The effect of imperfect CSI on the performance of random ad-hoc networks

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Abstract—The performance of Wireless Ad hoc Networks (WANETs) depends on the availability of Channel State Information (CSI) on all channels in the network. In this work we study the performance when each receiver has CSI only on the channel from its desired transmitter. We present a novel lower bound on the Ergodic Rate Density of a WANET with partial CSI. The novel bound is compared to a previously known bound and its superiority is demonstrated.

We also use the novel bound to study the effect of CSI availability on the WANETs’ performance. We show that for Rayleigh fading the CSI-gain can be up to 78%. But, if each network is optimized with respect to the active transmitter density, then the CSI gain is at most 13%. Simulation results show the practically of the novel bound and the accuracy of the CSI-gain expressions.

I. INTRODUCTION

Wireless ad-hoc networks (WANETs) offer simplicity and flexibility, which make them suitable for many practical applications. These networks do not depend on infrastructure such as base stations, and are primarily characterized by multi-hop communication. Channel access is typically coordinated by decentralized multiple access protocols such as ALOHA [1] or CSMA [2], [3].

In recent years, much insight on the performance and limits of WANETs was obtained by random networks analysis. Such analysis allows the characterization of WANETs performance, without the need to specify specific user locations. The most popular model for the positions of the users in random WANETs is the homogeneous Poisson Point Process (PPP), [4]. In this model the number of users in each finite area has a Poisson distribution, and their locations are uniformly distributed over the area.

Most works on random WANETs have assumed (explicitly or implicitly) complete Channel State Information (CSI) at the receiver, i.e., that the receiver is aware of the fading state from each transmitter in the network. Such analysis was performed either in an outage setup (e.g., [4], [5]) or in an ergodic setup (e.g., [6], [7]). However, complete CSI (and in particular knowledge of the fading state from the interfering transmitters) is not always available.

Recently, a novel upper bound on the ergodic rate density (ERD) of a random network was derived for the case of partial CSI at the receiver [8] (i.e., when the receiver knows the fading state from the desired transmitter, but not from the interfering transmitters). Such partial CSI typically characterizes the achievable information in wideband WANETs such as Frequency Hopping (FH), [8], or Orthogonal Frequency Division Multiplexing (OFDM), [9]. In such systems, the interference evaluation is often averaged over the entire bandwidth, which results in a loss of the information on each fading realization. A more detailed discussion on practical considerations of different WANETs and the resulting available CSI is given in appendix A.

In a fading environment, the loss of CSI on the instantaneous interference power results in a decrease in the WANET throughput. However, the amount of lost throughput has not been quantified so far for random WANETs.

In this work we present a novel lower bound on the ERD of a random ALOHA WANET with partial CSI (PCSI). The work focuses on the access protocol and the physical layer processing, and use simplifying assumptions on the operation of higher communication layers. The presented lower bound is an adaptation of the novel bound recently presented in [6], [7] to the PCSI scenario of [8]. This bound is tighter in most cases than the lower bound originally presented in [8]. We demonstrate that the novel lower bound, together with the upper bound of [8] can characterize the network ERD with good accuracy.

We also compare the novel lower bound (with PCSI) to the original lower bound of [7] (with complete CSI). We show that the performance loss in typical Rayleigh fading model is bounded, and we give an exact formula for this bound that depends only on the channel exponential decay factor. We further show that if the networks densities are optimized then the rate loss is at most 13%.

The rest of this paper is organized as follows: Section II describes the system model. Section III introduces the novel lower bound on ERD of WANETs with partial CSI and the gain of CSI. Section IV gives numerical and simulation results and Section V gives our concluding remarks.

II. SYSTEM MODEL

We assume a decentralized wireless ad-hoc network utilizing an ALOHA protocol (e.g., [4]). Assuming the operation of a routing mechanism, some of the nodes have data that needs to be transmitted to specific destinations. For simplicity we assume that the next destination for each message is located at fixed distance, d, from the transmission source. Nodes that have data to transmit randomly decide on an access time to the network. The distribution of the locations of the active transmitters is modeled as a two dimensional PPP with density of λ.
The power received at receiver $i$ from transmitter $j$ is:

$$W_{i,j} = \rho X_{i,j}^{-\alpha} V_{i,j}$$

where $\rho$ is the transmitted power, $\alpha > 2$ is the exponential decay factor, and $V_{i,j}$ and $X_{i,j}$ are the power fading and the distance between the $i$-th transmitter and the $j$-th receiver respectively (and for $i = j$ we have $X_{i,i} = d$). The power fading represents any random change in the power of the received signal. In this work we will focus on Rayleigh fading channels, in which case, $V_{i,j}$ have an exponential distribution. The fading variables $V_{i,j}$ are independent and identically distributed (i.i.d) and statistically independent of all distance variables. For normalization purposes, we will assume throughout the paper that $E[V_{i,j}] = 1$. In the following we also use the notation $V$ when we discuss the statistical nature of one of the Random Variables (RVs) $V_{i,j}$, bearing in mind that this is a single representative of the family of iid RVs.

We use the shift invariant property of the system, [10], to analyze the performance of the network using user 0 as a probe receiver. Without loss of generality, we assume that the probe receiver is located at the origin. For notational simplicity, in the following we drop the probe receiver index and define the set $\mathcal{A}$ as the set of all active transmitters, excluding the transmitter that is paired with the probe receiver.

Without loss of generality we assume that $d = 1$. We also assume that the network is operating at the interference limited regime and therefore neglect the contribution of the thermal noise.

We consider two types of typical receivers. The first type of receiver, termed a receiver with CSI, has perfect information on the channel state of all active transmitters, and in particular it has exact knowledge of the instantaneous signal-to-interference-ratio (SIR). The second type of receiver, termed receiver with partial CSI, has perfect information of the state of the desired channel but no knowledge of the fading state of the interfering channels. Assuming that all users employ a Gaussian codebook and single user decoders, the ERD of a network with a node density of $\lambda$ and receivers with CSI is given by:

$$R_{\text{CSI}}(\lambda) = \lambda \cdot E \left[ \log_2 \left( 1 + \frac{d^{-\alpha} V_0}{\sum_{j \in \mathcal{A}} X_j^{-\alpha} V_j} \right) \right]. \quad (2)$$

On the other hand, with only partial CSI the received interference is not Gaussian. Limiting the discussion to nearest neighbor decoders [11], the ERD of a network with node density of $\lambda$ and PCSI is given by [8]:

$$R_{\text{PCSI}}(\lambda) = \lambda \cdot E \left[ \log_2 \left( 1 + \frac{d^{-\alpha} V_0}{\sum_{j \in \mathcal{A}} X_j^{-\alpha} V_j} \right) \right]. \quad (3)$$

A WANEt achieves maximal ERD when its active users density is optimized (through the optimization of the channel access probability). The maximal ERD will be referred hereon as the WANEt’s max-throughput. In mathematical terms, the max-throughput of a WANEt with CSI is defined as:

$$T_{\text{CSI}} = \max_\lambda R_{\text{CSI}}(\lambda) \quad (4)$$

and for the a WANEt with partial CSI as:

$$T_{\text{PCSI}} = \max_\lambda R_{\text{PCSI}}(\lambda). \quad (5)$$

III. PERFORMANCE ANALYSIS

In order to analyse the affect of partial CSI on the network performance we first introduce the following novel lower bound:

A. Novel lower bound on the performance of WANET with partial CSI

**Theorem 1:** A lower bound on the ERD of a network with an active user density of $\lambda$ and partial CSI is:

$$R_{\text{PCSI}}^\text{LB}(\lambda) \geq R_{\text{PCSI}}^\text{LB}(\lambda)$$

where

$$R_{\text{PCSI}}^\text{LB}(\lambda) = \lambda e^{\frac{2}{\alpha} - 1} \cdot E \left[ \log_2 \left( 1 + \frac{V}{K_{\alpha} \cdot \lambda^2} \right) \right] \quad (7)$$

and

$$K_{\alpha} \triangleq \frac{2}{\alpha \left( \frac{\pi \alpha}{\alpha - 2} \right)^{\frac{2}{\alpha}}} \quad (8)$$

**Proof of Theorem 1:** The proof uses the lower bound of [6], [7]. This bound applies to the same model presented herein, but allows a different fading variable for the fading of the desired channel (denoted by $V$) and the fading of the interfering channels (denoted by $Y$). The bound of [6], [7] is given by:

$$R_{\text{CSI}}(\lambda) \geq \lambda e^{\frac{2}{\alpha} - 1} \cdot E \left[ \log_2 \left( 1 + \frac{V}{C_{\alpha} \cdot \lambda^2} \right) \right]. \quad (9)$$

and

$$C_{\alpha} \triangleq \frac{2}{\alpha \left( \frac{\pi \alpha}{\alpha - 2} \right)^{\frac{2}{\alpha}}} \left( \pi \alpha E \left[ Y^{\frac{2}{\alpha}} \right] \right)^{\frac{1}{\alpha}} \quad (10)$$

This bound is obviously a lower bound on (2), using $Y = V$. Note that in the case of Rayleigh fading, $V$ has an exponential distribution with a mean of 1, and $E[Y^{\frac{2}{\alpha}}] = \Gamma(1 + \frac{2}{\alpha})$.

To prove that (9) also lower bounds (3), we note that although the missing CSI results in a different statistical model of the received signal, the resulting performance metric in (3) is quite similar to the performance metric in (2). More precisely, this is the exact setup of the bound in (9) when the interference fading variable is set to $Y = 1$. Thus, substituting $V = 1$ into (9) leads directly to the bound in (7).

B. CSI Impact on the ERD and Max-Throughput

1) The ERD Gain from CSI: Let $R_{\text{LB}}^\text{CSI}$ be the lower bound on the ERD for a WANEt with CSI, which results from (9) by substituting $Y = V$, i.e.,

$$R_{\text{LB}}^\text{CSI}(\lambda) = \lambda e^{\frac{2}{\alpha} - 1} \cdot E \left[ \log_2 \left( 1 + \frac{V}{C_{\alpha} \cdot \lambda^2} \right) \right] \quad (11)$$

and

$$C_{\alpha} \triangleq \frac{2}{\alpha \left( \frac{\pi \alpha}{\alpha - 2} \right)^{\frac{2}{\alpha}}} \left( \pi \alpha E \left[ V^{\frac{2}{\alpha}} \right] \right)^{\frac{1}{\alpha}} \quad (12)$$
The following corollary characterizes the ratio between the lower bound with CSI and the lower bound with partial CSI, i.e., the gain in the ERD from the availability of complete CSI.

**Corollary 1:** The ERD gain from CSI is bounded by:

\[ 1 \leq \frac{R_{CSI}^{\text{LB}}(\lambda)}{R_{PB}^{\text{LB}}(\lambda)} \leq \left( E \left[ V^\frac{z}{2} \right] \right)^{-\frac{2}{\alpha}}. \]  

(13)

**Proof of Corollary 1:** The derivative of the the ERD gain ratio is:

\[
\frac{\partial}{\partial \lambda} \left( \frac{R_{CSI}^{\text{LB}}(\lambda)}{R_{PB}^{\text{LB}}(\lambda)} \right) = \frac{\partial}{\partial \lambda} \left( E \left[ \log_2 \left( 1 + \frac{Y}{C_{\alpha} \lambda^\frac{z}{2}} \right) \right] \right).
\]

(14)

where \( \Psi(x, c) \triangleq (c + x) \cdot \log (1 + c^{-1}x) \). We next prove that:

\[
\frac{\partial}{\partial \lambda} \left( \frac{R_{CSI}^{\text{LB}}(\lambda)}{R_{PB}^{\text{LB}}(\lambda)} \right) \geq 0 \forall \lambda
\]

(15)

by combining two facts: a) \( C_{\alpha} \leq K_\alpha \) and b) \( \Psi(x, c) \) is monotonic decreasing in c for any \( x > 0 \). a) is proved by noting that \( x^{-1} \) is concave in \( x \) for \( \alpha > 2 \), and hence Jensen’s inequality leads to \( E \left[ V^\frac{z}{2} \right] \leq (E[V])^\frac{z}{2} = 1 \). b) is proved by evaluating the derivative:

\[
\frac{d}{dc} \Psi(x, c) = \log(1 + c^{-1}x) - c^{-1}x < 0
\]

(16)

for any \( c, x > 0 \).

Comparing (7) and (11) and using (15) we have:

\[
\min_{\lambda} \frac{R_{CSI}^{\text{LB}}(\lambda)}{R_{PB}^{\text{LB}}(\lambda)} = \lim_{\lambda \to 0} \frac{R_{CSI}^{\text{LB}}(\lambda)}{R_{PB}^{\text{LB}}(\lambda)} = 1
\]

(17)

and

\[
\max_{\lambda} \frac{R_{CSI}^{\text{LB}}(\lambda)}{R_{PB}^{\text{LB}}(\lambda)} = \lim_{\lambda \to \infty} \frac{R_{CSI}^{\text{LB}}(\lambda)}{R_{PB}^{\text{LB}}(\lambda)} = K_{\alpha} C_{\alpha}^{-1}.
\]

(18)

Substituting (8) and (12) into (18) concludes the proof. \( \square \)

Note that the expression \( \left( E \left[ V^\frac{z}{2} \right] \right)^{-\frac{2}{\alpha}} \) in (13) is not necessarily upper bounded. This can be illustrated by the simple example of the on-off channel:

\[
V = \begin{cases} 1 & \text{w.p. } p \\ 0 & \text{w.p. } 1 - p \end{cases}
\]

(19)

where \( 0 \leq p \leq 1 \). For this channel the upper bound in (13) results with:

\[
\left( E \left[ V^\frac{z}{2} \right] \right)^{-\frac{2}{\alpha}} = p^{\frac{2}{\alpha}}
\]

(20)

which depends on the value of \( p \) and is clearly not upper bounded.

On the other hand, in practical systems, the ERD gain is typically bounded. For example, the case of Rayleigh fading channels, i.e., \( V \sim \text{Exp}(1) \), results in:

\[
\left( E \left[ V^\frac{z}{2} \right] \right)^{-\frac{2}{\alpha}} = \Gamma \left( 1 + \frac{2}{\alpha} \right).
\]

(21)

This expression is further illustrated in Section IV. It is also characterized by the following lemma.

**Lemma 1:** For Rayleigh fading, the CSI ERD gain is an increasing function in \( \alpha \) and is upper bounded by an ERD increases of 78%.

**Proof of Lemma 1:** See appendix B.

2) CSI impact on the max-throughput: 

**Corollary 2:** The max-throughput gain from the CSI is given by:

\[
\frac{T_{\text{CSI}}^{\text{LB}}}{T_{\text{PCSI}}^{\text{LB}}} = \left( E \left[ V^\frac{z}{2} \right] \right)^{-1}
\]

(22)

where

\[
T_{\text{CSI}}^{\text{LB}} = \max_{\lambda} R_{\text{CSI}}^{\text{LB}}(\lambda)
\]

(23)

and

\[
T_{\text{PCSI}}^{\text{LB}} = \max_{\lambda} R_{\text{PCSI}}^{\text{LB}}(\lambda).
\]

(24)

**Proof of Corollary 2:** Substituting (7) and (11) in (24) and (23), respectively, the max-throughput bound ratio is:

\[
\frac{T_{\text{CSI}}^{\text{LB}}}{T_{\text{PCSI}}^{\text{LB}}} = \frac{\max_{\lambda} \lambda \cdot E \left[ \log_2 \left( 1 + \frac{Y}{C_{\alpha} \lambda^\frac{z}{2}} \right) \right]}{\max_{\lambda} \lambda \cdot E \left[ \log_2 \left( 1 + \frac{Y}{K_{\alpha} \lambda^\frac{z}{2}} \right) \right]}
\]

\[
= \frac{C_{\alpha} \frac{z}{2} \cdot \max y \cdot E \left[ \log_2 \left( 1 + \frac{Y}{y^\frac{z}{2}} \right) \right]}{K_{\alpha} \frac{z}{2} \cdot \max y \cdot E \left[ \log_2 \left( 1 + \frac{Y}{y^\frac{z}{2}} \right) \right]}
\]

\[
= \left( E \left[ V^\frac{z}{2} \right] \right)^{-1}
\]

(25)

where the the second equality used the substitutions \( x = C_{\alpha} \frac{z}{2} \lambda \) and \( y = K_{\alpha} \frac{z}{2} \lambda \), and the last equality used the similarity of the max term in the nominator and the denominator, and the substitution of (8) and (12). For Rayleigh fading channels, \( E \left[ V^\frac{z}{2} \right] = \Gamma \left( 1 + \frac{2}{\alpha} \right) \). Note that in this case the maximum gain on the max-throughput due to the availability of complete CSI, is approximately 12.92% which is attained for \( \alpha \approx 4.33 \).

IV. NUMERICAL RESULTS

In this section we use simulation results to demonstrate the new lower bound and the difference in the performance of CSI and PCSI WANETs. The simulated expressions of the ERD and the max-throughput were evaluated using a network simulator with 50,000 nodes and averaging of 100,000 network

1Using \( \min \Gamma(x) \triangleq \Gamma(1.4616) \triangleq 0.8856 \), [12, p. 259].
realizations. Except when noted otherwise, the desired and interference channels are assumed to distribute as Rayleigh fading, i.e., $V \sim \text{Exp}(1)$.

Fig. 1 shows the novel bound of (7), depicted by a dashed line, and compares it to the bounds of Stamatiou et al. [8] (depicted by dotted lines and markers), as function of the active nodes density, $\lambda$. The figure is plotted for the case of partial CSI and $\alpha = 4$. In most of the range, the novel lower bound is tighter than the lower bound of Stamatiou et al. [8, Eq. 25]. Furthermore, the behaviour of the novel lower bound as function of the active nodes density is very similar to the behaviour of the ERD, and both achieve their maxima at nearly the same density. On the other hand, the Stamatiou lower bound, which is very tight for small nodes density, becomes very small at the optimal density, which make it less suitable for max-throughput evaluation. Thus, the combination of the novel lower bound and the upper bound of Stamatiou et al. [8, eq. 25] gives a good characterization of the W ANETs ERD.

We next wish to inspect the behavior of the CSI gain of the ERD as function of the active user density, $\lambda$. Equation (15) states that the ratio of the lower bounds is a non-decreasing function of $\lambda$. To test if this property holds also for the actual ERDs, Fig. 2 depicts ratio of the ERD with CSI, (2), to the ERD with partial CSI, (3), for path loss factors of $\alpha = 3$ and $\alpha = 4$. The figure shows that the ERD gain is indeed an increasing function of $\lambda$. Furthermore, Equation (21) anticipates that the maximum CSI gain for $\alpha = 3$ and $\alpha = 4$ is $1.17$ and $1.27$ respectively, which are the exact maximum measured CSI gains in Figure 2.

Fig. 3 depicts the maximum CSI gain, (upper bound in (13)), for three channel fading models: Rayleigh and Rician with $K_{\text{factor}}=1$ and with $K_{\text{factor}}=2$. For Rayleigh fading, the exact expression is given in (21), and Lemma 1 shows that the CSI gain is upper bounded by $78\%$. However, as can be noticed from Fig. 3, the CSI-gain does not exceed $41\%$ for $\alpha \leq 6$. Moreover, we can observe that the ERD of WANET with Rician fading is even less sensitive to the availability of complete CSI than in the Rayleigh fading case.

To conclude this section, Fig. 4 depicts the CSI gain for the max-throughput as function of $\alpha$. The star marks, obtained from simulation results, depict the ratio between the max-throughput in W ANET with complete CSI, (4), and the max-throughput in W ANET with partial CSI, (5). The solid line depicts the analytical lower bounds ratio curve derived in (25). As can be noticed, the two curves match perfectly, and (25) predicts the max-throughput gain with good accuracy.

V. CONCLUSIONS

We derived a novel lower bound on the ergodic rate density of a random W ANET with partial CSI. The novel bound is simple to evaluate and was shown to describe the ERD better than previously published bounds. Furthermore, the novel lower bound has much resemblance to the bound with complete CSI, and hence allows a convenient comparison between the complete and partial CSI case.

We defined and derived closed form expressions for the increase in the ERD due to availability of CSI. We showed that the CSI-gain of the ERD is unbounded in general. Yet, in the common case of Rayleigh fading, the CSI-gain was shown to be bounded by $78\%$. Simulation results verified that all CSI-gain expressions derived in this work are very accurate.

We further studied the CSI gain of the maximal-throughput of the network, when the active node density is optimized. We showed that in the case of Rayleigh fading, the gain from having complete CSI is at most $13\%$. This results shows the robustness of the network. It shows that nearly the same network throughput is achievable even when complete CSI is not available. Note that following [6], [7], the analysis herein can be easily extended to include more complicated system models such as threshold scheduling, transmit and/or receive beamforming, and any other model in which the interference is modeled as Rayleigh fading.

APPENDIX A

In this Appendix we present some of the practical aspects that lead to the different assumptions on the availability of CSI. We first need to note that in large networks, obtaining exact knowledge on the channels from each transmitter to each receiver is impractical. Hence, the notion of complete
CSI is mostly an abstraction. Yet, achieving the maximal ergodic rate of (2) actually requires each receiver to know only the amplitude and phase of the channel from its desired transmitter, and the instantaneous power of the sum of all interference. Thus, we use term complete CSI knowledge to describe the case that these three parameters can be estimated with good enough accuracy.

The amplitude and phase of the desired channel are typically estimated with good accuracy using properly placed pilot symbols and some filtering [13]. Once a good estimate of the desired channel has been obtained, the same pilot symbols can also be used to estimate the total interference power throughout the packet.

In narrowband networks, if the packet length is short compared to the channel coherence time, the estimated interference power will be affected by a single realization of all channels in the network. Hence, the estimated interference power will well represent the interference power required to achieve (2).

The estimation is more complicated if the packet length is much longer than the channel coherence time. or more commonly, in wideband networks when the transmission bandwidth is much larger than the channel coherence bandwidth. In such wideband network, the estimated interference power is affected by many realization of the channels fading (in various frequencies). Thus, estimating the instantaneous interference power at each frequency and in each time becomes very demanding, and in many cases impractical.

In such wideband networks, a more practical estimation goal is to estimate only the average interference power (averaged over all the whole bandwidth). This type of estimation evaluates the denominator of (3), and as stated above, allows to achieve the ergodic rate described by (3). In this work the knowledge of the average interference power is referred to as partial CSI. Thus, all performance analysis in this work that assumes complete CSI is typically achievable in narrowband networks, where the instantaneous interference power is typically available. On the other hand, all performance analysis in this work that assumes partial CSI is typically achievable in wideband networks, where only the average of the total interference power is typically available.

\[ \Gamma \left( 1 + \frac{2}{\alpha} \right)^{- \frac{z}{2}} \leq \lim_{\alpha \to \infty} \prod_{n=1}^{\infty} \left( \frac{n}{n+1} \right)^{\frac{n}{2}} \left( 1 + \frac{2}{\alpha n} \right)^{- \frac{n}{2}} \]

where (a) used \( \lim_{m \to \infty} \left( 1 + \frac{z}{m} \right)^{m} = e^{z} \), [12, eq. 4.2.21], and (b) used \( \lim_{m \to \infty} \sum_{j=1}^{m} \frac{1}{j} = -\ln(m) = \gamma \), [12, eq. 4.1.32], and \( \gamma \) is the Euler-Mascheroni constant (\( \gamma \approx 0.5772 \)).
REFERENCES


