difference. While in the frequency selective game the decisions for all \( k \) are taken at the same time at the beginning of the interaction, here, the actions are taken sequentially every coherence-time interval. This converts it into a repeated game, since at each stage (coherence-time) each player knows the other player’s actions up to the current one, and can take them into account. Nevertheless, it is still important to analyze the one-stage game, due to the well-known folk theorem which claims that every
point which Pareto dominates a NE point of the one-stage game is achievable using the grim trigger strategy (see e.g., [20]). This means that each NE point of the one-stage game defines an inner bound on the rate region. We therefore assume that player \( j \) is using a strategy of the form (19), which leads to an analysis identical to the one in the frequency selective game in Sec. 3.1. Thus, the \( \epsilon \)-NE point of Theorem 1 is also an \( \epsilon \)-NE point for fast fading channel, where the Tx does not observe the other player’s action when designing the codebook.

4. The BIG under channel estimation error

In this section, in addition to the fading and the incomplete CSIT that were addressed in the previous scenario (cf. Sec. 3), we include estimation errors. The \( \epsilon \)-NE of Theorem 1, which was derived under the assumptions of no estimation errors, provides higher rates for both users than the pure FS NE. Our objective is to determine whether this remains the case under estimation error. The key question is whether a small estimation error will drive the system out of stability. The following definition extends the well-known notion of robustness of NE points [21, Definition 12.1] to \( \epsilon \)-NE point.

**Definition 6.** An \( \epsilon \)-NE point \((\hat{s}_1, \hat{s}_2)\) with payoff \(u_1, u_2\) is robust if for every \(\delta > 0\) there exists \(\eta > 0\) such that for every \(\hat{u}_1, \hat{u}_2\) which satisfies \(\max_{i,j} |u_i - \hat{u}_i| < \eta, i, j \in \{1, 2\}, i \neq j\), the point \((\hat{s}_1, \hat{s}_2)\) is an \((\epsilon + \delta)\)-NE of the perturbed game; i.e., the same game as the BIG, but with perturbed payoff \(\hat{u}_1, \hat{u}_2\) rather than the original payoff \(u_1, u_2\).
In other words, this definition states that an $\epsilon$-NE point is robust if small payoff perturbations turn it to an $(\epsilon + \delta)$-NE. Because $\epsilon$ and $\delta$ may be arbitrarily small, the $\epsilon$-NE and the $(\epsilon + \delta)$- are essentially the same. This is an important property since if an equilibrium point varies drastically due to small payoff perturbations, it is completely useless in practice, since players can gain significantly by deviating from that original point. In what follows we show that the $\epsilon$-NE point in Theorem 1 is robust to perturbations due to estimation error.

The first step towards this goal is to analyze the effect of the estimation error on the utility. The communication setup here is identical to the one discussed in Sec. 3, only now user $i$ only knows the estimated values $\hat{X}_{i,k}, \hat{Y}_{i,k}$ instead of the true values $X_{i,k}$ and $Y_{i,k}$. Thus, its spectrum allocation policy must be based on $\{\hat{X}_{i,k}, \hat{Y}_{i,k}\}_{k=1}^K$, rather than the true values. As in Sec. 3, we assume that user $j$ is using a strategy of the form (19), but here $\hat{X}_{j,k}, \hat{Y}_{j,k}$ are substituted instead of $X_{j,k}, Y_{j,k}$. The resulting strategy $S_j(\hat{X}_{j,k}, \hat{Y}_{j,k})$ will sometimes be denoted $S_{j,k}$ for brevity. Given that user $j$ uses the strategy $S_j(\hat{X}_{j,k}, \hat{Y}_{j,k})$, user $i$ observes a fading channel similar to (17) whose state $Q_{i,k} = \{X_{i,k}, Y_{i,k}, S_j(\hat{X}_{j,k}, \hat{Y}_{j,k})\}$, with partial CSIR given by $\{E_k\}_{k=1}^K$ where $E_k = \{\hat{X}_{i,k}, \hat{Y}_{i,k}, S_j(\hat{X}_{j,k}, \hat{Y}_{j,k})\}$ and CSIT $\{U_{i,k}\}_{k=1}^K$, where $U_{i,k} = \{\hat{X}_{i,k}, \hat{Y}_{i,k}\}$. Since $S_j$ is a memoryless frequency-invariant strategy strategy, User $i$ observes a memoryless channel, which falls into the category of Proposition 1 [39]. The maximum achievable rate is

$$\bar{R}_i(s_i, S_j) = \max_{F_{V_i|X_i, S_j}(v_i|\hat{x}_i, \hat{y}_i)} \int I(V_i; W_i|S_i = s_i, S_j, \hat{X}_i = \hat{x}_i, \hat{Y}_i = \hat{y}_i) dF_{\hat{X}_i, \hat{Y}_i}(\hat{x}_i, \hat{y}_i),$$

(24)
where $V_i = \{ V_i^{(1)}, V_i^{(2)} \}$, and we omit the index $k$ due to stationarity. Denote

$$
\dot{u}(\hat{x}_i, \hat{y}_i, s_i, s_j) = I(V_i; W_i | S_i = s_i, S_j = s_j, \hat{X}_i = \hat{x}_i, \hat{Y}_i = \hat{y}_i);
$$

then, taking into account that $S_j$ is either 1 or 1/2, user $i$’s best response, in terms of maximum achievable rate, to strategy $S_j$ is to maximize

$$
\hat{\pi}_i(s_i, S_j, \hat{x}_i, \hat{y}_j) = E\left\{ \dot{u}(\hat{x}_i, \hat{y}_i, s_i, S_j) | \hat{X}_i = \hat{x}_i, \hat{Y}_i = \hat{y}_i \right\}
$$

where $a_j = P(S_j = 1)$. Since maximizing $\hat{\pi}_i$ in (26) for every $k$, will maximize the achievable rate (24), we choose $\hat{\pi}_i$ as the payoff function in the case of estimation errors. The block-fading BIG is robust to estimation errors if the strategy profiles of the $\epsilon$–equilibrium points without estimation errors remain $\epsilon$–equilibrium points under estimation errors.

The following lemma shows that $\dot{u}(\hat{x}_i, \hat{y}_i, s_i, s_j)$ is “continuous” with respect to the channel estimation error.

**Lemma 2.** Assume that the channel gains have finite first and second moments and that the channels are estimated by a sequence of estimates $\{ \hat{H}_{iq}^l \}_{l=1}^{\infty}$, $1 \leq i, q \leq 2$, all defined on the same probability space $(\Omega, \mathcal{F}, P)$. Assume further that $\mathcal{F}_l \subseteq \mathcal{F}_{l+1}$ where $\mathcal{F}_l = \sigma(\hat{H}_{iq}^l)$, $\sigma(X) = \{ A \in \mathcal{F} : A = X^{-1}(B), B \in \mathcal{B}(\mathbb{R}) \}$, and $\mathcal{B}(\mathbb{R})$ is the Borel $\sigma$–algebra on $\mathbb{R}$. Then, for every $\gamma$, the $\dot{u}$ in (25) satisfies

---

4Intuitively, this condition implies that $\hat{H}_{iq}^{l+1}$ exploits the measurements used by its predecessor $\hat{H}_{iq}^l$ and additional measurements. This is a very mild condition which is satisfied by every reasonable estimator.
\[ a) \quad \hat{u} \left( \hat{X}^l_i, \hat{Y}^l_i, s_i, s_j \right) = u \left( \hat{X}^l_i, \hat{Y}^l_i, s_i, s_j \right) \xrightarrow{l \to \infty} 0 \quad \text{a.s.,} \]

\[ \forall s_i, s_j \in \{1/2, 1\}, \]

where \( \hat{X}^l_i = \gamma |\hat{H}^l_{ii}|^2, \hat{Y}^l_i = \gamma |\hat{H}^l_{ij}|^2; \) and \( u, \hat{u} \) are defined in Table 1 and (25), respectively.

\[ b) \quad \hat{u}(\hat{X}_i, \hat{Y}_i, s_i, s_j) \text{ is a.s. lower bounded by the function given in Table 2.} \]

**Proof:** To prove part a, we will derive lower and upper bounds on the utility and show that the two bounds coincide in the limit. We begin with the utility in the case where both players choose FS. Similar to [40], a lower bound can be derived as follows,

\[ \hat{u}(\hat{X}_i, \hat{Y}_i, FS_i, FS_j) = I(V_i, W_i | \hat{X}_i = \hat{X}^l_i, \hat{Y}_i = \hat{Y}^l_i) \]

\[ = h(V_i | \hat{X}_i = \hat{X}^l_i, \hat{Y}_i = \hat{Y}^l_i) - h(V_i | W_i, \hat{X}_i = \hat{X}^l_i, \hat{Y}_i = \hat{Y}^l_i) \]

\[ = h(V_i | \hat{X}_i = \hat{X}^l_i, \hat{Y}_i = \hat{Y}^l_i) - h(V_i - \alpha W_i | W_i, \hat{X}_i = \hat{X}^l_i, \hat{Y}_i = \hat{Y}^l_i) \]

\[ \geq h(V_i | \hat{X}_i = \hat{X}^l_i, \hat{Y}_i = \hat{Y}^l_i) - h(V_i - \alpha W_i | \hat{X}_i = \hat{X}^l_i, \hat{Y}_i = \hat{Y}^l_i) \]

\[ = \log (2\pi e \bar{p}) - h(V_i - \alpha W_i | \hat{X}_i = \hat{X}^l_i, \hat{Y}_i = \hat{Y}^l_i) \]

\[ \geq \log (2\pi e \bar{p}) - \log \left( 2\pi e \times \text{var} \left( V_i - \alpha W_i | \hat{X}^l_i, \hat{Y}^l_i \right) \right) \]

where the last inequality follows because Gaussian random variables have maximum entropy. In order to tighten the above lower bound, we choose

\[ \alpha = \frac{E\{V_i W_i | \hat{X}^l_i, \hat{Y}^l_i \}}{E\{W_i^2 | \hat{X}^l_i, \hat{Y}^l_i \}}, \]

since \( \alpha W_i \) is the linear mean square error estimate of \( V_i \). Denote \( \hat{H}^l_{iq} = E\{H_{iq} | \hat{H}_{iq} \} \) and \( \sigma^2_{H_{iq} | \hat{H}_{iq}} = \text{var} \left\{ H_{iq} | \hat{H}_{iq} \right\} \); then,

21
Table 2: Lower bound on user \( i \)'s payoff \( u_i(\hat{X}_i, \hat{Y}_i, s_i, s_j) \)

<table>
<thead>
<tr>
<th>( s_i = 1 )</th>
<th>( s_j = 1 )</th>
<th>( s_i = 1/2 )</th>
<th>( s_j = 1/2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( FDM )</td>
<td>( \frac{1}{2} \log_2 \left( 1 + \frac{E{X</td>
<td>\hat{X}<em>i}}{\sigma</em>{Hi}^2} \right) )</td>
<td>( \frac{1}{2} \log_2 \left( 1 + \frac{E{X</td>
</tr>
<tr>
<td>( FS )</td>
<td>( \frac{1}{2} \log_2 \left( 1 + \frac{E{X</td>
<td>\hat{X}<em>i}/2}{\sigma</em>{Hi}^2} \right) )</td>
<td>( \frac{1}{2} \log_2 \left( 1 + \frac{E{X</td>
</tr>
</tbody>
</table>

\[
\text{var} \left( V_i - \alpha W_i \mid \hat{X}_i, \hat{Y}_i \right) = \text{var} \left( V_i \right) - \frac{E\{W_i V_i \mid \hat{X}_i, \hat{Y}_i \}}{E\{X_i^2 \mid \hat{X}_i, \hat{Y}_i \}} \]  \( (30) \)

where \( X_i = \gamma \hat{H}_{iq} \) and \( Y_i = \gamma \hat{H}_{iq} \) and \( \sigma_{Hi}^2 = \frac{\sigma_{Hi}^2 + \sigma_{H_i}^2}{2} \). Therefore, for every \( \gamma \)

\[
\hat{u}_i(\hat{X}_i, \hat{Y}_i, s_i, s_j) \geq \log \left( 1 + \frac{\hat{X}_i^2}{\hat{Y}_i^2 + 3\sigma_{Hi}^2} \right), \text{ a.s.} \]  \( (31) \)

Because \( \hat{H}_{iq} \overset{t \to \infty}{\to} H_{iq} \) a.s., \( H_{iq} \) is \( \mathcal{F}_\infty \)-measurable, and since \( \{ \hat{H}_{iq}, \mathcal{F}_t \}_{t \geq 1} \)

\( i, q \in \{1, 2\} \), are Doob Martingales\footnote{An important property of Doob Martingales is that if \( X \) is \( \mathcal{F}_\infty \)-measurable (where \( \mathcal{F}_\infty = \cup_{t \geq 1} \mathcal{F}_t \)), then \( X_t \overset{t \to \infty}{\to} X \) a.s. and in \( L^1 \) [see e.g. \( \text{[11]} \), Theorem 13.3.7].}

\[
\hat{H}_{iq} \overset{t \to \infty}{\to} H_{iq}, \text{ a.s., } \forall i, q \in \{1, 2\} \]  \( (32) \)

\[ \mathbf{[11]} \), Theorem 13.3.7\] and therefore

\[
\hat{H}_{iq} - \hat{H}_{iq} \overset{t \to \infty}{\to} 0, \text{ a.s., } \forall i, q \in \{1, 2\}. \]  \( (33) \)
Moreover, because $H_{iq}$ has finite second moment, $\sigma^2_{H_{iq}|H_{iq}} = \mathbb{E}\{H^2_{iq}|\hat{H}_{iq}\} - E^2\{H_{iq}|\hat{H}_{iq}\}$, $q \in \{1, 2\}$ is also a Doob Martingale with respect to the filtration $\{\mathcal{F}_l\}_{l \geq 1}$, and since $H_{iq}$ is measurable with respect to $\mathcal{F}_\infty$, so is $H^2_{iq}$ and therefore $\sigma^2_{H_{iq}|H_{iq|l}} \xrightarrow{l \to \infty} 0$ a.s. Next, by continuity, for every $\gamma$

$$\log \left(1 + \frac{\hat{R}_l/2}{\hat{R}_l/2 + \sigma^2_{H_{iq}|\hat{H}_{iq}+1/\gamma}}\right) - \log \left(1 + \frac{\hat{R}_l/2}{\hat{R}_l/2 + 1/\gamma}\right) \xrightarrow{l \to \infty} 0, \text{ a.s.},$$

(34)

hence, for every $\gamma$

$$\liminf_{l \to \infty} \mathbb{E}\left(\hat{X}_l, \hat{Y}_l, FS_i, FS_j|\mathcal{F}_l\right) - \log \left(1 + \frac{\hat{X}_l}{\hat{Y}_l/2+1}\right) \geq 0,$$

(35)

which establishes a lower bound.

We now derive an upper bound on the utility as

$$\hat{u}_i(\hat{X}_l, \hat{Y}_l, FS_i, FS_j) = I(V_i; W_i|X_i = \hat{X}_l, Y_i = \hat{Y}_l)$$

$$\leq I(V_i; W_i|X_i, Y_i, \hat{X}_i = \hat{X}_l, \hat{Y}_i = \hat{Y}_l)$$

$$= \mathbb{E}\left\{\log \left(1 + \frac{\hat{X}_l}{\hat{Y}_l/2+1}\right) | \hat{X}_l, \hat{Y}_l\right\}$$

$$\leq \log \left(1 + \mathbb{E}\left\{\frac{\hat{X}_l}{\hat{Y}_l/2+1} | \hat{X}_l, \hat{Y}_l\right\}\right)$$

$$= \log \left(1 + \frac{\hat{X}_l}{2} \mathbb{E}\left\{\frac{1}{\hat{Y}_l/2+1} | \hat{X}_l, \hat{Y}_l\right\}\right)$$

(36)

Note that the function $v(y) = 1/(1 + y)$ is measurable and bounded for every $y \geq 0$. Thus, for every $\gamma$, $\mathbb{E}\{|v(Y_i)|\} < \infty$, and it follows that $\{\mathbb{E}\{v(Y_i)|\hat{Y}_l\}|\mathcal{F}_l\}_{l \geq 1}$ is a Martingale. Because $Y_i$ is $\mathcal{F}_\infty$ measurable, so is $v(Y_i)$ and therefore $\mathbb{E}\{v(Y_i)|\mathcal{F}_\infty\} = v(Y_i)$ a.s. Thus, $\mathbb{E}\{v(Y_i)|\hat{Y}_l\} \xrightarrow{l \to \infty} v(Y_i)$ [II, Theorem 13.3.7] and consequently $\mathbb{E}\{v(Y_i)|\hat{Y}_l\} - v(\hat{Y}_l) \xrightarrow{l \to \infty} 0$. Combining the latter limit with (36), it follows that for every $\gamma$

$$\limsup_{l \to \infty} \hat{u}_i(\hat{X}_l, \hat{Y}_l, FS_i, FS_j) \leq \log \left(1 + \frac{X_i/2}{\hat{Y}_l/2+1}\right), \text{ a.s.}$$

(37)

23
The latter inequality and (35) imply that for every \( \gamma \),
\[ u_i(\hat{X}_l^i, \hat{Y}_l^i, FS_i, FS_j) - u_i(\hat{X}_l^i, \hat{Y}_l^i, FS_i, FS_j) \rightarrow 0, \text{ a.s.} \]
which establishes the desired result in the case where both players choose FS.

To complete the proof it is sufficient to show (27) is satisfied in the case
where both players choose FDM. Similar to the derivation of (31),
\[ u_i(\hat{X}_l^i, \hat{Y}_l^i, FDM_i, FDM_j) \geq \frac{1}{2} \log \left( 1 + \frac{\hat{Y}_l^i}{\sigma_{H_l}|H_l|^\gamma + 1} \right), \text{ a.s.} \] (38)
Also, similar to the derivation of (35),
\[ \liminf_{l \rightarrow \infty} u_i(\hat{Y}_l^i, \hat{X}_l^i, FDM_i, FDM_j) - \log \left( 1 + \frac{\hat{X}_l^i/2}{\hat{Y}_l^i/2 + 1} \right) \geq 0, \text{ a.s.} \] (39)
and similar to (10)
\[ \hat{u}_i(\hat{X}_l^i, \hat{Y}_l^i, FDM_i, FDM_j) = I(W_l; V_l|\hat{X}_l = \hat{X}_l^i, \hat{Y}_l = \hat{Y}_l^i) \]
\[ \leq I(W_l; V_l|X_l, Y_l, \hat{X}_l = \hat{X}_l^i, \hat{Y}_l = \hat{Y}_l^i) \]
\[ = E \left\{ \log (1 + X_l/2) | \hat{X}_l^i, \hat{Y}_l^i \right\} \]
\[ \leq \log \left( 1 + E \left\{ X_l/2 | \hat{X}_l^i, \hat{Y}_l^i \right\} \right), \text{ a.s.} \] (40)
Therefore
\[ \limsup_{l \rightarrow \infty} \hat{u}_i(\hat{X}_l^i, \hat{Y}_l^i, FDM_i, FDM_j) \leq \log (1 + X_l/2), \text{ a.s.} \] (41)
Thus, \( \hat{u}_i(\hat{X}_l^i, \hat{Y}_l^i, FDM_i, FDM_j) - u_i(X_l, Y_l, FDM_i, FDM_j) \rightarrow 0, \text{ a.s.} \)
This establishes (27) for the case of mutual DFM. The proof for the rest of the
utility entries is identical; i.e., for \( u_i(\hat{X}_l^i, \hat{Y}_l^i, FDM_i, FS_j) \) the proof is the same as the one we provided for \( u_i(\hat{X}_l^i, \hat{Y}_l^i, FS_i, FS_j) \) since the signal level
is only scaled by a factor of 1/2 and the interference remains the same.
In the case of \( u_i(\hat{X}_l^i, \hat{Y}_l^i, FS_i, FDM_j) \), the utility is the sum of two utilities
resulting from: (i) an interference-free channel which is similar to the case of \(u_i(\hat{X}_i^l, \hat{Y}_i^l, FDM_i, FDM_j)\), (ii) an interference channel which is similar to the case of \(u_i(\hat{X}_i^l, \hat{Y}_i^l, FS_i, FS_j)\).

It remains to show part b; which follows immediately from \((31), (38)\) and by deriving similar equations to the case where one user chooses FDM while the other chooses FS. \(\square\)

In the next theorem, it is shown that BIG’s spectrally efficient equilibrium point is robust to estimation error.

**Theorem 3.** Let \(\hat{S}_i^l\) be the strategy profile in Theorem 1 with \(\hat{X}_i^l, \hat{Y}_i^l\) substituted for \(X_i, Y_i\). Then, under the conditions of Lemma 2, the BIG’s non pure-FS \(\epsilon\)-NE point given in Theorem 1 is robust to estimation error. That is, for every \(\epsilon\), there exists \(\gamma_0\), such that for every \(\gamma > \gamma_0\) there exists \(L\) such that for every \(l > L\), \((\hat{S}_1^l, \hat{S}_2^l)\) is an \(\epsilon\)-NE of the perturbed BIG; i.e., the BIG but with information \(\hat{X}_i^l, \hat{Y}_i^l\) and utility \(\hat{u}_i\) instead of \(X_i, Y_i\) and \(u_i\), respectively.

Note that the robustness of Theorem 3 is different form Definition 6 in that it is restricted to perturbation due to estimation error.

**Proof:** By substituting \(\hat{X}_i^l, \hat{Y}_i^l\) for \(X_i, Y_i\) into \((12), (13)\) one obtains the following strategy profile

\[
\hat{S}_1^l = \hat{S}_1(\hat{X}_1^l, \hat{Y}_1^l, \hat{a}_2) \\
\hat{S}_2^l = \hat{S}_2(\hat{X}_2^l, \hat{Y}_2^l, \hat{a}_1)
\]

where \(\hat{a}_1, \hat{a}_2\) are the solutions to \((14), (15)\) and \(\hat{S}\) is given in \((8)\). Let \(\hat{a}_j^l\) be the probability that player \(j\) chooses FDM under estimation error; i.e.,

\[
\hat{a}_j^l = P\left(\hat{S}_j(\hat{X}_j^l, \hat{Y}_j^l, \hat{a}_2) = 1\right).
\]
Then player $i$’s best response is

$$\hat{S}_i(\hat{X}_i^l, \hat{Y}_i^l, \hat{a}_j) = \begin{cases} s_i = 1, & \text{if } \hat{e}(\hat{X}_i^l, \hat{Y}_i^l, \hat{a}_j) > 0 \\ s_i = 1/2, & \text{otherwise} \end{cases}$$

(44)

where

$$\hat{e}(\hat{X}_i^l, \hat{Y}_i^l, a_j)
= a_j \left( \hat{u}_i(\hat{X}_i^l, \hat{Y}_i^l, FDM_i, FDM_j) - \hat{u}_i(\hat{X}_i^l, \hat{Y}_i^l, FS_i, FDM_j) \right) + (1 - a_j) \left( \hat{u}_i(\hat{X}_i^l, \hat{Y}_i^l, FDM_i, FS_j) - \hat{u}_i(\hat{X}_i^l, \hat{Y}_i^l, FS_i, FS_j) \right)$$

(45)

In what follows, it will be shown that for a sufficiently small estimation error, player $i$ can gain no more than $\epsilon$ by unilaterally deviating from the strategy profile in (42); i.e., for every $\gamma$ and $\epsilon$ there exists some $L$, such that for every $l > L$,

$$|\Delta \hat{\pi}_i(\hat{X}_i^l, \hat{Y}_i^l, \hat{S}_i^l, \hat{S}_j^l)| < \epsilon \text{, a.s.}$$

(46)

where, $\hat{S}_i^l = \hat{S}_i(\hat{X}_i^l, \hat{Y}_i^l, \hat{a}_j)$ and

$$\Delta \hat{\pi}_i(\hat{X}_i^l, \hat{Y}_i^l, \hat{S}_i^l, \hat{S}_j^l) = E \left\{ \hat{u}_i(\hat{X}_i^l, \hat{Y}_i^l, \hat{S}_i^l, \hat{S}_j^l) \big| \hat{X}_i^l, \hat{Y}_i^l, \hat{S}_i^l \right\} - E \left\{ \hat{u}_i(\hat{X}_i^l, \hat{Y}_i^l, \hat{S}_i^l, \hat{S}_j^l) \big| \hat{X}_i^l, \hat{Y}_i^l, \hat{S}_i^l \right\}. $$

(47)

Because

$$E \left\{ \hat{u}_i(\hat{X}_i, \hat{Y}_i, S_i, S_j) \big| \hat{X}_i, \hat{Y}_i, S_i \right\}
= P(S_j = 1)\hat{u}_i(\hat{X}_i, \hat{Y}_i, S_i, FDM_j)
+ P(S_j = 0)\hat{u}_i(\hat{X}_i, \hat{Y}_i, S_i, FS_j),$$

(48)

it follows that

$$\Delta \hat{\pi}_i(\hat{X}_i^l, \hat{Y}_i^l, \hat{S}_i^l, \hat{S}_j^l)
= \hat{a}_j \left( \hat{u}_i(\hat{X}_i^l, \hat{Y}_i^l, \hat{S}_i^l, FDM_j) - \hat{u}_i(\hat{X}_i^l, \hat{Y}_i^l, \hat{S}_i^l, FDM_j) \right)
+ (1 - \hat{a}_j) \left( \hat{u}_i(\hat{X}_i^l, \hat{Y}_i^l, \hat{S}_i^l, FS_j) - \hat{u}_i(\hat{X}_i^l, \hat{Y}_i^l, \hat{S}_i^l, FS_j) \right).$$

(49)
where we used (43).

Note that \( \Delta \hat{\pi}(X_i^l, Y_i^l, S_i^l, \bar{S}_i^l) = \hat{\epsilon}(X_i^l, Y_i^l, \hat{a}_i^l) \) if \( S_i^l \neq \bar{S}_i^l \) and \( \Delta \hat{\pi}(X_i^l, Y_i^l, S_i^l, \bar{S}_i^l) = 0 \) otherwise. Thus, to show that (46) is satisfied, we rewrite \( \Delta \hat{\pi} \) as

\[
\Delta \hat{\pi}(X_i^l, Y_i^l, S_i^l, \bar{S}_i^l) = \hat{\epsilon} \left( \hat{X}_i^l, \hat{Y}_i^l, \hat{a}_i^l \right) I_{\hat{D}_i^l \Delta \bar{D}_i^l} \left( \hat{X}_i^l, \hat{Y}_i^l \right),
\]

where \( \hat{D}_i^l = \{(x, y) : \hat{\epsilon}(x, y, a) > 0\} \) and \( \bar{D}_i^l = \{(x, y) : e(x, y, a) > 0\} \). Note that \( \Delta \hat{\pi} \neq 0 \) iff \( \hat{X}_i^l, \hat{Y}_i^l \in \hat{D}_i^l \Delta \bar{D}_i^l \) which means that \( e(\hat{X}_i^l, \hat{Y}_i^l, \hat{a}_j) \leq 0 \) and \( \hat{\epsilon}(\hat{X}_i^l, \hat{Y}_i^l, \hat{a}_j) > 0 \) or vice versa. Assume that \( \hat{\epsilon}(\hat{X}_i^l, \hat{Y}_i^l, \hat{a}_j) > 0 \), then, from Lemma 2 it follows that for every \( \gamma \)

\[
eq (\hat{X}_i^l, \hat{Y}_i^l, \hat{a}_j) - (\hat{X}_i^l, \hat{Y}_i^l, \hat{a}_j) \xrightarrow{l \to \infty} 0, \text{ a.s.} \quad (51)
\]

Denote \( \hat{a}_j = P(\hat{s}_j(X_j, Y_j, \hat{a}_i) = 1) \), then by the fact that \( \hat{X}_j^l \xrightarrow{l \to \infty} X_j, \) a.s. \( \hat{Y}_j^l \xrightarrow{l \to \infty} Y_j, \) a.s. and because \( e(x, y, a) \) is a continuous function of \( x, y, a \), it follows\(^6\) that for every \( \gamma \), \( \hat{a}_j \xrightarrow{l \to \infty} \hat{a}_j \). By \( [38, \text{Lemma } 2] \) \( \hat{a}_j \xrightarrow{\gamma \to \infty} \hat{a}_j \); combining the latter two limits, it follows that for every \( \delta > 0 \), there exists a \( \gamma_0 \) such that for every \( \gamma > \gamma_0 \) there exists \( L \) such that \( |\hat{a}_j - \hat{a}_j| < \delta \) for every \( l > L \). In other words, \( \lim_{\gamma \to \infty} \lim_{l \to \infty} \hat{a}_j = \hat{a}_j \). Thus, for every \( \epsilon > 0 \), there exists a \( \gamma_0 > 0 \), such that for every \( \gamma > \gamma_0 \) there exists \( L \), such that for every \( l > L \)

\[
|e(\hat{X}_i^l, \hat{Y}_i^l, \hat{a}_j) - e(\hat{X}_i^l, \hat{Y}_i^l, \hat{a}_j)| < \epsilon, \text{ a.s.} \quad (52)
\]

This implies that there exists a \( \gamma_0 \) such that for every \( \gamma > \gamma_0 \) there exists \( L \) such that for every \( l > L \)

\[
eq (\hat{X}_i^l, \hat{Y}_i^l, \hat{a}_j) < 0, \text{ a.s.} \quad (53)
\]

\(^6\)Recall that \( \hat{s}_j(x, y, a) = 1 \) is equivalent to \( e(x, y, a) > 0 \).
Combining (53) with (51) it follows that there exists a \( \gamma_0 \) such that for every \( \gamma > \gamma_0 \) there exists \( L \) such that for every \( l > L \)

\[
|\hat{\epsilon}(\hat{X}_l^i, \hat{Y}_l^i, \hat{a}_l^i)| < \epsilon, \ a.s. \tag{54}
\]

This establishes the desired result.

5. Simulation Results

While Theorem 4 shows robustness to estimation errors, it does not indicate how small the error must be. We now address this question via simulation. Consider the BIG in a Rayleigh flat-fading channel, in which every coherence period user \( i \) obtains unbiased estimates \( |\hat{H}_{iq}|^2, q = 1, 2 \). These estimates are then substituted into the non-pure FS \( \epsilon \)-NE strategy profile of Theorem 4, instead of the actual channel gains \( |H_{iq}|^2, q = 1, 2 \). We assume that during the estimation phase, players coordinate to transmit their training signals in disjoint sub-bands. Thus, each player observes

\[
W_{iq}^k = H_{iq}^k d_k + N_k, \ q = 1, 2, \tag{55}
\]

where \( k \) is the channel coherence-interval index. The vector \( W_{ii}^k \in \mathbb{C}^{M \times 1} \) is used for the direct channel estimation and \( W_{ij}^k \) is used for the interference estimation. Also, \( d_l = [d_1^k, ..., d_M^k]^T \) is a known training signal and \( N_k = [N_1,k, ..., N_{M,k}] \) is a white circular Gaussian noise vector with covariance \( \sigma_N^2 I \).

\footnote{This assumption is made to simplify the analysis. In general, since the BIG assumes no interference cancellation, only the interference power is needed. Thus, players can use an energy detector without knowing their opponent’s training sequence.}
The channels \( H_{iq}^k, q = 1, 2 \) distribute as \( H_{iq}^k \sim CN \left( 0, \sigma_{iq}^2 \right) \). We use the following channel model

\[
H_k = \sigma \Phi_k, \tag{56}
\]

where \( \Phi_k \) is an i.i.d. random sequence, whose distribution is \( CN(0, \sigma^2) \), and \( \sigma \) is a known deterministic [12]. In the simulation, the unknown \( H_k \) (or equivalently \( \Phi_k \)) is estimated at each coherence time \( k \) from \( W_{iq}^k \) (see (55)) via the MMSE estimator

\[
\hat{H}_k = \frac{d^H W_{iq}^k}{\sigma_n^2 \left( \sigma^2 + |d|^2 / \sigma_n^2 \right)^{1/2}}. \tag{57}
\]

Two scenarios are studied. The first is a symmetric game; i.e., \( \sigma_{i,q} = \sigma, \forall i, q \in \{1, 2\} \), and the second is a non-symmetric weak-strong interference setup with \( ISR_1 = -5, ISR_2 = 5 \), where \( ISR_i = \sigma_{ij}^2 / \sigma_{ii}^2 \). Each scenario is evaluated for high and low SNR; i.e., \( \sigma_{ii}^2 = -90 \text{dB} \) (high) and, \( \sigma_{ii}^2 = -110 \) (low) \( \forall i \in \{1, 2\} \) with a noise floor of -121 dBm and a transmit power of \( \bar{p} = 0 \) dBm. The training signal \( d_t \) is chosen to be a constant vector of ones.

The simulation is carried out as follows. In each setup, we draw samples of flat-fading channels with the corresponding set of path-losses, where each player \( i \) estimates its own channel \( \hat{H}_{ii} \) and the interference channel \( \hat{H}_{ij} \). In the presence of estimation errors, there is no closed-form expression for the BIG payoff. Therefore, in order to evaluate the performance loss, we use the lower bound in Table 2, with the MMSE’s conditional variance

\[
\sigma_{H_{iq}^k \hat{H}_{iq}^k} = \text{var} \left( H_{iq}^k | \hat{H}_{iq}^k \right) = \frac{\sigma_{iq}^2 \sigma_n^2}{|d|^2 \sigma_n^2 + \sigma_n^2} = \frac{\sigma_{iq}^2 \sigma_n^2}{|d|^2 / \sigma_n^2 + 1}. \tag{58}
\]

We begin with simulation results for the symmetric game. Figure 5 depicts the lower bound of Table 2 (solid curves), as a function of the training
size $M$, against the payoff in the case of no estimation error, given in Table 1 (dashed curves). The black curves represent the non-pure FS equilibrium point of Theorem 1, and the blue curves represent the payoff in the case of deviation; i.e., of the player who deviates from the non-pure FS equilibrium point and uses FS, while its opponent sticks to it. The figure also presents the payoff for the mutual-FS NE point (green lines). It is shown that in both the high and low SNR regimes, the non-pure FS $\epsilon$-NE point of Theorem 1 approaches the corresponding point of the game with no estimation errors. Even for a short training length of one, the performance loss due to estimation error is only 17% and 6% in the low and high SNR regimes, respectively. The figure further shows that the non-pure FS equilibrium point is robust against deviations, since for a training length of one, deviation does not increase the payoff, and for larger values of $M$, it leads to a lower payoff. In addition, the non-pure FS equilibrium point Pareto dominates the FS-NE for all training lengths. Similar conclusions can be drawn concerning the weak-strong game from Fig. 1, which shows the expected payoff of the strong and weak users in the low and high SNR regimes. Similar to the symmetric case, the non-pure FS equilibrium point is robust, for all training lengths and for both low and high SNR. Furthermore, the non-pure FS equilibrium point Pareto dominates the FS NE for every training length, for both the weak and the strong users and for both weak and high SNRs.

6. Summary

In this paper, we studied the two-user Bayesian interference game in the case of block fading channels, and examined its robustness to estimation er-
In the case of time-invariant channels, with perfect channel estimation, the BIG is known to have a non-pure FS $\epsilon$-NE point which is more spectrally-efficient than the FS-NE point since it yields a higher payoff for both users; a property known as Pareto dominance. It was shown that the strategy profile that yields the non-pure FS $\epsilon$-NE point in the case of time-invariant channels leads to an identical equilibrium point in time-varying, or frequency selective block-fading channels. In addition, it was shown that the non-pure FS equilibrium point is robust to estimation errors. The numerical results showed that the non-pure FS equilibrium point is robust even when the channels are estimated via a training sequence as short as one, and that it maintains its Pareto dominance over the FS NE point.
Figure 6: Weak-strong scenario in the low SNR regime

\[ \sigma_{iq}^2 = -110\text{dB} \forall i, q \in \{1, 2\} \], and SNR regime \( \sigma_{iq}^2 = -90\text{dB} \forall i, q \in \{1, 2\} \). The weak user’s ISR is 5dB where the strong user’s ISR is -5dB. The solid and dashed lines represent the same quantities as in Fig. 5.
References


[16] J. Lunden, V. Koivunen, H. V. Poor, Spectrum exploration and ex-


