Coherent Combining and Phase
Locking of Fiber Lasers

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Abstract

Our investigations and developments on passive phase locking and coherent combining of fiber lasers are presented. These include the effects of amplitude, noise and time delays, the relation between phase locking and chaos synchronization, stable and practical configurations that can potentially generate high output powers together with good output beam quality, and the effects of up scaling to a large number of fiber lasers.

1. Introduction

Fiber lasers are commonly comprised of doped double clad fibers as the gain medium, and high reflecting and low reflecting fiber Bragg gratings as the mirrors. The fiber lasers are usually pumped from the rear using multimode diode lasers but can be also pumped from the side with special combiners. Typically the bandwidth of fiber lasers is determined by the bandwidth of the Bragg grating which ranges from 10nm to as narrow as 0.1nm. The electricity to light conversion efficiency of fiber lasers can exceed 50% and since light is confined inside the fiber, the lasers are highly robust [1]. Yet, the light is confined in the small core of the fiber, so the output power from a single fiber laser is limited due to non-linear effects and the danger of damage to the fiber [2].

In order to overcome the limitation of output power from single fiber lasers, several low power fiber lasers could be combined. Such a combination could be performed either incoherently or coherently. When the field distributions of several laser output beams are incoherently combined, the resulting beam-quality factor ($M^2$) is relatively poor with low optical brightness. Nevertheless, the incoherent approach is pursued quite actively because the number of lasers that can be combined efficiently is relatively large. When the field distributions are coherently added, with the proper phase relations, the combined beam quality factor can be as good as that of a single low-power laser, while the combined power is greater by a factor equal to the number of the combined lasers.
When coherently combining two or more fiber laser output fields three major difficulties are encountered [3, 4]. The first results from the need to properly couple the individual laser fields, so as to establish mutual coherence and enable relative phase locking between them. Such coupling typically introduces excessive losses to each laser field, and requires accurate relative alignment. The second (and somewhat related) difficulty results from the need to accurately control the relative phase and amplitudes between the different fiber laser fields, so as to ensure constructive interference between them in the far field. This requires that the distances between the participating optical components be very accurately controlled, causing the output power to be extremely sensitive to thermal drifts and acoustic vibrations. The third difficulty results from the need to efficiently combine many separate fiber lasers output beams into one single beam.

During the past decade, we extensively investigated new approaches for passive phase locking and combining of several lasers. These involved the development of unique intra-cavity elements and laser configurations in order to obtain efficient phase locking and combining of solid state lasers as well as fiber lasers [5-26]. In general, our results indicate that robust and practical laser systems that could potentially have high output powers concomitantly with very good output beam quality can be developed. Here we present some of our recent developments on fiber lasers. Specifically, we present our configurations and results on coherent combining of two and then four fiber lasers, our investigations and results on the effects of noise, longitudinal modes and time delays, possibilities for up scaling the number of lasers that can be combined, and finally our configurations and results where up to 25 fiber lasers were phase locked which were also exploited for studying extreme value statistics.

2. Passive phase locking and coherent combining of small arrays

In this section we describe the configurations and present the results of our investigations on passive phase locking and coherent combining with a small number of fiber lasers [5-13]. We start with phase locking and coherent combining of two fiber lasers, and then of four fiber lasers arranged in a 2-dimensional array.

2.1 Efficient coherent combining of two fiber lasers

We investigated phase locking and coherent combining of two fiber lasers using two free-space Vernier-Michelson configurations of end-pumped fiber lasers, schematically shown in Fig. 1. The first, shown in Fig. 1(a), is an intra-cavity configuration [5]. One end of each fiber is attached to a high reflection fiber Bragg grating (FBG) and the other fiber end is cleaved at an angle of 8° to suppress any reflections back into the fiber cores, so that each fiber is essentially an amplifier. The light emerging from both fiber lasers is coherently added in free space by means of a 50% beam splitter and a common output coupler with 4% reflection. This configuration is analogous to the inner-fiber configurations except that the combined beam now propagates only in free space.

The second configuration, shown in Fig. 1(b), is an outer-cavity configuration. One end of each fiber is again attached to a FBG, but the other fiber end is now cleaved
at 0° which reflect 4% of the light back into the fiber core, so that each fiber behaves as an independent fiber laser resonator. The light emerging from each fiber laser is coherently added in free space by means of the same beam splitter and output coupler as in the first configuration.

![Diagram of basic configurations for coherent combining of two fiber lasers.](image)

**Figure 1:** Basic configurations for coherent combining of two fiber lasers. (a) intra-cavity addition. (b) outer-cavity addition. FBG - High reflection Fiber Bragg Grating. BS - 50% Beam Splitter.

In both configurations phase locking of the lasers results in constructive interference toward the output channel direction and destructive interference toward the loss channel direction. However, to decide which configuration is superior we can either measure the coherent combining efficiency or the phase locking efficiency.

The efficiency of the combined output power as a function of the coupling strength $\kappa$ in both configurations is presented in Fig. 2. It is normalized such that 1 denotes 100% efficiency and corresponds to twice the output power of a single fiber laser, measured without the beam splitter. As evident from Fig. 2, even at coupling as low as 1%, an efficiency of 90% is obtained for both configurations. Thus, only a small amount of light needs to be reflected back into the fibers, so the risk of optical damage is low. At low coupling, the outer-cavity configuration is much more efficient than the intra-cavity one. For example, in the intra-cavity configuration the combined output power reduces to 75% for coupling of 0.6%, while in the outer-cavity configuration the coupling can be low as 0.2% for the same output power.

Such a large difference is due to the fact that the coupling effects are different for the two configurations. In the outer-cavity configuration, reducing coupling keeps the power of each laser nearly unchanged, and only decreases the efficiency of coherent combining of the two lasers. However, in the intra-cavity configuration, reducing the coupling decreases the power of each fiber laser, long before the reduction of the efficiency of coherent combining occurs.

![Graph showing efficiency as a function of coupling strength for intra-cavity and outer-cavity configurations.](image)

**Figure 2:** Efficiency as a function of coupling strength for intra-cavity and outer-cavity configurations [10].
On the other hand, for the intra-cavity configuration, phase locking occurs for much lower values of coupling than for the outer-cavity configuration. To determine the level of phase locking, we measured the fringe visibility of the interference pattern of the light emerging from the two lasers (see the insets of Fig. 3), according to

\[ v_f = \frac{(I_{\text{max}} - I_{\text{min}})}{(I_{\text{max}} + I_{\text{min}})} \]

where \( I_{\text{max}} \) and \( I_{\text{min}} \) are the maximum and minimum intensities along a cross section of the fringes. The results of fringe visibility as a function of coupling strength for the intra-cavity and outer-cavity configurations are presented in Fig. 3. As evident, a high fringe visibility level occurs at a significantly lower coupling strength, for the intra-cavity configuration. Specifically, the transition from zero phase locking to almost complete phase locking for the intra-cavity configuration occurs over a range of 0.001 coupling strengths, whereas for the outer-cavity configuration the range is 0.01 [5].

In order to obtain realistic calculated fringe visibility results that can be directly compared with our experimental results, we added a noise term to the laser equations. The results are presented in Fig. 3, where the solid curves denote the numerically obtained results and the dashed curves denote corresponding analytically calculated results [11].

![Figure 3](image_url)

Figure 3: Experimental and calculated fringe visibility as a function of coupling strength. Stars denote experimental results for the intra-cavity configuration. Dots denote experimental results for the outer-cavity configuration. Solid curves denote corresponding numerically calculated results, and dashed curves denote corresponding analytically calculated results [11].

In order to obtain realistic calculated fringe visibility results that can be directly compared with our experimental results, we added a noise term to the laser equations. The results are presented in Fig. 3, where the solid curves denote the numerically obtained results and the dashed curves analytically obtained results [6].

### 2.2 Compact coherent combining of four fiber lasers

In order to obtain practical combining systems it is best to resort to more compact configurations. In these configurations coherent combining is performed by means of interferometric combining elements that are compact, leading to overall configuration that can be readily up scaled [7-12]. The configuration for free-space coherent combining of four fiber lasers is presented schematically in Fig. 4 [13]. It includes four fiber lasers and two intra-cavity interferometric combiners. Each fiber laser consists of polarization maintaining Erbium doped fiber of about 7 meters in length, where one end is attached to a high reflection fiber Bragg grating (FBG) of 5nm spectral bandwidth that serves as a back reflector mirror and the other end is spliced to a collimating graded index (GRIN) lens with anti-reflection layer to suppress any reflections back into the fiber cores, and a flat output coupler of 20% reflection that is common to all fiber lasers. Each fiber laser is pumped with a diode laser of 911 nm wavelength and 5 W maximum output power, which is spliced at the back of the FBG. Each interferometric combiner is a planar substrate, where half of the front surface is coated with an antireflection layer and the other half with a 50%
beam-splitter layer, while half of the rear surface is coated with a highly reflecting layer and the other half with an antireflection layer. When there is phase locking among the fiber lasers, then the first interferometric combiner transforms efficiently four light beams to two beams and the second interferometric combiner transforms the two beams into one nearly Gaussian beam.

![Image](image_url)

**Figure 4:** Basic intra-cavity configuration for coherent combining of four fiber lasers in free space using two orthogonally oriented interferometric combiners [13].

We placed the first interferometric combiner in order to coherently add horizontally the light from the four fiber lasers each with a power of 35mW to form two beams each with a power of 66mW. This corresponds to a combining efficiency of over 90%. Then, we added the second interferometric combiner to coherently add vertically the two remaining beams into one beam with a power of 121mW. This corresponds to an overall combining efficiency (i.e. the ratio between the overall power with the interferometric combiners to that without the combiners) of 86%. We also placed CCD cameras to detect the intensity distributions without the interferometric combiners, after the first interferometric combiner and after the second one. The results are presented in Fig. 5. As evident, the initial four beams first coherently transform to two beams after the first interferometric combiner and then to one beam after the second interferometric combiner.

![Image](image_url)

**Figure 5:** Experimental intensity distributions for the intra-cavity configuration. (a) Before the interferometric combiners; (b) after the first interferometric combiner; (c) after the second interferometric combiner [13].

### 2.3 Efficient coherent combining of four fiber lasers operating at 2 µm

We also performed experiments for passive coherent combining of four fiber lasers operating at 2 µm. The experimental configuration was similar to that shown in Fig. 4. It consisted of four single mode Thulium doped fibers, two interferometric
coupling assemblies and a common output coupler [13]. The rear end of each fiber was attached to a high reflection fiber Bragg grating (FBG) and the other fiber end was attached to a collimator with anti-reflection coating to suppress any reflections back into the fiber cores so that each fiber was essentially an amplifier. Each fiber was end-pumped by a diode laser operating at 790 nm wavelength through the FBG. In order to align the four fiber lasers to be exactly parallel, each was aligned individually such that it lases with a single parallel output coupler located in front of the array. This ensures that all the fiber lasers are perpendicular to the same plane hence exactly parallel to each other. The beams emerging from the fiber lasers were coherently combined in free space by means of two interferometric combiner assemblies (horizontal and vertical). Each interferometric combiner assembly was composed of a 50% beam splitter and a high reflecting mirror properly displaced from it. We first measured the parallelism of the lasers and confirmed that the angle between them was less than 0.5 mrad.

The results are presented in Fig. 6. Figure 6(a) shows the near field light intensity distribution, detected with a 2 µm CCD camera. As evident, the four individual beams have indeed near Gaussian distributions, are equally spaced and have nearly the same size, i.e. all the properties needed for efficient coherent combining. Next, we placed the first interferometric combiner assembly in order to combine coherently four lasers into two. Then, we added the second interferometric combiner to coherently add the two remaining beams into one. The combined output beam is presented in Fig. 6(b) and indicates that the near Gaussian distribution of the individual lasers is indeed preserved after combining. We measured an overall coherent combining efficiency of about 92%, i.e. the light efficiency for coherently combining four different beams into one. Finally, we also confirmed coherence between any two individual laser beams by directing them at an angle onto the CCD camera and detecting the resulting interference fringes between them. The results are presented in Fig. 6(c). The high contrast interference fringes correspond to very strong and stable phase locking between the beams, leading to the high combining efficiency that was obtained.

Figure 6: Experimental intensity distributions when coherently combining four fiber lasers operating at 2 µm. (a) Intensity distributions at the outputs of the four individual fiber lasers; (b) intensity distribution of the combined output beam; (c) high contrast fringes obtained by interfering two of the four coupled fiber lasers [13].

3. Effects of amplitude dynamics, noise, longitudinal modes and time delayed coupling
In this section we describe our investigations and results about the effects of amplitude dynamics, noise, longitudinal modes and time delayed coupling on phase locking and coherent combining with two fiber lasers [14-20].

3.1 Effects of amplitude dynamics

The phase locking and coherence properties between two weakly coupled lasers are presented [14]. We show how the degree of coherence between the two lasers can be enhanced by nearly an order of magnitude after taking into account the effects of coupling on both their phases as well as their amplitudes. Specifically, correlations between synchronized spikes in the amplitude dynamics and the phase dynamics of the lasers allow for an interference pattern with a fringe visibility of 90%, even when the coupling strength is far below the critical value and they are not phase locked.

Many different schemes for coupling lasers have been extensively investigated over the past decades [1-25]. In all, phase locking can only occur when the coupling strength $\kappa$ exceeds some critical value $\kappa_c$. In general, theoretical models that deal with the effects of coupling on the coherence and phase locking between weakly coupled lasers, i.e. $\kappa < \kappa_c$ only take into account the effects of the phase difference between the lasers, while neglecting the effects that the lasers amplitudes may have on the coherence and phase locking properties of the lasers.

We investigate the dynamics of two weakly coupled lasers, where $\kappa < \kappa_c$. We find that by including the effects of lasers amplitudes in addition to the phase difference, the coherence between the two lasers can be enhanced by up to an order of magnitude. Such enhanced coherence is a direct result of correlation between the amplitude and phase dynamics.

In order to determine the dynamics of two coupled lasers, we begin with the rate equations that are used for describing a broad range of coupled lasers [16, 21, 25], as

$$\frac{dE_{1,2}}{dt} = \frac{1}{\tau_c} \left[ (G_{1,2} - \alpha_{1,2})E_{1,2} + \kappa E_{2,1} \right] + i \omega_{1,2} E_{2,1},$$

$$\frac{dG_{1,2}}{dt} = \frac{1}{\tau_f} \left[ P_{1,2} - (I_{1,2} - 1)G_{1,2} \right],$$

where $E_{1,2}$ are the complex electric fields of laser 1 and laser 2, $\tau_c$ the photon cavity round trip time, $\tau_f$ the fluorescence life time, $\omega_{1,2}$ the frequency detuning from a mean optical frequency for each laser, $\kappa$ the coupling strength, and for each laser $\alpha_{1,2}$ is the round trip loss, $G_{1,2}$ the round trip gain, $P_{1,2}$ the pump strength, and $I_{1,2}$ the intensity in units of the saturation intensity. Now, with $E_{1,2} = A_{1,2} \exp(i \varphi_{1,2})$ and separating the equations into real and imaginary parts, it is possible to obtain relations for the amplitudes and phases of the two coupled lasers. The real part yields relations for amplitudes $A_1$ and $A_2$ as

$$\frac{dA_{1,2}}{dt} = \frac{1}{\tau_c} \left[ (G_{1,2} - \alpha_{1,2})A_{1,2} + \kappa A_{2,1} \cos \varphi \right].$$

The imaginary part yields the relation for the phase difference $\varphi$, as

$$\frac{d\varphi}{dt} = \Omega - \frac{\kappa}{\tau_c} \beta \sin \varphi,$$

$$\beta = \frac{A_2}{A_1} - \frac{A_1}{A_2}.$$
where \( \varphi = \varphi_2 - \varphi_1 \) is the phase difference between the two lasers and \( \Omega = \omega_2 - \omega_1 \) is the frequency detuning between the two lasers. With both lasers having the same pump strength and the same round trip losses, phase locking can only occur for \( \kappa > \kappa_c = \Omega \tau_c / 2 \) [25].

The degree of phase locking between two lasers is usually quantified by determining the fringe visibility of the intensity interference pattern formed by two interfering lasers [16, 22, 25]. This fringe visibility \( v_{fv} \) is

\[
v_{fv} = \frac{i_{\text{max}} - i_{\text{min}}}{i_{\text{max}} + i_{\text{min}}}
\]

where \( i_{\text{max}} \) and \( i_{\text{min}} \) are the maximal and minimal values of the time averaged intensities in the interference pattern. Writing \( i_{\text{max}} \) and \( i_{\text{min}} \) in terms of the lasers amplitudes and the relative phase difference between the lasers then the relation for the fringe visibility \( v_{fv} \) is

\[
v_{fv} = \frac{2\sqrt{(A_1A_2 \cos \varphi)^2 + (A_1A_2 \sin \varphi)^2}}{(A_1^2 + A_2^2)}.
\]

According to this equation, even when the amplitudes of the lasers are identical, the fringe visibility still depends on the amplitude dynamics as well as the phase difference. Consequently, the degree of phase locking cannot be always quantified by the fringe visibility. Accordingly, more suitable way to quantify phase locking is by resorting to an alternative phase locking parameter, defined by \( v_{pl} \), as

\[
v_{pl} = \sqrt{(\cos \varphi)^2 + (\sin \varphi)^2}.
\]

Note that \( v_{pl} = v_{fv} \) only when the amplitudes of both lasers are identical and their dynamics are uncorrelated with those of \( \varphi \), i.e. \( \langle A_1 A_2 \cos \varphi \rangle = \langle A_1^2 \rangle \langle \cos \varphi \rangle \).

We numerically solved the coupled lasers rate equations using the typical time scales for Yb-doped fiber lasers [16, 22] of \( \tau_c = 30 \) ns and \( \tau_f = 230 \) \( \mu \)s, frequency detuning \( \Omega = 200 \) kHz. Figure 7 shows the fringe visibility \( v_{fv} \) and the phase locking parameter \( v_{pl} \) as a function of the coupling strength \( \kappa \) normalized by the critical coupling \( \kappa_c \). As shown, the phase locking parameter monotonically increases with coupling strength until reaching a value of 1 at \( \kappa = \kappa_c \). However, the fringe visibility is characterized by sharp increases and sharp drops as coupling strength increases. The first sharp increase of the fringe visibility, \( v_{fv} \approx 0.1 \) at point A to \( v_{fv} \approx 0.86 \) at point B. The inset shows a simulated intensity interference pattern with a fringe visibility corresponding to that of point B.

To clarify and explain the unusual behavior of the fringe visibility shown in Fig. 7, we calculated the lasers amplitudes and phase difference at points A-F as a function of time. The results are presented in Fig. 8. Since the amplitudes dynamics are identical, we only present the amplitudes of one of the lasers. Figure 8 (A) shows the results corresponding to point A. As evident, the lasers amplitudes have small fluctuations around a mean value and \( \varphi \) monotonically increases with time. These indicate that \( v_{pl} = v_{fv} \) and poor fringe visibility because \( \varphi \) continually varies. Figure 8 (B) shows the results corresponding to point B. Here \( \varphi \) is essentially the same but the lasers amplitudes are dramatically different, characterized by short and intense pulses with a repetition rate equal to \( \Delta \omega \). The phase difference accumulated between adjacent pulses is \( 2\pi \), so effectively the coupled lasers experience a relatively constant phase difference between them, as if they are phase locked at all times. Since the lasers operate in short intense pulses, the average fringe visibility is
essentially the same as the instantaneous fringe visibility lasting the duration of one pulse, so it is relatively high at point B.

Figure 7: Fringe visibility \( v_{fu} \) and phase locking parameter \( v_{pl} \) as a function of the coupling strength \( \kappa \) normalized by the critical coupling strength \( \kappa_c \). Inset shows, the intensity pattern of two interfering Gaussian beams with fringe visibility corresponding to that of point B [14].

Figure 8: Lasers amplitudes and phase difference as a function of time, for different values of \( \kappa/\kappa_c \) corresponding to points A, B, C, D, E and F of Fig.7. Solid blue denotes amplitudes and dashed green denotes the phase difference [14].

As the coupling strength increases, the pulses become narrower and more intense, as shown in Fig. 8(C), \( v_{fu} \) increases to 0.99. A slight further increase of \( \kappa \) leads to a pulse break-up of the intensity where two wider pulses occur in each 2\( \pi \) phase cycle,
as shown in Fig. 8(D). This break-up leads to a sharp drop of $v_f$ at point D. As coupling is increased further, the process of pulse width narrowing that leads to increasing visibility and then to pulse break-up is repeated again and again. Specifically, Fig. 8(E) shows a break-up into three pulses for each $2\pi$ phase cycle, and Fig. 8(F) shows a break-up into seven pulses for each $4\pi$ phase cycle. As coupling approaches its critical value, the pulse break-up becomes more sensitive to variations, eventually leading to chaos [26].

To conclude, we showed that the coherence between two weakly coupled non-phase locked lasers can be substantially enhanced as a result of amplitude and phase correlated dynamics. Such enhancement was obtained for a wide range of laser parameters and also appears quite robust to parameter mismatch and noise. We believe that amplitude enhanced coherence may have potential applications in the field of coherent combining of lasers, where our numerical results showed that a combining efficiency of 90% can be achieved for coupling strengths as low as 20% of the critical coupling strength.

3.2 Effects of quantum noise

Phase locking between two simple linear and noiseless oscillators, is entirely dependent on the interplay of frequency detuning and coupling strength between them. However, in coupled laser oscillators there are additional factors that affect phase locking such as multi-mode operation, and noise [15, 16].

Typically, the quantum noise is much weaker than other sources of noise, but when the lasers operate very close to the threshold, the quantum noise can become dominant due to the inherent high spontaneous emission [15]. In order to characterize the quantum noise, we first determined that the threshold of each laser occurs at a pumping current of 1.016A for an output power of 69.28\(\mu\)W. This was determined under controlled environment and stable conditions, in order to reduce acoustic noise and Brillouin scattering, so the quantum noise was the most dominant. Then, we measured the output power spectra close to the threshold for different pump powers, and verified that the spectrum has a Lorentzian shape as would be expected for quantum noise. The results are presented in Fig. 9. It shows the FWHM bandwidth of the power spectra as a function of the output power, near threshold, for each fiber laser, along with representative results of a typical power spectrum. Fitting the power spectrum to a Lorentzian shape gave a measure of 0.98, while fitting to a Gaussian shape gave a measure of only 0.70. As evident from Fig. 9, the bandwidth increases as the laser output power decreases, indicating a typical behavior of the spectrum of quantum noise [15].
Figure 9: Bandwidth of the power spectra as a function of the output power for a single laser. Inset shows a representative results of one power spectrum for laser output power of 96.3 µW [15].

When operating the lasers very close to threshold, there is a significant fluctuation of the output powers. Accordingly, we simultaneously measured the output power from each laser while detecting the fringe visibility. Thus, we obtained phase locking as a function of the lasers output powers for different coupling strengths. The results of the measurements near threshold for two different coupling strengths are presented in Fig. 10. Figure 10(a) shows the phase locking as a function of the laser output powers for a coupling strength of 4.8%, whereas Fig. 10(b) shows it for a coupling strength of 1.8%. As is evident, there is not only the well known dependence of the phase locking on the coupling strength, but also a strong dependence on the laser output powers. It should be emphasized that this dependence occurs only when operating the lasers very close to the threshold, supporting the hypothesis that it is a result of a quantum phenomenon.

Figure 10: Fringe visibility of interference between two fiber lasers as a function of laser output power and quantum noise. (a) Coupling strength of 4.8%. (b) Coupling strength of 1.8%. Dots denote experimental results and solid curves denote analytic results [15].

The laser output powers can be related to the quantum noise by resorting to the Schawlow-Townes equation, which relates the lasers output powers to the bandwidth of each of their longitudinal modes. For our lasers, which are three level lasers, the bandwidth of each longitudinal mode is

$$\Delta f = \frac{2\pi \hbar \omega \epsilon^2 n}{P_{out}},$$

where $\hbar$ is the reduced Planck constant, $\omega = 1780$ THz is the light frequency, $\epsilon \omega = 50$ MHz is the cold cavity bandwidth, $n = 17000$ is the number of longitudinal modes, and $P_{out}$ is the power level at which the lasers oscillates. At around threshold, the bandwidths of the longitudinal modes correspond to the noise from the spontaneous emission. Thus, it is now possible to relate the fringe visibility (phase locking) directly to the quantum noise, as presented in Fig. 10.

To determine the phase locking as a function of quantum noise, we introduced a Langevin noise term into the coupled lasers rate equation to yield

$$\frac{d\phi}{dt} = \Omega + \frac{\kappa}{\tau_\epsilon} \left[ A_{2,1} + A_{1,2} \right] \sin \left( \phi - \frac{\pi}{2} \right) + \sqrt{\epsilon} \eta(t),$$

where $\sqrt{\epsilon} \eta(t)$ is a white noise source corresponding to the spontaneous emission, with noise amplitude $\epsilon = \Delta f$, and $\langle \eta(t_1) \eta(t_2) \rangle = \delta(t_2 - t_1)$ with $\delta$ the delta function.
Next, we developed an analytic expression for the fringe visibility as a function of the quantum noise. We introduced an effective potential \( U \) to yield

\[
\frac{d\varphi}{dt} = -\frac{dU(\varphi)}{d\varphi} + \sqrt{\epsilon} \eta(t),
\]

Since this equation describes a viscous motion, the phase \( \varphi \) has no inertia, and since the quantum noise \( \eta(t) \) has a \( \delta \) time correlation function, the motion of \( \varphi \) is only governed by \( U \) and by the instantaneous value of \( \eta(t) \). If the instantaneous value of \( \eta(t) \) is smaller than a certain critical noise, the system will be phase-locked. On the other hand, if it is larger, the system will not be phase-locked. Thus, the noise term \( \sqrt{\epsilon} \eta(t) \) can be added to \( U \) to yield a stochastic time-dependent potential \( U(t) \) as

\[
U(\varphi, t) = -\Omega(t)\varphi - \frac{2\kappa}{\tau_c} \cos \left( \varphi + \frac{\pi}{2} \right),
\]

where \( \Omega(t) = \Omega_0 + \sqrt{\epsilon} \eta(t) \) is the time dependent stochastic detuning. Accordingly

\[
\frac{d\varphi}{dt} = -\frac{dU(\varphi, t)}{d\varphi}.
\]

This indicates that phase locking is determined by the instantaneous potential \( U(\varphi, t) \). Since the instantaneous detuning is a rapidly varying stochastic variable, the time average fringe visibility (measured in our experiments by a slow CCD camera) is equal to the probability for phase locking to occur. Specifically, when \( \Omega(t) \) is smaller than the critical detuning of \( \Omega_c = 2\kappa/\tau_c \), phase locking occurs. The fringe visibility, namely, contrast \( C \), can be calculated by integrating over the probability from negative critical detuning \( -\Omega_c \) to positive critical detuning \( +\Omega_c \) as

\[
C = \int_{-\Omega_c}^{\Omega_c} P(\Omega) d\Omega,
\]

where \( P(\Omega) \) is the probability distribution of the detuning. Assuming a Lorentzian distribution for \( P(\Omega) \) with bandwidth \( \Delta f \), leads to a fringe visibility \( C \) as a function of the lasers output powers \( P_{\text{out}} \), as

\[
C(P_{\text{out}}) = \frac{\Delta f(P_{\text{out}})}{2\pi} \int_{-\frac{2\kappa}{\tau_c}}^{\frac{2\kappa}{\tau_c}} \frac{d\Omega'}{\left( \Omega' + \frac{\Delta f(P_{\text{out}})}{2} \right)^2} = \frac{2}{\pi} \arctan \frac{2\kappa P_{\text{out}}}{\pi \hbar \omega^2 \tau_c}.
\]

The calculated results based on this equation are also presented in Fig. 10 as the solid curve, without any fitting parameters and neglecting the frequency detuning. The results are essentially independent of the frequency detuning as long as it is much smaller than the bandwidth. As is evident, the analytic results are in good agreement with the experimental results.

We also investigated how the coupling strength that is needed for phase locking is affected by the quantum noise when the lasers operate near threshold. Representative experimental and calculated results at a fringe visibility of 50% of maximum are shown in Fig. 11. The results show a linear behavior, as was predicted from our analytic model, and that the coupling strength must be increased as the quantum noise increases.
3.3 Effects of many longitudinal modes

When trying to increase the number of phase locked fiber lasers we encounter a limit [16]. To understand this limit we must take into account the spectral properties of phase locked lasers. In order to phase lock two fiber lasers, it is necessary that they have a common frequency in the two cavities. Each cavity has a comb of frequencies (longitudinal modes) which satisfy it. This comb is set by the length of the cavity, and the spacing between each pair of adjacent longitudinal modes is \( \Delta \omega = c/2l \), where \( c \) is the speed of light and \( l \) is the length of the cavity. When introducing coupling between the cavities the spectral properties of the lasers change. Here we show how the phase locking between the two lasers and their longitudinal mode spectrum vary as a function of the coupling strength [17].

The experimental configuration for determining the phase locking and the spectrum of longitudinal modes for two coupled fiber lasers as a function of the coupling strength between them is presented in Fig. 12. Each fiber laser was comprised of a polarization maintaining ytterbium-doped fiber, where one end was attached to a high reflection fiber Bragg grating (FBG), with a central wavelength of 1064 nm and a bandwidth of about 1 nm, that served as a back reflector mirror, and the other end was attached to a collimating GRIN lens with antireflection coating to suppress any reflections back into the fiber cores, and an output coupler (OC) with reflectivity of 20% common to both lasers. The lasers were pumped with 915 nm diode lasers of 300 mW from the back end through the FBG. The two fiber lasers were forced to operate in orthogonal polarizations by using a calcite beam displacer in front of a common output coupler, and the coupling strength between the lasers was controlled by an intra-cavity quarter-wave plate (QWP). The optical length of the cavity of one fiber laser was 10 m, while the optical length of the other was 11.5 m, so each fiber has 20,000 longitudinal modes within the FBG bandwidth. The combined output power of about 200 mW was detected by a fast photo detector, which was connected to an RF spectrum analyzer, to measure the beating frequencies and determine the longitudinal mode spectrum at the output [5]. We also measured the phase locking between the two fiber lasers by detecting the interference of small part of the light from each laser with a CCD camera and determining the fringe visibility [22]. The longitudinal mode spectrum was measured first when \( \theta = 0 \) (\( \kappa = 0 \)), and the measurement was sequentially repeated after rotating the QWP by 1° steps until we reached \( \theta = 45° \) (\( \kappa = 1 \)).
We developed a model for calculating the distribution of longitudinal modes and phase locking for the two coupled lasers. For each laser, the effective reflectivity \( R^\text{eff} \) of self reflection and the light coupled into it from the other laser was calculated self-consistently. The longitudinal mode spectrum was then derived from the total effective reflectivity of the two lasers. Our model can account for the full range of coupling strength between the lasers. The effective reflectivity resulting from the coupling to the other laser, for each laser, can be shown to be

\[
R^\text{eff}_{1,2} = \left(1 - r(1 - \sqrt{\kappa}) - \frac{r^2 \kappa e^{i l_{2,1} k}}{1 - r(1 - \sqrt{\kappa}) e^{i l_{2,1} k}}\right)^{-1},
\]

where \( k \) denotes the propagation vector of the light, \( \kappa \) the coupling strength between the two lasers, \( l_{2,1} \) the length of each laser, and \( r \) the reflectivity of the output coupler. In our calculations we used \( r=0.55 \) rather than the experimental value of \( r=0.2 \) to ensure that the width of the calculated longitudinal modes fits those of the experimental modes. This is justified because cold cavity models do not take into account gain competition, which tends to narrow the width of the longitudinal modes. We then sum over the round-trip propagations to obtain the self-consistent field for each laser, as

\[
R^\text{eff}_j e^{i k l_j} + \left(R^\text{eff}_j e^{i k l_j}\right)^2 + \cdots = \left(1 - R^\text{eff}_j e^{i k l_j}\right)^{-1},
\]

where \( j=1,2 \) is the laser number. Figure 13 shows the experimental and calculated longitudinal mode spectra as functions of coupling strength \( \kappa \). Figure 13(a) shows the experimental results of the longitudinal mode spectrum as a function of coupling strength over a 200 MHz range, and Fig. 13(b) the corresponding calculated results. The experimental and calculated results are also shown in greater detail for four specific coupling strengths (\( \kappa=0,0.28,0.7,1 \)) in Figs. 13(c)–13(f), respectively. Without coupling (i.e., \( \kappa=0 \)) two independent sets of frequency combs exist simultaneously; one corresponds to the 10 m long fiber laser (15 MHz separation between adjacent longitudinal modes), while the other corresponds to the 11.5 m long fiber laser (13 MHz separation), as is also seen in Fig. 13(c). Each seventh longitudinal mode of the 10-m-long laser is very close to the eighth mode of the other, so they are essentially common longitudinal modes. When \( \kappa \) is increased from 0 to 0.3 the longitudinal modes that are not common gradually disappear according to their detuning while transferring their energy to the remaining ones via the homogenous broadening of the gain. The longitudinal modes with the larger detuning disappear first, while the ones with smaller detuning disappear for larger values of \( \kappa \), and only the common
longitudinal mode remains, as is also seen in Fig. 13(d), indicating that at this coupling strength there is full phase locking. As the coupling strength increases above 0.3 the longitudinal modes gradually reappear, as is also seen in Fig. 13(e). The longitudinal modes with the smaller detuning reappear first, and the ones with larger detuning reappear for larger values of $\kappa$. Finally, when $\kappa$ approaches unity, whereby all the light from one laser is transferred to the other, new longitudinal modes appear between adjacent longitudinal modes, as is also seen in Fig. 13(f), corresponding to a single combined laser cavity whose length is the sum of the two lasers.

Figure 13 reveals a good quantitative agreement between the experimental and calculated results. In particular, the observed gradual disappearance of non-common longitudinal modes as the coupling is increased, their gradual reappearance when the coupling is further increased, and finally the doubling of the frequency comb at near-unity coupling strength are all accurately reconstructed by our model.

![Figure 13: Experimental and calculated distributions of longitudinal mode beating frequencies in the output power for two coupled lasers as a function of the coupling strength $\kappa$.](image)

**3.4 Effects of time delayed coupling**

It is well known that stable phase locking and synchronization can occur in two coupled lasers where the time delay coupling between them is relatively short [30 - 32]. Yet, there are some situations, such as in secure communications, that require long time delayed coupling [33, 34]. Accordingly, there have been extensive theoretical and experimental investigations on time delayed coupling [35 - 39]. The emphasis has been on intensity synchronization of coupled lasers but very little on phase locking with long time delay coupling. In particular, although phase locking of lasers with long time delayed coupling was theoretically predicted some time ago [37], so far no experimental confirmation was reported. Phase locked lasers with
long time delays can be useful for applications where two distant lasers require a well defined relative phase. For example, in synchronizing optical clocks for time standard setting, in long baseline interferometry for optical telescopes with large effective apertures and in other interferometric applications such as detection of gravity waves.

When coupling lasers, it is necessary to take into account the time it takes for the coupled light from one laser to reach the others. Two coupling regimes exist for time delayed coupling. One is where the coupling delay time is shorter than the coherence time of the lasers so that the delay time has very little, if any, effect. The other is where the coupling delay time is longer than the coherence time of the lasers so that the effect of the delay time cannot be neglected. Here we investigate the effects of long coupling delay times on phase locking of two coupled fiber lasers. Specifically, we compare two different arrangements for coupling, and demonstrate that phase locking can occur even with time delayed coupling of 20 µs (delay line of 4km). Such delays are much longer than the coherence length of our fiber lasers measured as 10 cm (0.3ns) [18-20]. We find that with long time delayed coupling, phase locking requires delayed self feedback in addition to the delayed coupling signal [18].

Our basic experimental arrangements for investigating time delayed coupling between two fiber lasers are shown schematically in Fig 14. Figure 14(a) shows one that contains a delayed self feedback signal in addition to the delayed coupling signal. Figure 14(b) shows the other that contains delayed coupling only. Each fiber laser included a 10m long Yb-doped double-clad polarization maintaining fiber with a fiber Bragg grating (FBG) at the rear. The fiber lasers were end-pumped with a diode laser through the FBG and the front ends of the fibers were cleaved at an angle to suppress any back reflections into the fiber core. Individual output couplers with R=20% reflectivity were placed at the front of each fiber resulting in two independent lasers [18].

![Figure 14: Experimental arrangements for phase locking two fiber lasers with time delayed coupling. (a) Delayed coupling with delayed self feedback; (b) delayed coupling only.](image)

For the arrangement shown in Fig 14(a), the beams from the two lasers perfectly overlapped in angle and position after a 50% beamsplitter. The light was then coupled into a long single mode delay fiber. At the end of this fiber a common output coupler reflected the light back into the delay fiber and then equally into both lasers. The time it takes the light to propagate from one laser to the other, namely the coupling delay time $\tau_d$ is approximately $\tau_d = 2nL_{\text{fiber}}/c$, where $L_{\text{fiber}}$ the length of the delayed fiber, $n$ is the refractive index and $c$ the speed of light. For the arrangement shown in Fig. 14(b), each laser was coupled into a different end of the delay fiber. The light from each laser propagated through the delay fiber and was then re-injected into the other laser. The coupling delay time in this arrangement was thus approximately $\tau_d = nL_{\text{fiber}}/c$. The coupling strength was
controlled by varying the amount of light injected into the delay fiber. For both arrangements, the coupling strength between the lasers (the relative amount of energy that was transferred from one laser to the other) was set to be the same (approximately 8%). The time delay was controlled by varying the length of the delay fiber from 60 cm to 2 km, so as to get time delays from 0.01 µs to 20 µs. Some light from each laser was directed towards a CCD camera. From the fringe visibility of the interference pattern, we deduced the degree of phase locking [5-11].

First we measured the fringe visibility $V$ as a function of the coupling delay time for the first arrangement. In these measurements, we ensured that the distances from the camera to each one of the lasers is the same. Thus, a stable interference pattern indicates that both lasers are phase locked isochronally (at the same time). The experimental and numerical results of the normalized fringe visibility (normalized by the fringe visibility value with no delay) as a function of the delay time is presented in Fig. 15. Also shown in the inset is a representative example of the interference pattern of two coupled lasers with a coupling delay time of 20 µs, which corresponds to a delay fiber length of 2 km. The actual experimental values of the fringe visibility ranged from 0.4 to 0.5 rather than nearly 1 as we found earlier at short coupling delays [22]. This is mainly due to the fact that the FBG in the two lasers in our current experiment differed, so their central wavelengths were not the same. Thus, part of the light that was detected by the camera did not participate in the actual phase locking and just served as a bias. Nevertheless, the fringe visibility and fringe position remained constant for long periods of time over the entire range of delay times [18-20].

Figure 15: Results of normalized fringe visibility as a function of coupling delay time obtained for the arrangement that includes self feedback. Points and bars (red) denote experimental results and solid (blue) curve denote the numerical results. Inset shows the interference pattern of two lasers coupled by a 4 km long delay line.

For the second arrangement, the fringe visibility was very poor, indicating no isochronal (at the same time) phase locking. Thus, we resorted to achronal (at different times) phase locking, which is indicated by a stable interference pattern of the light from one laser with delayed light from the other. Experimentally, this was done by letting the optical distance from one of the lasers to the camera be longer than that of the other laser, as shown in Fig. 14(B). In our experiment we used the same delay fiber for delaying the coupling signal as well as for the main delaying of the detection signal.

Using a long delay fiber of length of $L_{delay} = 200 m$, we measured the fringe visibility for several values of the camera delay line. The results are presented in Fig.
As evident, the maximal phase locking occurs at a distance of $L_{\text{delay}}$, whereby $L_{\text{delay}}$ is the overall optical distance along which the coupling signal propagates. Also shown in Fig. 16, is the fringe visibility decay width of several cm. The fringe visibility decay width corresponds to the finite coherence length of a single fiber laser, which was independently measured, with the aid of a Michelson interferometer, to be about 10 cm.

Figure 16: Experimental results for the fringe visibility as a function of the camera delay line, for the arrangement that contain delayed coupling only.

4. Up scaling the number of phase locked fiber lasers

In order for coupled fiber lasers to phase lock they must have at least one longitudinal mode within the bandwidth of their gain that is common to all lasers. As the number of coupled lasers increases, the probability for having such common longitudinal modes drops exponentially. Accordingly there are several theoretical predictions which set upper limits of 8 to 12 fiber lasers that can be efficiently phase locked [16]. Nevertheless, experimentalists are attempting to exceed such upper limits but so far with no success. In this section we describe our developments, investigation and results on up scaling the number of lasers that can be passively phase locked and combined. We start with a simultaneous coherent and spectral combining approach that could potentially lead to significant up scaling, and then actual phase locking of twenty five fiber lasers which allowed us to investigate in some detail the phase locking efficiency as a function of number of fiber lasers and the connectivity arrangement between them. Some of our results with twenty five fiber lasers also have a bearing on extreme value statistics.

4.1 Simultaneous spectral and coherent combining

One way to overcome the limitation on the number of fibers that can be coherently combined is to simultaneously use spectral combining. In spectral combining, the beams emerging from the individual lasers, each operating at a slightly different wavelength, are combined incoherently by means of a linear diffraction grating, so all the lasers must usually be aligned along one dimension [21].
Here, we present a configuration in which we simultaneously perform spectral and coherent combining so as to enable combining of a two-dimensional fiber laser array. In one dimension we obtain coherent combining by means of an interferometric combiner in free space [7-11], while in the other dimension we obtain spectral combining by means of a diffraction grating [40]. This simultaneous approach of coherent and spectral combining opens an alternative route to up scaling to very large arrays of lasers, overcoming the up scaling limitation of each individual approach.

Our configuration is presented schematically in Fig. 17. It contains four end-pumped single-mode Erbium-doped CW fibers arranged in two by two square array with a distance of 3.5 mm between adjacent fibers. Each fiber has 10 µm core diameter with numerical aperture of 0.19 and a length of about 7 m. One end of each fiber is attached to a fiber Bragg grating (FBG) that serves as the rear mirror. The other fiber end is connected to a collimator coated with anti-reflection layers to suppress any reflections back into the fiber. The four parallel beams emerging from the fiber collimators are first coherently added in the horizontal direction by means of an intra-cavity passive interferometric combiner to obtain two parallel beams [5]. These beams are then deflected in the vertical direction by means of a lens so they exactly overlap on the surface of a 1200 lines/mm blazed diffraction grating and diffracted towards a common planar output coupler of 20% reflectivity. The wavelength of each beam is self-selected to ensure normal incidence on the output coupler and consequently exact self-reflection back into the fibers in order to obtain lasing [40]. Thus, the two beams are spectrally added into a single output beam, maintaining the high beam quality of each of the individual lasers.

![Figure 17: Basic configurations for adding four fiber lasers. In the horizontal direction, coherent combining is performed by means of an interferometric combiner, and in the vertical direction, spectral addition is performed by means of a linear diffraction grating. The insets show the four beams emerging from the fibers, the two beams after the interferometric combiner, and the single combined output beam after the grating [21].](image)

We measured the beam quality factor $M^2$, the combining efficiency, and the spectra of the individual fiber lasers outputs and combined output. To determine $M^2$, two CCD cameras and a Spiricon laser beam analyzer were used for detecting and characterizing the near- and far-field intensity distributions and then $M^2$ was calculated in accordance with $M^2 = \sigma_n f \sigma_f \pi \lambda F$, where $F$ is the focal length of the lens used to detect the far field and $\sigma_n$ and $\sigma_f$ are the second moments of the near and far fields, respectively. We found that $M^2 = 1.15$ for the combined output beam, essentially identical to that of each individual fiber laser.

The combining efficiency of the coherent combining only was determined by measuring the power of the overall light with and then without the interferometric
combiner, and then calculating their ratio. The overall combining efficiency, after both coherent combining and spectral addition, was similarly measured and found to be 82%. We attribute the reduction of efficiency from 100% mainly to losses to undesired diffraction orders from the blazed grating and also to small residual misalignment between the fiber lasers.

The spectra of the fiber lasers and the combined output were measured by means of a grating spectrometer. The results are presented in Fig. 18. Figure 18(a) shows the expected single spectrum when only the upper pair of lasers are coherently added. Figure 18(b) shows the corresponding spectrum when only the lower pair of lasers are coherently added. As evident, each pair of lasers operates at a different wavelength, with a spectral separation of 1.3nm. Finally, Fig. 18(c) shows the spectrum of the combined output beam. As evident, there is a simple addition of the individual spectrum of each pair of lasers, indicating that there is no interaction between them. Accordingly, both coherent combining and spectral addition simultaneously occur.

![Figure 18: The spectra of the fiber lasers and the combined output, using a blazed diffraction grating of 1200 lines/mm for spectral addition. (a) Spectrum when only upper pair of lasers are coherently added. (b) Spectrum when only lower pair of lasers are coherently added. (c) Spectrum of the spectrally combined output.](image)

### 4.2 Phase locking 25 fiber lasers

When trying to phase lock fiber lasers whose length cannot be accurately controlled the probability for having common longitudinal modes rapidly decreases as the number of fiber lasers increases. Nevertheless, we coupled 25 fiber lasers in order to check it experimentally. The experimental configuration is described in details in [22]. We measured the phase locking level as a function of time for different number of lasers in the array and for different connectivities. This was done by continuously detecting the far-field intensity distribution of the total output light from the array with a CCD camera, determining the maxima and minima intensities, and calculating the average fringe visibility along the x and y directions. The fringe visibility provides a direct measure for the phase locking level that ranges from 0% to 100%. The measurements were performed over a period of 10 hours to obtain about 300,000 measurements. These measurements were then repeated for different numbers of lasers and connectivities. We used the same effective reflectivity model to calculate the maximal effective reflectivity as a function of number of lasers in the array. This model takes into account the reflections from the coupling mirrors and from the
front FBGs. Then we repeated the calculations 1000 times, each time choosing a different random realization of the fiber lasers lengths, and determined the average of the results.

Figure 19 shows the average phase locking level (crosses) and the maximal phase locking level (dots) as a function of the number of lasers in the array for the 2D connectivity. It also includes representative far-field intensity distributions of two fiber lasers [inset (a)], of 25 fiber lasers with average low phase locking level [inset (b)], and of 25 fiber lasers with instantaneous maximal phase locking level [inset (c)]. These results indicate that as the number of lasers in the array increases, the probability to find common longitudinal modes rapidly drops as predicted. However, since the length of each fiber laser fluctuates randomly (modulus $\lambda$) due to thermal and acoustic variations, there is a certain probability for briefly obtaining a common longitudinal mode for all fiber lasers. We found that the probability for obtaining phase locking levels above 90% drops rapidly. Specifically, the probability is 0.1% for 12 lasers, 0.012% for 16 lasers, 0.004% for 20 lasers and 0.001% for 25 lasers. These results could probably be improved due to non-linear effects by resorting to better alignment of component and higher power [2].

Figure 19: Experimental and calculated results of the average and the maximal phase locking levels as a function of number of lasers in the array. Asterisk - maximal phase locking level; crosses - average phase locking level; solid curve - calculated average phase locking level using effective reflectivity model. Insets show the far field intensity distributions corresponding to specific data points [23].

We also determined how the average phase locking level is related to the connectivity of the fiber lasers in the array [23, 24]. Specifically, we measured the average phase locking level of an array of 25 fiber lasers as a function of the average number of coupled neighbors to each fiber laser, for different coupling connectivities. The results are presented in Fig. 20. We started with 1D connectivity of the full array, shown at the left inset, in which the average number of coupled neighbors to each fiber laser is only 1.9. Then, we varied the connectivity and increased the average number of coupled neighbors and measured the average phase locking level of the array in each case, up to a 2D connectivity where the average number of coupled neighbors is 3.2. As evident, there is a monotonic increase in the average phase locking level of the array from 21% up to 29%. These results manifest that connectivity influences the phase locking level, consistent with the expected increase of an order parameter with dimensionality for all coupled oscillators.
Another interesting aspect is the total phase locking of the array as a function of time and not just the average. We measured the fringe visibility of the far field pattern of the 25 fiber lasers. The fringe visibility provides a direct measure for the phase locking level that ranges from 0 to 1. The correlation time of the phase locking level is shorter than 100ms, so over a 10 hours period we acquired about 370,000 uncorrelated measurements of the fringe visibility.

Due to thermal and acoustic fluctuations, the length of each fiber laser and its corresponding Eigen frequencies changes rapidly and randomly. Phase locking minimizes loss in the array, so mode competition will favor frequencies that maximize the size of the phase locked clusters at each moment [22]. Since the distribution of the phase locking level for different frequencies is Gaussian, the statistics of the maximum phase locking level should be described by the Gumbel distribution function.
To check for possible correlations hidden in the experimental results, we fitted the phase locking level distribution with the Bramwell-Holdsworth-Pinton (BHP) distribution using a single fitting parameter $C_1$ [39]. For highly correlated systems, the parameter $C_1$ approaches $\pi/2$, while for uncorrelated systems the parameter $C_1$ has an integer value. In particular, when $C_1=1$, the BHP distribution reduces to the Gumbel extreme-value distribution. The functional form of the BHP distribution is given by

$$P(x) = e^{C_1 \left( \frac{x-\mu}{\sigma} - e^{\frac{x-\mu}{\sigma}} \right)},$$

where $\mu$ denotes the mean value, $\sigma$ the width of the distribution and $c$ the measure for correlations. After fitting the measured probability distribution of the phase locking level of 25 coupled fiber lasers to the BHP distribution, we obtained $C_1=1.03$, indicating a Gumbel distribution.

Fitting the Gumbel distribution to the experimental results for 12, 16, and 20 fiber lasers was not as good as for the 25 fiber lasers. The distribution of the experimental results for low number of lasers is clamped because $\mu$ and $\sigma$ are higher, and the distribution of the phase locking level is bound between 0 and 1. Therefore, we resort to the generalized extreme value (GEV) distribution, that contains one extra parameter - the shape parameter $\xi$. When $\xi=0$, the GEV distribution reduces to the Gumbel distribution, but as $\xi<0$ the GEV distribution is clamped and approaches the Weibull distribution. The GEV distribution is given by

$$P(x) = \frac{1}{\sigma} \left[ 1 + \xi \left( \frac{x-\mu}{\sigma} \right) \right]^{-\frac{1}{\xi}} e^{-\left[ 1+\xi \left( \frac{x-\mu}{\sigma} \right) \right]^{-1/\xi}},$$

where $\mu$, $\sigma$ and $\xi$ were calculated from the experimental data by maximum-likelihood parameter estimation. We expect that as the number of fiber lasers in the array decreases from 25, the mean value and standard deviation of the phase locking level will increase, and the $\xi$ parameter should vary from 0 to a negative value. The measured phase locking level histograms for laser arrays with 12, 16, 20, and 25 fiber lasers and the corresponding GEV distributions with the calculated parameters and their 95% confidence intervals are presented in Fig. 22. As evident, there is a very good agreement between the experimental results and the GEV distributions, extending over three decades. As expected, as the number of fibers increases the values of $\mu$ and $\sigma$ decrease and the $\xi$ parameter approaches 0. For 25 fiber lasers the $\xi$ parameter reach 0.01, close to the expected zero value where the GEV distribution reduces to the Gumbel distribution.
5. Conclusion

We have presented our investigations of passive phase locking and coherent combining of fiber lasers and demonstrated high combining efficiency and good quality of the combined output beams for a variety of configurations and wavelengths. Since the relative phase between the lasers is self-adjusted to minimize losses of the coupled laser system, phase passive phase locking is rather robust even under variable environmental conditions as long as common frequencies exist among all the fiber lasers. This typically occurs when the number of lasers is smaller than ten and for higher number of lasers some control of the length of each laser will be required. Vardit Eckhouse, Ami Ishaaya, Liran Shimshi, Eitan Ronen and Rami Pugatch contributed to the work presented here.
References: