Weighted Max-Min Resource Allocation for Frequency Selective Channels

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***Abstract*—In this paper[1], we discuss the computation of weighted max-min rate allocation among N users, using joint FDM/TDM strategies under a Power Spectral Density (PSD) mask constraint. We show that the weighted max-min solution allocates the rates according to a predetermined rate ratio defined by the weights, a fact that has considerable implications for telecommunication service providers. Furthermore, we show that the problem can be efficiently solved using a linear programming technique, where at most $N-1$ frequency bins are shared by more than one user. We also address the resource allocation problem in the mixed services scenario where certain users have the required rates, while the others have flexible rate requirements. The solution is applicable to many communication systems that are limited by a power spectral density mask constraint such as UWB. The theoretical results are followed by simulations for various allocation schemes.**

***Index Terms*—Power allocation, multi-carrier systems, and rate control.**

I. Introduction

Orthogonal Frequency Division Multiple Access (OFDMA) is becoming a ubiquitous technique for wireless multiple access schemes in communication systems such as UWB, WLAN, WiMAX and LTE, due to its high spectral efficiency. OFDMA waveforms provide the flexibility of allocating subcarriers to combat frequency selective fading. These standards operate under two types of power constraints: total power constraint and Power Spectral Density (PSD) mask constraint. The PSD mask constraint is imposed by the regulator on the radio transmitter. The total capacity of the OFDMA can be optimized by dynamically allocating subcarriers among users according to channel conditions. However, the operator must satisfy the subscribers' demands to provide a reasonable level of Quality of Service (QoS). The standards define several different services that allow QoS differentiation. The major challenges facing QoS in wireless networks are the dynamics of the channels, bandwidth allocation, and handoff support. It is important to guarantee QoS at each layer so that the network stays flexible. Bandwidth and bit rates play a major role because they need to be allocated efficiently. In some systems, data services and voice services must be supported simultaneously. These services can conflict because voice services are highly delay-sensitive and require real-time service. By contrast, data services are less delay-sensitive but are highly sensitive to loss of data and require almost error-free transmission. Thus, both factors must be taken into account when providing QoS for voice and data services.

In [1], a power adaptation method was suggested to maximize users' total data rate in downlinks of an OFDM system. The transmitted power adaptation scheme was derived by solving the maximization problem in two steps involving subcarrier assignment of users and power allocation of subcarriers. The outcome was that the data rate of a multiuser OFDM system is maximized when each subcarrier is assigned to only one user with the best channel gain for that subcarrier, and the transmit power is distributed over the subcarriers by a water-filling policy. However, fairness was not factored into this approach. In the extreme case, most of the spectrum is allocated to a small group of subscribers with high average channel gains. In [2], the problem of resource allocation of the OFDMA system was addressed. A heuristic scheduling algorithm was proposed under the constraint that each subscriber must obtain a preset data rate. Rhee and Cioffi [3] derived a multiuser convex optimization problem under the total power constraint to find a max-min suboptimal subcarrier allocation, where equal power is allocated to the subcarriers. A max-min rate allocation algorithm maximizes the data rate of the worst user, such that all users operate at a similar data rate. However, this solution is unworkable when the operator has to provide different levels of service. Shen et al. [4] proposed a suboptimal proportional fairness resource sharing mechanism that provides multiple service levels under a total power constraint while maximizing the total data rate. The algorithm involves two steps. First, the subcarriers are allocated under the assumption that the power is equal on each subcarrier. In the second step, the power is distributed among the allocated subcarriers to maximize the total rate while maintaining proportional fairness constraints. Ihyoung et al. [5] and Yui et al. [6] addressed the joint problem of resource allocation, bit loading and power allocation under a total power constraint. They proposed *suboptimal approaches* that perform channel allocation and bit loading separately. In [7] the author suggested a resource allocation based on maximizing the weighted sum rate under a total power constraint. This approach does not guarantee fairness, since the users with the best channels get most of the resources. An alternative approach to the resource allocation problem is based on bargaining by using game theoretic solutions such as the Nash bargaining solution under total power constraint (see e.g., [8], [9]), [10] or under a PSD mask constraint [11] as well as the Kalai-Smorodinski solution [12], [13], [14].

Here, we describe an alternative optimization problem. We suggest optimizing the resource allocation under weighted

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max-min rate criteria and a PSD mask constraint. The PSD mask constraint imposes a restriction on the amount of interference a transmitter is allowed to generate in each frequency bin. The operators' objective is to maximize the utility of the spectrum by transmitting the maximum data rate in each frequency bin in the shortest time period from the central station (base station, or access point). Therefore, the central station, which is not power limited, operates very close to the PSD mask constraint as was set by the FCC regulator [15], and the wireless standards such as WLAN [16], WiMAX [17], UWB [18], and LTE [19]. Note that the rate that each user can obtain in each frequency bin is limited by the transmitted power and the channel response (Shannon capacity equation). We introduce the mechanisms to enable explicit subcarrier allocation for multiple users in wireless systems when the following conditions must be fulfilled:

1) Differentiated service levels must be supported. A wireless operator has the flexibility to specify differentiated service levels (or weights). The available radio resource has to be partitioned proportionally to the weights.
2) Voice service is supported using a fixed data rate.
3) Computational and signaling overhead must be minimal. One primary design goal of an efficient resource allocation algorithm is to minimize the communication and the computational load of feedback iterations. Algorithms have to be designed to calculate the allocation that puts a minimal load on the system. Specifically, the time it takes to calculate the fair rate must be minimal.

To achieve the above goals, we show how the weighted max-min fairness design criterion can assist operators in network optimization, at multiple target rates. Here, we use a model similar to [4] but employ a PSD mask rather than a total power constraint. It is well known that the total data throughput of a zero-margin system is close to capacity even with a flat transmit PSD as long as the energy is only poured into subcarriers with high SNR gains. A good algorithm will not assign power to bad subcarriers. Furthermore, a flat transmit (PSD) might be necessary if the PSD mask constraint is tighter than the total power constraint.

We analyze the weighted max-min solution under a PSD mask constraint and show that irrespective of the number of frequencies, for any set of weights, a solution exists where the N users share at most $N-1$ frequencies in time, and all other frequencies are allocated to a single user. When the solution is unique this follows from results reported in Mjelde [20]; however we provide a simpler proof relying on the fact that any point on the boundary of the rate region is the solution of a weighted max-min problem and show that such a solution can be found even when the solution is non-unique. This result is important for systems with large numbers of sub-carriers operating over random channels, since in practical terms it suggests that the optimal FDM allocation is near optimal in this case. Furthermore, since any point on the boundary of the FDM/TDM rate region is the solution of a weighted max-min problem, we obtain that the same holds for other game theoretic solutions which cannot be solved using linear programming such as the Nash Bargaining solution [11], the Generalized NBS and the Kalai-Smorodinsky solution [13].

The paper is organized as follows. In Section II, we describe the general model for the wireless system and derive a solution for the weighted max-min resource allocation problem. Section III focuses on analyzing the structure of the optimal solution showing that at most $N-1$ frequencies should be shared, based on a theorem that is proved in APPENDIX A. In section IV a closed-form solution for the two user case is discussed together with a simple algorithm for computing the weighted max-min solution. Simulation results are presented in Section V. We end with some concluding notes and discussion of possible extensions.

II. Resource allocation using the weighted max-min solution

In this section, we show that under a PSD mask constraint a max-min fair solution can be computed using linear programming. This is simpler than the total power constraint where general convex programming is necessary. Assume that we have N users, operating over N by N frequency selective Gaussian interference channels. Let the K channel matrices[2] at frequencies $k = 1, ..., K$ be given by

$$\mathbf{H}_k = \begin{bmatrix} h_{11}(k) & \dots & h_{1N}(k) \\ \vdots & \ddots & \vdots \\ h_{N1}(k) & \dots & h_{NN}(k) \end{bmatrix}, \qquad k = 1, ..., K. \quad (1)$$

The noise at the n receiver in frequency bin k has zero mean and variance $\sigma_n^2(k)$. Each user is allowed to transmit using a maximal power $p(k)$ in the k'th subcarrier. In this paper, we limit ourselves to a joint FDM and TDM scheme in which an assignment of disjoint portions of the frequency band to the various transmitters can be different at each time instance as is the case in WiMAX. In FDM/TDM scheme, we have the following:

1. User n transmits using a PSD limited by $\langle p_n(k) : \ k = 1, ..., K\rangle$.
2. Each user n is allocated a relative time vector $\boldsymbol{\alpha} = [\alpha_{n1}, ..., \alpha_{nK}]^T$ where α_k is the proportion of time allocated to user n at the k'th frequency channel. This is the FDM/TDM part of the scheme.
3. For each k, $\sum_{n=1}^{N} \alpha_{nk} = 1$. This is a Pareto-optimality requirement.
4. The rate obtained by user n is given by

$$R_n(\boldsymbol{\alpha}_n) = \textstyle\sum_{k=1}^{K} \alpha_{nk} R_{nk}, \quad (2)$$

where

$$R_{nk} = \log_2 \left(1 + \frac{|h_{nn}(k)|^2 p_n(k)}{\sigma_n^2(k)} \right), \quad (3)$$

and the subcarrier bandwidth is normalized to 1. Interference is avoided by time sharing at each frequency band; i.e., only a single user transmits at a given frequency bin at any time. Furthermore, since at each time instance each frequency is used by a single user, each user will transmit using the maximal power. Note that we can replace the instantaneous

[2]These can be the uplink, downlink or multiple source-destination pairs within the network.

rates by the long term averages using well-known coding theorem for fading channels [21]. This allows for much slower information exchange and makes the proposed approach more practical in real wireless systems.

A. Problem Formulation

The weighted max-min fair solution with weights $\gamma_1, ..., \gamma_N, \gamma_n \in (0, \infty)$, is given by solving the following equation:

$$R_{\max\min} = \max_{\boldsymbol{\alpha}_1,...,\boldsymbol{\alpha}_N} \min_{1 \le n \le N} \gamma_n R_n(\boldsymbol{\alpha}_n). \tag{4}$$

To solve this equation, we rephrase it as a linear programming problem: Let c be the value of the weighted max-min rate. We maximize c under the constraints $\gamma_n R_n \ge c$, for all $1 \le n \le N$. Since each R_n depends linearly on $\boldsymbol{\alpha}_n$, we require

$$\max_{\boldsymbol{\alpha}_1,...,\boldsymbol{\alpha}_N, c} c, \tag{5}$$

$$\text{s.t.} \begin{cases} 0 \le c, \\ \frac{c}{\gamma_n} \le \sum_{k=1}^{K} \alpha_{nk} R_{nk}, \qquad n = 1, ..., N\ , \\ \sum_{n=1}^{N} \alpha_{nk} = 1, \qquad k = 1, ..., K. \end{cases} \tag{6}$$

The Lagrangian is given by:

$$\begin{aligned} f(\boldsymbol{\alpha}, \boldsymbol{\delta}, \boldsymbol{\mu}, \boldsymbol{\lambda}, c) = \quad & -c - \sum_{n=1}^{N} \delta_n \left(\sum_{k=1}^{K} \alpha_{nk} R_{nk} - c/\gamma_n \right) \\ & - \sum_{n=1}^{N} \sum_{k=1}^{K} \mu_{nk} \alpha_{nk} \\ & + \sum_{k=1}^{K} \lambda_k \left(\sum_{n=1}^{N} \alpha_{nk} - 1 \right) - \beta c. \end{aligned} \tag{7}$$

To better understand the problem, we first derive the KKT conditions [22]. Taking the derivative with respect to the variables α_{nk} and c, we obtain

$$\begin{cases} -\mu_{nk} + \lambda_k - \delta_n R_{nk} = 0, \\ -1 + \sum_{n=1}^{N} \frac{\delta_n}{\gamma_n} - \beta = 0, \end{cases} \tag{8}$$

with the complementarity conditions:

$$\begin{cases} \lambda_n \left(\sum_{n=1}^{N} \alpha_{nk} - 1 \right) = 0, \\ \delta_n \left(\sum_{k=1}^{K} \alpha_{nk} R_{nk} - c/\gamma_n \right) = 0, \\ \mu_{nk} \alpha_{nk} = 0, \\ \beta c = 0, \mu_{nk} \ge 0, \beta \ge 0, \delta_n \ge 0. \end{cases} \tag{9}$$

Note that this problem is always feasible by choosing $c = 0$. Based on (8)-(9), we can easily see that the following theorem holds:

Theorem 2.1: The Lagrange multipliers in (9) satisfy the following claims:

1. If there is a non-zero feasible solution, then $\beta = 0$.
2. For each user with total rate equal to $c/\gamma_n > 0$, $\delta_n > 0$, and $\beta = 0$. Therefore, $\sum_{n=1}^{N} \delta_n/\gamma_n = 1$. Otherwise, $\delta_n = 0$.
3. If $\alpha_{nk} > 0$, then $\mu_{nk} = 0$ and $\lambda_k = \delta_n R_{nk}$.
4. If $\alpha_{nk} = 0$, then $\mu_{nk} \ge 0$ and $\lambda_k \ge \delta_n R_{nk}$.

We can view λ_k as a threshold level of frequency k, and $\delta_n R_{nk}$ is the normalized rate of user n in frequency k. Thus, only users whose normalized rate in frequency k is equal to threshold λ_k, can use frequency bin k. Note, that $\{\lambda_k\}$ and $\{\delta_n\}$ are the variables of the dual optimization problem. We now can obtain the following theorem:

Theorem 2.2: If all the users have a non-zero PSD mask in all frequency bins, the weighted max-min fair solution is achieved if all users have equal weighted rates; i.e., the optimal c satisfies $c = \gamma_n R_n, \forall n$.

Proof: Let c be the optimal value. Assume that there is a user n with a rate higher than c and let k be a frequency such that $\alpha_{nk} > 0$. Define $\alpha'_{nk} = \alpha_{nk} - \varepsilon, \varepsilon > 0$, and for $m \ne n$: $\alpha'_{mk} = \alpha_{mk} + \varepsilon/(N-1)$. Obviously the weighted rate for all other users is increased. Choosing $\varepsilon \le \gamma_n \sum_{k=1}^{K} \alpha_n(k) R_{nk} - c$, ensures that $\gamma_n R_n > c$. Since by construction all users $m \ne n$ achieve a rate higher than c, we obtain a contradiction to the optimality of c. ■

This claim is an important result from a network planning perspective. The achieved rates are proportional to $1/\gamma_n$; in other words, users with rates γ_m, γ_n will receive rates satisfying $R_m/R_n = \gamma_m/\gamma_n$. This is desirable since the utility typically scales with $\log R$, so that doubling the rate results in a fixed increase in the total utility.

B. Voice and data rate allocation

In networks carrying mixed services, it is important to be able to allocate a fixed bandwidth to constant-bit-rate and latency-sensitive services such as voice services. The weighted max-min formulation can be easily generalized to this case. Voice users (fixed rate) will get at least R_{min}, while, other variable-bit-rate users will get the weighted max-min rate according to their respective service levels. We have two groups of users (V, and D), and the optimization becomes:

$$\max_{\boldsymbol{\alpha}_1,...,\boldsymbol{\alpha}_N, c} c, \tag{10}$$

$$\text{s.t.} \begin{cases} 0 \le c, \\ c \le \sum_{k=1}^{K} \alpha_{ik} R_{nk}, \qquad i \in D, \\ R_{\min} \le \sum_{k=1}^{K} \alpha_{ik} R_{nk}, \qquad i \in V, \\ \sum_{i=1}^{N} \alpha_i(k) = 1, \qquad k = 1, ..., K. \end{cases} \tag{11}$$

Here, we solve the optimization problem by first assuming that set D is empty. This will confirm that there is a feasible solution for the voice users. If there is a feasible solution for the set V, then we know that there is a feasible solution to the general problem. A simple version of this scenario is analyzed in Example II in Section V.

We now show that the feasibility of a given rate allocation can be tested by solving a simple weighted max-min problem, where the weights are given by the inverse of the desired rates. By Theorem 2.2, the solution to the weighted max-min problem with weights given by $\gamma_n = 1/R_n^d$ where R_n^d is the desired rate for user n, provides the largest c such that for each user $cR_n^d = R_n$. Hence the rate vector $\left(R_1^d, ..., R_N^d\right)$ is feasible if and only if the solution satisfies $1 \le c$. Otherwise the rate vector is infeasible. This completes the solution of the feasibility problem.

Since the discovery of the Simplex Method in the 1940s, extensive work has been done on algorithms for solving Linear Programming, (LP). Large numbers of optimization algorithm have been developed,including variants on Simplex

Method, the Ellipsoid Method, and the Primal-Dual Interior-Point Method. Khachiyan [?] proved in 1979 that Linear Programming is polynomially solvable; namely, that an LP problem with rational coefficients, m inequality constraints and n variables can be solved in $O(n^3(n+m)L)$ arithmetic operations, L[3] being the input length of the problem, i.e., the total binary length of the numerical data specifying the problem instance. In our case (the primal dual problem) we have $n = KN + 1$ variables and $m = K + N$ inequality constraints. Note that the matrix in our case is almost unimodular, and sparse, thus the worst case complexity is on the order of $O(K^6 N^5)$. Thus, from a practical viewpoint, all algorithms are efficient with fairly low empirical complexity. This is why these methods are able to solve very large-scale real world LP problems in reasonable time. In practice, the algorithms converge faster than the worst case bound. A more extensive discussion on complexity can be found in [?].

III. The minimal set of shared frequency bins

In this section, we will prove that under a PSD mask constraint there is always an optimal weighted max-min resource allocation solution where at most $N-1$ frequency bins are shared by more than one player. For the specific case where there is a unique allocation this result follows from the results in [20], we go further and show that even when there is no unique solution there is always a solution where at most $N-1$ frequencies are shared.

Based on Theorems 2.1 and 2.2, we can conclude that there is always a feasible solution and the solution is given by a set of N parameters $\delta_n, n = 1, ..., N$, that satisfy the following relations:

1. $\sum_{n=1}^{N} \delta_n = 1$.
2. For each frequency bin, there is a threshold λ_k.
3. If $\alpha_{nk} > 0$, then $\lambda_k = \delta_n R^*_{nk}$; otherwise, $\alpha_{nk} = 0$.
4. The weighted rates of all players are equal; i.e., $\sum_{k=1}^{K} \alpha_{nk} R^*_{nk} = \sum_{k=1}^{K} \alpha_{n'k} R^*_{n'k} = c, \forall n, n' \in \{1, .., N\}$, where $R^*_{nk} = \gamma_n R_{nk}$.
5. If one or more players are sharing the frequency bin, the α's must satisfy items 3 and 4.

The solution can be described graphically as a bipartite graph[4] whose vertices are divided into two disjoint sets, S_N and S_K. S_N is the set of players, and S_K is the set of frequency bins. A vertex from the set S_N is connected by edges to a subset of vertices in the set S_K. An edge that connects a vertex $n \in S_N$ to a vertex $k \in S_K$ has a weight α_{nk}. The aggregate weight of the edges that enter a vertex $k \in S_K$ is 1.

For example, let us assume that there are three players $\{1, 2, 3\}$ and three frequency bins $\{A, B, C\}$, that have to be shared with a weight vector $\{7.5, 6, 10\}$. The utility matrix $\mathbf{R} = \{R_{nk}\}$ is assumed to be

$$\mathbf{R} = \begin{pmatrix} 1\,0\,3 \\ 2\,3\,0 \\ 0\,1\,2 \end{pmatrix}. \tag{12}$$

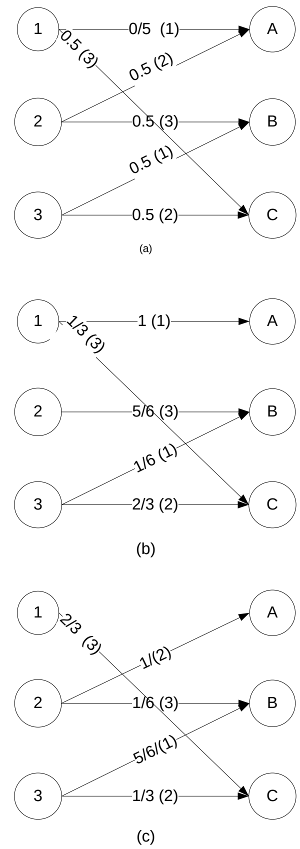

Fig. 1. Three options for sharing the frequency bins with weights $\{7.5, 6, 10\}$. (a) Sharing three frequency bins. (b) Sharing two frequency bins. (c) Sharing two frequency bins.

In Figure 1, three alternatives for sharing the frequency bins are depicted. The first number on each edge denotes the value of α_{nk}, and the second number denotes the maximum feasible rate that a user can get from this channel, namely R_{nk}. In option a, user 1 gets frequency bins A and B half of the time and he shares it with the other users. The total rate that he gets is $2 = 0.5 \times 1 + 0.5 \times 3$. In option b, user 1 gets frequency bins A and shares frequency C half of the time with user 3. The total rate that he gets is $2 = 1 \times 1 + 3/3$. As can be seen, there is no unique solution, and the other options provide the same rate for each of the users where only two frequency bins are shared. In what follows, we will formulate sufficient conditions for a unique solution, and derive an upper bound on the number of frequency bins that are shared by more than one user.

The following lemma provides an essential condition for a unique solution.

Lemma 3.1: If there is a unique solution, then any pair of users share at most a single frequency.

[3]In our case K(N-1) coefficients of the matrix A are zeros or ones; thus the value of L is bounded by, where $L = \sum_{i,j} \log_2(a_{i,j} + 1) + \log_2(nm) + (nm + m) = O(K^2 N)$

[4]The bipartite graph here, is defined differently than in [23], where a vertex represents a player, and two vertices are adjacent if some of the frequency bins are shared by the two players.

Proof: Without loss of generality, we assume that in the optimal solution, two frequency bins m and ℓ are shared by users 1 and user 2, respectively. We also assume that $\gamma_1 = \gamma_2 = 1$ (since this can be achieved by multiplying user n's rates by γ_n). Then,

$$\begin{array}{rcl} R_1(\boldsymbol{\alpha}_1) & = & \sum_{k=1}^{K} \alpha_{1k} R_{1k} \\ & = & R_1^{-(m,\ell)} + \alpha_{1,m} R_{1,m} + \alpha_{1\ell} R_{1,\ell} \\ & = & R_2^{-(m,\ell)} + \alpha_{2,m} R_{2,m} + \alpha_{2,\ell} R_{2,\ell} \\ & = & \sum_{k=1}^{K} \alpha_{2k} R_{2k} = R_2(\boldsymbol{\alpha}_2) \end{array} \tag{13}$$

and

$$R_i^{-(m,\ell)} = \sum_{k \neq m,\ell}^{K} \alpha_{ik} R_{ik}. \tag{14}$$

Here, $R_i^{-(m,\ell)}, i = 1, 2$ is the weighted sum rate of user i from all other frequency bins with user $j \neq i$. We will now show by contradiction that this scenario is not feasible if the solution is unique. Assuming that we increase the value of $\alpha_{1,m}$ by $\delta > 0$ as shown in Figure 2a. Then, in order not to change the rate of user 1, $\alpha_{1,\ell}$ must decrease by $\delta \frac{R_{1,m}}{R_{1,\ell}}$. Thus, the value of the $\alpha_{2,\ell}$ increases by $\delta \frac{R_{1,m}}{R_{1,\ell}}$, and the value of $\alpha_{2,m}$ decreases by $\delta \frac{R_{1,m} R_{2,\ell}}{R_{1,\ell} R_{2,m}}$. Now, since the original allocation is unique, this new allocation is not feasible. This implies that the increase of $\alpha_{1,m}$ is greater than the decrease in $\alpha_{2,m}$; i.e., $\delta > \delta \frac{R_{1,m} R_{2,\ell}}{R_{1,\ell} R_{2,m}}$, or in other words,

$$\frac{R_{1,m}}{R_{2,m}} < \frac{R_{1,\ell}}{R_{2,\ell}}. \tag{15}$$

Similarly, if we decrease the value of $\alpha_{1,m}$ by $\delta > 0$, and apply the same type of calculations we can show that the value of $\alpha_{1,\ell}$ has to increase by $\delta \frac{R_{1,m}}{R_{1,\ell}}$. The value of $\alpha_{2,\ell}$ has to decrease by $\delta \frac{R_{1,m}}{R_{1,\ell}}$, and $\alpha_{2,m}$ increases by $\delta \frac{R_{1,m} R_{2,\ell}}{R_{1,\ell} R_{2,m}}$. Since the original allocation is unique, this is an infeasible solution. This implies that, $\delta < \delta \frac{R_{1,m} R_{2,\ell}}{R_{1,\ell} R_{2,m}}$, or

$$\frac{R_{1,m}}{R_{2,m}} > \frac{R_{1,\ell}}{R_{2,\ell}}. \tag{16}$$

This contradicts (15). Therefore, if there is a unique solution, then at most a single frequency is shared by the same two users.

Hence, if there is an optimal solution where the two users (1 and 2) share two frequency bins (m and ℓ), then $\frac{R_{1,m}}{R_{2,m}} = \frac{R_{1,\ell}}{R_{2,\ell}}$. This is not a unique solution, since by changing the value of α's, there is an alternative solution where the two users only share a single frequency bin. ■

Before proving the general theorem, we will introduce the following two definitions.

Definition 3.1: A group of r users $\{n_1, n_2, ..., n_r\}$ is called *a connected group of order* r, if there is a set of r shared frequency bins $k_{n_1,n_2}, k_{n_2,n_3}, ..., k_{n_r,n_1}$, where $k_{n_i,n_{i+1}}$ is the frequency bin that is shared between user n_i and user n_{i+1}.

Definition 3.2: A *minimal connected group of order* t is the minimal set of t users (not necessarily unique) that is a connected group. Specifically, there are no connected groups of $r < t$ users.

Theorem 3.2: If there is a unique optimal solution to the weighted max-min problem, then there is no minimal connected group of users in the solution.

This result was proved by Mjelde [20]. However, our proof is simpler since we only consider the weighted max-min optimal allocations, and we present it in the appendix for completeness.

Lemma 3.3: There is always an optimal solution where at most $N - 1$ frequency bins are used by more than a single user.

Proof: Since there are N users, if there are more than $N - 1$ shared frequencies, there must be at least one connected group. However, Theorem 3.2 implies that there is no minimal connected group, if there is a unique solution. Then, at most $N - 1$ frequency bins can be shared by more than one user.

On the other hand, if the solution is not unique, there is a minimal connected group. This is feasible only if

$$\frac{R_{n_1}(k_{n_t,n_1})}{R_{n_t}(k_{n_t,n_1})} \prod_{i=1}^{t-1} \frac{R_{n_{i+1}}(k_{n_i,n_{i+1}})}{R_{n_i}(k_{n_i,n_{i+1}})} = 1. \tag{17}$$

Thus, by gradually increasing (see Figure 3) $\alpha_{n_1}(k_{n_1,n_2})$, we decrease the value of $\alpha_{n_2}(k_{n_1,n_2})$ and increase the value of $\alpha_{n_2}(k_{n_2,n_3})$. This process affects all the frequencies in the connected group. The value of $\alpha_{n_p}(k_{n_p,n_{p+1}})$ increases and the value of $\alpha_{n_p}(k_{n_{p-1},n_p})$ decreases. Since $0 \leq \alpha_{n_p}(k_{n_{p-1},n_p}), \alpha_{n_p}, (k_{n_p,n_{p+1}}) \leq 1, p \in 1, ..., N$, the connected group will be broken when one of the α's is assigned a value of one or zero. Thus, the number of shared frequency bins is less than N. We now address the case in which there are J independent connected groups in the graph with N_j players in each connected group. In this case, each connected group can be broken, and the number of shared frequency bins in each group will be $N_j - 1$. Since the groups are independent we get that $\sum_{j=1}^{J} (N_j - 1) < N - J$. ■

The following comments are in order concerning the above properties:

1) It is impractical to check a-priori that there will be a unique solution, since it must be confirmed that there is no loop that satisfies the equality in (17). Nevertheless, if there is a solution with a connected group, the connected group can be disconnected by changing the values of α's gradually until the group is disconnected.
2) In order to guarantee a unique solution, we can add small random variables to $\{R_{nk}\}$. Thus, the probability that the (17) holds is very unlikely, and the probability of getting a solution with a connected group is very small.

Finally, the following important consequence needs to be emphasized. Recently, cooperative approaches derived from game theory have been used for efficient radio resource allocation. The most popular approaches are based on the Nash Bargaining Solution (NBS) [24], the Generalized Nash Bargaining Solution (GNBS) [25], and the Kalai-Smorodinsky solution [26]. All of these cooperative approaches provide Pareto optimal solutions that are located on the boundaries of the achievable rate region. The following theorem provides an upper bound on the number of frequencies that are shared between users.

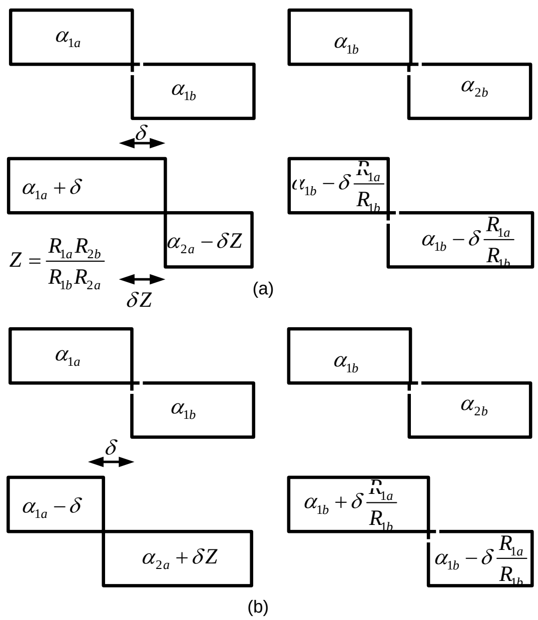

Fig. 2. Impact of a change by δ (a) a small increase of α_{1a}, (b) a small decrease of α_{1a}

Theorem 3.4: Any point on the boundary of the achievable rate region for an orthogonal allocation with no self interference can be obtained using joint FDM/TDM frequency allocations where at most $N-1$ frequencies are shared using TDM.

Proof: Any point on the boundary of the achievable rate region $(R_1, ..., R_N)$ (for orthogonal allocation and no self interference) can be obtained by assigning a proper weight vector $W = (1/R_1, ..., 1/R_N)$ and solving the corresponding weighted max-min optimization problem. Therefore, there are at most $N-1$ frequencies shared by more than one player. ■

Corollary 3.1: For all game theoretic solutions such as Nash Bargaining, Generalized Nash Bargaining, Kalai-Smorodinski and the proportionally fair solution, where the solution is on the boundary of the achievable rate region with joint FDM/TDM frequency allocations, then at most $N-1$ frequencies are shared using TDM.

This observation improves the result reported in [11].

A. Approximating the optimal FDM solution

The FDM problem is a hard combinatorial problem. We now describe a simple modification of the optimal joint FDM/TDM allocation and show that the proposed modification provides a simple approximation to the optimal FDM solution. Furthermore, this approximation is asymptotically optimal with number of tones. This is of practical interest, since implementing an FDM solution is simpler than implementing a joint FDM/TDM solution. The proof of theorem 3.2 is the key to approximating the optimal FDM solution. By the proof of the theorem, there is no connected group of frequencies. Define the following graph:

Definition 3.3: Let $G = (V, E)$, where, V is the set of users $\{$1,...,N$\}$ and $(i, j) \in E$ iff users i, share a frequency.

Since there is no connected group of frequencies, G is a union of trees (a forest). For any tree in the forest, designate one of the users as the root (by choosing the user with largest capacity on a shared link, the bound below can be slightly improved). We begin with the root and allocate to the root all the frequencies which it shares. Since there are no circles any user sharing a frequency with the root loses at most a single frequency. Now we inductively allocate to each node of depth $d+1$ all the frequencies it shares, which have not been allocated at previous steps. It is easy to see that each user loses at most a single frequency, which is the frequency allocated to its predecessor in the tree. This discussion shows that we can modify the optimal joint FDM/TDM solution into an FDM solution, where each user loses at most a single frequency it shares with others. If we assume i.i.d channel coefficients, the loss per user is bounded by the rate of its best frequency, which is the maximal order statistics of the capacity of K i.i.d channel. This implies that the maximal loss per user compared to the joint FDM/TDM max-min solution is bounded by the order statistics of the best channel out of NK channels. Assume that $N << K$ which is the case in OFDMA with grouping, and that the channels are i.i.d., Rayleigh fading channels. The capacity of the best channel among all channels of all users is bounded by [27]

$$C_{\max} \leq \log\left(1 + \log(NK)SNR\right). \tag{18}$$

Using the fact that each user receives at least the average rate of K/N random channels, the maximal relative loss is bounded by

$$\frac{\log\left(1 + \log(NK)SNR\right)}{\frac{K}{N}\log\left(1 + SNR\right)}. \tag{19}$$

In the low SNR regime this reduces to

$$\frac{N\left(\log(N) + \log(K)\right)}{K} \tag{20}$$

In the high SNR regime which is of interest to modern systems (employing high order modulations), this reduces to

$$\frac{N\left(1 + \log\log(N) + \log\log(K)\right)}{K} \tag{21}$$

which is $O\left(\frac{N\log\log(K)}{K}\right)$. Note that this is a pessimistic bound, and in practice the difference is even smaller. This simple analysis shows that the proposed solution also serves as a good approximation to the hard combinatorial problem of FDM design.

IV. The two user case

In this section, we address the special case of two users. In this case, the optimization problem can be dramatically simplified. Using $1-4$ in Theorem 2.1, the following rules apply:

1) $\frac{\delta_1}{\gamma_1} + \frac{\delta_2}{\gamma_2} = 1$ (item 2 in Theorem 2.1).
2) If $\delta_1 R_{1k} > \delta_2 R_{2k}$, frequency bin k is allocated to user 1.
3) If $\delta_1 R_{1k} < \delta_2 R_{2k}$, frequency bin k is allocated to user 2.

4) If $\delta_1 R_{1k} = \delta_2 R_{2k}$, frequency bin k is shared between the users such that they both get the same total rate, based on item 3 in Theorem 2.1.

Based on the above properties, we suggest a simple sorting algorithm (therefore the complexity is $O(K \log K)$) motivated by our analysis of the Nash Bargaining Solution (NBS) for a frequency selective interference channel [11], [28]. Extensions to the total power constraint are possible, similar to the solution to the NBS [10]. We show that at most a single frequency may be shared between the two users. To that end, let $\alpha_{1k} = \alpha_k$, and $\alpha_{2k} = 1 - \alpha_k$, and without loss of generality, we set $\gamma_1 = 1$, and $\gamma_2 = \gamma$. The ratio $\Gamma = \frac{\delta_2}{\delta_1} = \frac{1-\delta_1}{\delta_1 \gamma}$ is a threshold that is independent of the frequency and is set by the optimal assignment. Although Γ is a-priori unknown, it exists. We also assume that the rate ratios $L(k) = R_{1k}/R_{2k}, 1 \le k \le K$ are sorted in decreasing order; i.e., $L(k) \ge L(k'), \forall k \le k'$. [5] Using Theorem 2.2, we obtain

$$\sum_{k=1}^{K} \alpha_k R_{1k} = \gamma \sum_{k=1}^{K} (1 - \alpha_k) R_{2k}. \tag{22}$$

We are now ready to define the optimal assignment of the α_k's.

Let Γ_k be a moving threshold defined by

$$\Gamma_k = \frac{A_k}{B_k \gamma} \tag{23}$$

where

$$A_k = \sum_{m=1}^{k} R_{1m},\ B_k = \sum_{m=k+1}^{K} R_{2m}. \tag{24}$$

A_k is a monotonically increasing sequence, whereas B_k is monotonically decreasing. Hence, Γ_k is also monotonically increasing. A_k is the rate of user 1, when frequencies $1, ..., k$ are allocated to him. Similarly B_k is the rate of user 2 when frequencies $k+1, ..., K$ are allocated to him. Let

$$k_{\min} = \min_k \{k : A_k \ge B_k \gamma\}. \tag{25}$$

We are interested in a feasible solution such that the ratio of the accumulated rate of the users is equal to γ. Thus, frequency bin k_{min} has to be split between the users, and $\alpha_{k_{min}}$ is given by solving:

$$A_{k_{min}-1} + \alpha_{k_{min}} R_{1k_{min}} = \gamma \left(B_{k_{min}-1} - \alpha_{k_{min}} R_{2k_{min}}\right), \tag{26}$$

or

$$\alpha_{k_{min}} = \frac{\gamma B_{k_{min}-1} - A_{k_{min}-1}}{R_{1k_{min}} + \gamma R_{2k_{min}}}. \tag{27}$$

It easy to confirm that $0 \le \alpha_{k_{min}} \le 1$.

The outline of the algorithm is given in Table I.

V. Examples and Simulations

To illustrate the algorithm, we compute the weighted max-min solution in the following example.

Example I: Consider two users communicating over a 2x2 memoryless Gaussian channel with 6 frequency bins. The weights of user 1 and 2 are 1 and 1.25, respectively. The interference free user rates in each frequency bin (sorted according to L_k) are given in Table II. We now compute the values of A_k and B_k for each user. Since, $\Gamma_3 > 1$, we conclude that $k_{min} = 4$ and $\alpha_{k_{min}} = 0.8$. Thus, user 1 is using subcarriers $1, 2, 3$, and sharing subcarrier 4 with user 2. The total rate of users 1 and 2 are 45 and 36, respectively. We can also provide a geometric interpretation to the solution. In Figure 4, we depict the feasible total rate that user 1 can obtain as a function of the total rate of user 2. The enclosed area in blue is the achievable rates set. Since the subcarriers are sorted according to L_k, the set is convex. Point $(45, 36)$ is the operating point of the weight max-min with $\gamma = 1.25$. A change in the value of γ will move the solution on the boundaries of achievable rate set.

TABLE I
Algorithm for computing the 2x2 weighted max-min

Initialization: Sort the ratios $L(k)$ in decreasing order. Calculate the values of A_k, B_k and Γ_k.
Calculate k_{min} using (25). Calculate $\alpha_{k_{min}}$ using (26). User 1 gets bins $1 : k_{min-1}$ and $\alpha_{k_{min}}$ of bin k_{min}. User 2 gets bins $k_{min+1} : K$ and $1 - \alpha_{k_{min}}$ of bin k_{min}.

TABLE II
User rates in each frequency bin after sorting, and the values of Γ_k.

k	1	2	3	4	5	6
R_1	14	18	5	10	9	3
R_2	6	10	5	15	17	16
$L(k)$	2.33	1.80	1.00	0.67	0.53	0.19
A_k	14	32	37	47	56	59
B_k	63	53	48	33	16	0
Γ_k	.178	.483	.617	1.14	2.80	∞

Next, we demonstrate simulation results for rate allocation of various weight values in two cases. In both cases, the users communicate over a frequency selective Rayleigh fading channel with variance 1. The number of frequency bins is 64. The first case simulates two groups of users, where each group is of size 8. This is a typical scenario where one group has higher priority. The weight for one data group is γ, and for the second data group it is $1 - \gamma$, where $0 \le \gamma \le 1$. For each value of γ, we performed 10000 tests. The SNR values of the two data groups are 20dB and 10 dB, respectively. Figure 5 presents the distribution of the feasible rates for various values of γ. It is clear that for a given value of γ the feasible rate will be along a ray with an angle $\phi = \arctan \frac{\gamma}{1-\gamma}$ relative to the x axis. Figure 6 presents a histogram of the ray with $\gamma = 0.1$. Figure 7 presents the average value of the feasible rate for group 1 vs. the average rate of group 2. Figure 8 shows the outage regions for an outage probability of 0.1 and 0.05. It is clear that by reducing the outage probability from 0.1 to 0.05, there is a significant impact on the achievable rate of group 2 from 1.6 to 1.

Example II: The second case simulates three groups of users: a voice group of size 4 and two data groups each of size 8. The SNR value of the voice group is 5dB and the SNR of the two data groups is 20dB. Figure 9 shows the outage regions for an outage probability of 0.05, 0.1 and 0.5.

[5] This can be achieved by sorting the frequencies according to $L(k)$.

VI. Conclusion and extensions

In this paper, we described a simple rate allocation technique, called weighted max-min, for multiple-access OFDMA systems by applying joint FDM/TDM subchannel allocation. The method is applicable whenever a central access point or base station is available. The allocation can be executed using a standard linear programming algorithm with polynomial complexity. However, in the special case of two users we induce a sorting algorithm with a complexity of $O(K \log K)$. Note that the allocation can be made using channel statistics instead of the actual channel realization, whenever the channel condition changes rapidly. A new and important result is that any point on the boundary of the achievable rate region for an orthogonal allocation with no self interference can be obtained using joint FDM/TDM frequency allocations where at most $N-1$ frequencies are shared using TDM.

Appendix A
Proof of Theorem 3.2

Assume that there is a minimal connected group of order t (see Figure 3) in the unique optimal solution of the weighted max-min problem. Then, any small change of $\alpha_{n_1}(k_{n_1,n_2})$ by $\delta > 0$ must cause a series of changes in the values of $\alpha_{n_i}(k_{n_i,n_{i+1}}), \alpha_{n_i}(k_{n_{i-1},n_i}), i = 2,..,t-1$ and $\alpha_{n_t}(k_{n_t,n_1}), \alpha_{n_1}(k_{n_t,n_1})$. Since $0 < \alpha_{n_i}(k_{n_i,n_{i+1}}) < 1, \forall i$, a sufficiently small increase (decrease) of $\alpha_{n_i}(k_{n_{i-1},n_i})$ can be compensated for by a small decrease (increase) in the value of $\alpha_{n_i}(k_{n_i,n_{i+1}})$ while maintaining the total rate of user n_i. Since there is a unique solution to the max-min problem, any increase in the value of $\alpha_{n_1}(k_{n_1,n_2})$ by any $\delta > 0$ must result in an infeasible solution. This can only occur if the value of $\alpha_{n_1}(k_{n_t,n_1})$ has to decrease more than $\delta \frac{R_{n_1}(k_{n_1,n_2})}{R_{n_1}(k_{n_t,n_1})}$; otherwise, the rate of at least one user will be larger than c. Thus, we get the condition.

$$\delta \frac{R_{n_t}(k_{n_{t-1},n_t})}{R_{n_t}(k_{n_t,n_1})} \prod_{i=2}^{t-1} \frac{R_{n_i}(k_{n_{i-1},n_i})}{R_{n_i}(k_{n_i,n_{i+1}})} > \delta \frac{R_{n_1}(k_{n_1,n_2})}{R_{n_1}(k_{n_t,n_1})} \quad (28)$$

or in another form,

$$\frac{R_{n_1}(k_{n_t,n_1})}{R_{n_t}(k_{n_t,n_1})} \prod_{i=1}^{t-1} \frac{R_{n_{i+1}}(k_{n_i,n_{i+1}})}{R_{n_i}(k_{n_i,n_{i+1}})} > 1. \quad (29)$$

In a similar way, any decrease in the value of $\alpha_{n_1}(k_{n_1,n_2})$ by $\delta > 0$ must lead to an infeasible solution. This can only occur if the value of $\alpha_{n_1}(k_{n_t,n_1})$ increases less than $\delta \frac{R_{n_1}(k_{n_1,n_2})}{R_{n_1}(k_{n_t,n_1})}$; otherwise, the rate of at least one user will be larger than c. Thus, we get the condition.

$$\frac{R_{n_1}(k_{n_t,n_1})}{R_{n_t}(k_{n_t,n_1})} \prod_{i=1}^{t-1} \frac{R_{n_{i+1}}(k_{n_i,n_{i+1}})}{R_{n_i}(k_{n_i,n_{i+1}})} < 1. \quad (30)$$

Since, inequalities (29) and (30) are contradictory, we conclude that there is no minimal connected group for any value of t.

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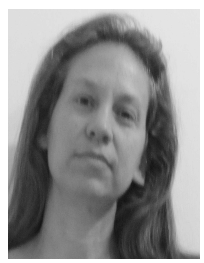

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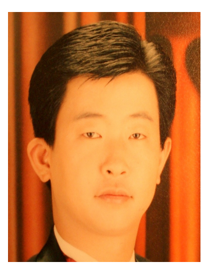

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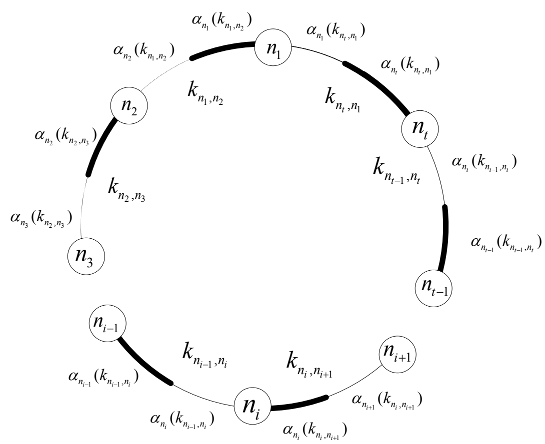

Fig. 3. A connected group of t users $n_1, ..., n_t$

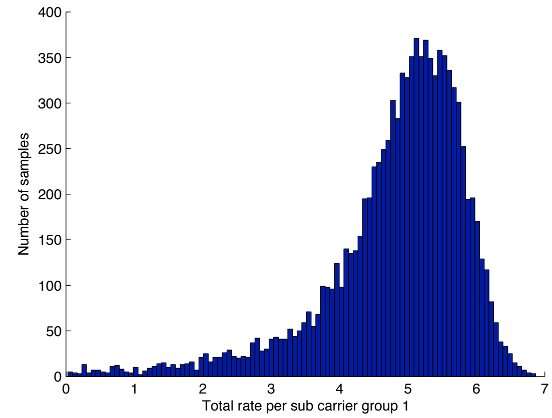

Fig. 6. A histogram of group 1 rates for $\gamma = 0.1$ and $[SNR_1, SNR_2] = [20dB, 10dB]$.

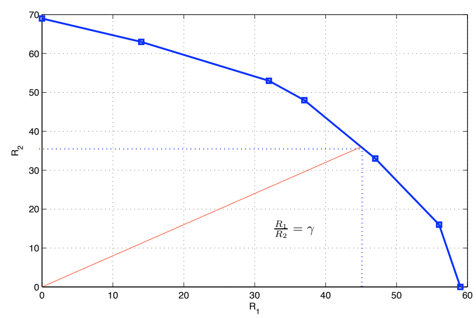

Fig. 4. The feasible total rate of user 1 vs. the feasible total rate of user 2.

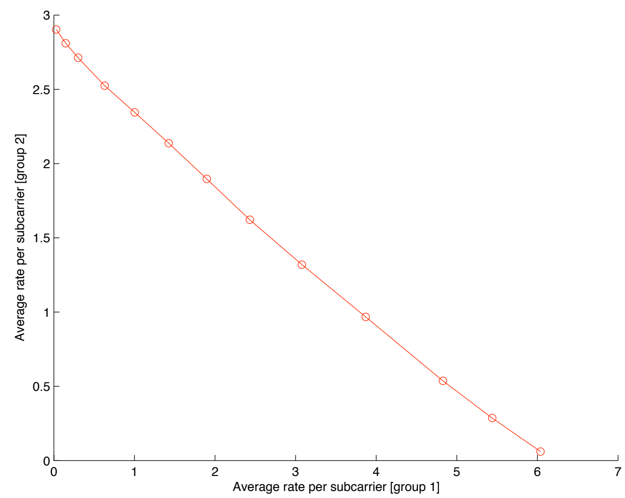

Fig. 7. The average rate of group 2 vs. the average rate of group 1 for $[SNR_1, SNR_2] = [20dB, 10dB]$.

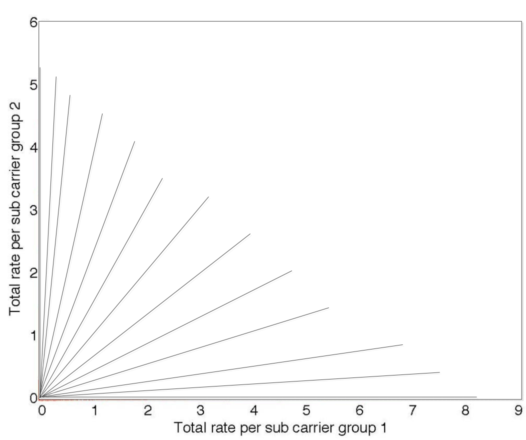

Fig. 5. The distribution of feasible rates for each value of γ. $[SNR_1, SNR_2] = [20dB, 10dB]$.

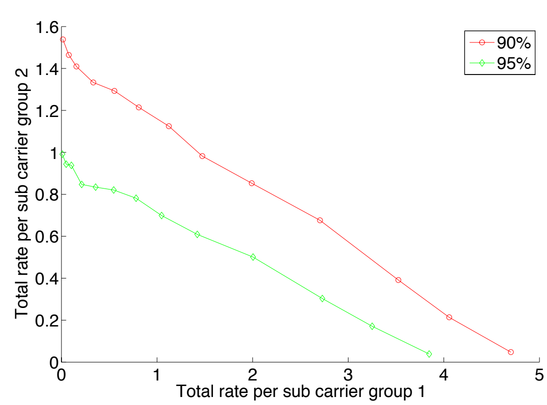

Fig. 8. The rate of group 2 vs. the rate of group 1 for outage probabilities of 10% and 5%. $[SNR_1, SNR_2] = [20dB, 10dB]$.

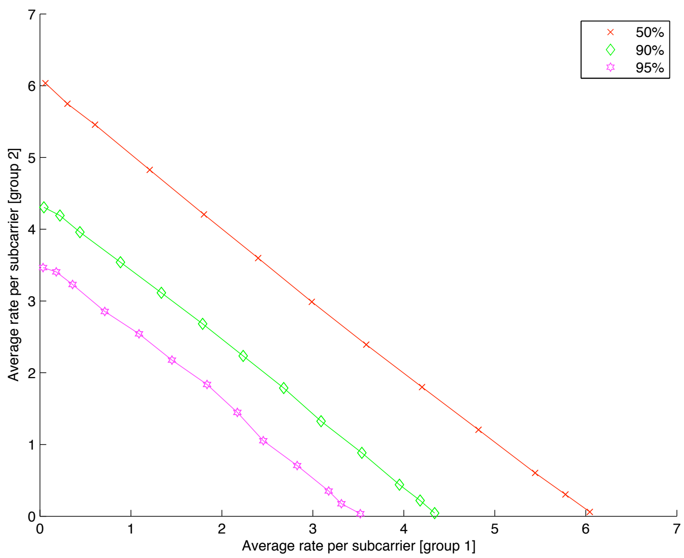

Fig. 9. The rate of data group 2 vs. the rate of data group 1 for $SNR = 20$ (voice group $SNR = 5$), and outage probabilities 0.05,0.1 and 0.45.