Arbitrary partial FEXT cancellation in adaptive precoding for multichannel downstream VDSL

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Abstract

In this paper we present and analyze a simplified, adaptive precoder for arbitrary partial far-end cross talk (FEXT) cancellation in the downstream of multichannel VDSL. The precoder is based on error signal feedback and is computationally efficient. Furthermore, the partial precoding makes it possible to operate the precoder at any desired complexity, and the system’s performance increases with any increase in system complexity. Unlike previous works, in which each user experienced either complete FEXT cancellation or no cancellation at all, the presented precoder can mitigate the FEXT from any desired subset of interfering users for each user. We derive sufficient conditions for convergence of the adaptive precoder, for any partial cancellation scheme, and provide closed form steady state error analysis. The precoder’s performance and convergence properties are also demonstrated through simulations.

I. INTRODUCTION

Far-end crosstalk (FEXT) is currently the greatest bottleneck in very high-speed digital subscriber Line (VDSL) systems. In a typical DSL topology the optical fiber ends at an optical network unit (ONU) and the data are further distributed to the users over the existing copper infrastructure. In the downstream, the receiving modems are located in different customer premises and there is no way to cancel FEXT at the end users’ side. On the other hand, the transmitting modems are co-located at the ONU, and hence FEXT cancellation is feasible for the downstream if proper precoding is employed at the ONU. Most research on downstream precoding is based

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on the assumption that accurate enough channel state information (CSI) is available at the ONU, based on channel estimate feedback from the end-user.

Ginis and Cioffi [2], [3], considered the problem of a digital subscriber line (DSL) system with coordinated transmission and uncoordinated receivers, and proposed a transmitter precoding scheme based on (generalized) zero-forcing equalization and Tomlinson-Harashima precoding [4], [5], [6], which might be interpreted as a suboptimal implementation of the zero-forcing dirty-paper precoding scheme proposed in [7] and studied in [8]. Their decision-feedback structure, based on the Tomlinson-Harashima precoder (THP), was shown to operate close to the single-user bound [3]. An improvement of the scheme of [2] was proposed in [9], where Tomlinson-Harashima precoding is replaced by more efficient trellis precoding schemes. Cendrillon et. al [10], [11] [12], noted that it is sufficient to use linear precoding due to the diagonal dominance property of the FEXT coupling matrix. Still, their zero-forcing (ZF) solution requires matrix inversion at each tone of the multichannel DSL system (and current DSL systems use thousands of tones). Leshem and Li [13], [14], proposed a simplified approximate precoder, based on a first or second order approximation of the ZF precoding matrix, which significantly reduces the computationally complexity.

An efficient alternative adapts the precoder by transmitting error symbols from the end-users to the ONU [15]. Bergel and Leshem [16] analyzed a simplified version of this adaptive precoder and presented convergence conditions and a steady state performance analysis. They also suggested implementing partial FEXT cancellation in which users with low SNR do not apply FEXT cancellation. However, in practical systems partial FEXT cancellation is important because of complexity limitations. In many cases, the number of multiplications required for complete FEXT cancellation is too high and for a given user, the system can only cancel the FEXT generated by a subset of the other users. For example, Cendrillon et al. [17] presented a method to calculate the partial FEXT cancellation matrix for given subsets of cancelled FEXT terms. They also analyzed the resulting performance as a function of system complexity, and presented several methods to choose cancellation subsets. In this paper we present a novel partial FEXT cancellation version of the adaptive precoder proposed in [16]. This precoder only adapts the matrix elements that correspond to the selected FEXT cancellation subsets. We prove its convergence to the partial FEXT cancellation precoder described in [17]. We also derive a convergence analysis and a steady state error analysis. The paper is structured as follows:
in section II we describe the mathematical model for multi-pair DSL systems, and discuss the partial FEXT cancellation precoder presented by Cendrillon et al. [17], and then present the novel adaptive partial FEXT precoder. In section III we provide a convergence analysis, including convergence conditions. In section IV we analyze steady state performance. Section V reports simulation results. The conclusions discusses the implications of partial FEXT cancellation.

II. SIGNAL MODEL

We consider a discrete multi-tone (DMT) multichannel pre-coded system where transmission takes place independently over many narrow sub-bands. Following VDSL2 conventions, we assume that the system operates in a frequency division duplex mode (FDD), where upstream and downstream transmissions operate at separate frequency bands, and all transmissions in the binder are synchronized. Due to synchronization, near end cross talk (NEXT) is eliminated. On the other hand, FEXT can significantly reduce the system capacity. To reduce the FEXT, the system coordinates the transmission of all \( u \) twisted pairs in a binder. Focusing on a single frequency bin, the \( n \)-th received symbol for all pairs can be written in vector form as:

\[
x_n = H F_n s_n + v_n,
\]

where

\[
H = \begin{bmatrix} h_{11} & \cdots & h_{1u} \\
\vdots & \ddots & \vdots \\
h_{u1} & \cdots & h_{uu} \end{bmatrix}
\]

is the channel matrix, \( s_n = [s_1^n \ldots s_u^n]^T \) and \( v_n = [v_1^n \ldots v_u^n]^T \), are the vectors containing the transmitted symbols and sampled noise for all pairs respectively, and \( F_n \) is the precoding matrix. We assume that all users have identical PSD and denote the transmitted power in the analyzed frequency bin by \( P_s \). We also assume that the symbols transmitted to all users are statistically independent and satisfy:

\[
E[s_n s_n^H] = P_s \cdot I, \quad E[s_n s_n^H s_n s_n^H] = (u + L - 1) P_s^2 \cdot I,
\]

where \( L = E[|s_n^2|^4]/P_s^2 \), which ranges from 1 in QPSK modulation to 2 in Gaussian modulation. The noise is additive white Gaussian (AWGN); i.e., the noise samples are independent.
identically distributed (iid) Gaussian random variables with:

\[ E[v_n] = 0, \quad E[v_n \cdot v_n^H] = \sigma_v^2 \cdot I. \]  \hspace{1cm} (4)

The precoder aims to minimize the FEXT measured by all users (for no precoding we use \( F_n = I \)). For example, the popular ZF linear Pre-coder uses:

\[ F_{ZF} = H^{-1}D, \]

where the diagonal matrix \( D \) is:

\[
D = \begin{bmatrix}
h_{11} & & \\
& \ddots & \\
& & h_{uu}
\end{bmatrix}. \hspace{1cm} (5)
\]

This precoder achieves a FEXT free channel:

\[ x_n = Ds_n + v_n, \]

typically at the cost of a small reduction in the received power level. A useful precoder for partial FEXT cancellation was presented by Cendrillon et al. [17]. With a minor modification we describe this precoder as follows: Let \( G_j = \{g_j^1, \ldots, g_j^{r_j}\} \subseteq \{1, 2, \ldots, u\} \) be the set of receivers that need FEXT cancellation from the \( j \)-th user. In the \( j \)-th column of the precoding matrix only the elements indicated by the set \( G_j \) are not set to zero. Denote the set size by \( 0 \leq r_j = |G_j| \leq u \) (i.e., \( G_j \) can also be an empty set). We define the partial precoder \( F_{P} \) as:

\[ F_{P} = \hat{F}_{P} + I, \]

where \( I \) is the identity matrix and the matrix \( \hat{F}_{P} \) satisfies \( (\hat{F}_{P})_{ij} = 0 \) for any \( i \notin G_j \). The elements with non-zero values in \( \hat{F}_{P} \) are constructed according to a generalization of the ZF principle. In this way we completely cancel the FEXT generated by the \( j \)-th user to all users \( i \in G_j \).  \(^1\) To simplify the precoder analysis we define the diagonal \( j \)-th selection matrix:

\[
(\Gamma_j)_{m,i} = \begin{cases} 
1 & i = m \in G_j \\
0 & \text{otherwise}
\end{cases}, \hspace{1cm} (8)
\]

and the minimized \( j \)-th \((r_j \times u)\) selection matrix:

\[
(\tilde{\Gamma}_j)_{m,i} = \begin{cases} 
1 & i = g_j^m \\
0 & \text{otherwise}
\end{cases}. \hspace{1cm} (9)
\]

\(^1\)This precoder is identical to the precoder presented in [17], but, we also allow the case in which that \( G_j \) does not necessarily include \( j \).
(Note that \( \tilde{\Gamma}_j^T \tilde{\Gamma}_j = \Gamma_j \)). We also define the global selection matrix, which selects all rows that were selected by at least one of the \( \Gamma_j \)'s. Setting: \( G_{\text{max}} = \bigcup_{j=1}^{u} G_j \), the \( u \times u \) diagonal global selection matrix is given by:

\[
(\Gamma_{\text{max}})_{m,j} = \begin{cases} 
1 & i = m \in G_{\text{max}} \\
0 & \text{otherwise} 
\end{cases}.
\]

The \( j \)-th column of the precoder is constructed to satisfy:

\[
\Gamma_j H \hat{F}_P^{(j)} = \Gamma_j D^{(j)},
\]

where \( A^{(j)} \) is the \( j \)-th column of the matrix \( A \). Note that \( \hat{F}_P^{(j)} \) has zeros in the rows that do not belong to the set \( G_j \), therefore:

\[
\hat{F}_P^{(j)} = \Gamma_j \hat{F}_P^{(j)} = \tilde{\Gamma}_j^T \tilde{\Gamma}_j \hat{F}_P^{(j)},
\]

and the non zero elements of the \( j \)-th column of the precoder can be calculated by solving:

\[
\tilde{\Gamma}_j H \hat{F}_P^{(j)} = \tilde{\Gamma}_j D^{(j)}.
\]

Using (7) and (12), the left side of (13) is:

\[
\tilde{\Gamma}_j H \hat{F}_P^{(j)} = \tilde{\Gamma}_j H \left( \hat{F}_P^{(j)} + I^{(j)} \right) = \tilde{\Gamma}_j H \left( \tilde{\Gamma}_j^T \tilde{\Gamma}_j \hat{F}_P^{(j)} + I^{(j)} \right) = \tilde{H}_j \tilde{\Gamma}_j \hat{F}_P^{(j)} + \tilde{\Gamma}_j H^{(j)}
\]

where \( \tilde{H}_j = \tilde{\Gamma}_j H \tilde{\Gamma}_j^T \) is an \( r_j \times r_j \) non singular matrix formed by the elements of the channel matrix that are located in the rows and columns that belong to \( G_j \) (i.e. by \( (H)_{m,i} : m, i \in G_j \)). Substituting (14) back into (13) the non-zero elements of \( \hat{F}_P^{(j)} \) are then calculated by:

\[
\tilde{\Gamma}_j \hat{F}_P^{(j)} = \tilde{H}_j^{-1} \tilde{\Gamma}_j \left( D^{(j)} - H^{(j)} \right),
\]

and the actual precoder is given by (7). Note that for full FEXT cancellation \( \tilde{\Gamma}_j = I \), and \( F_P \) is equal to the ZF precoder \( (F_P = F_{ZF}) \). Partial precoder results with residual FEXT whose powers (for all users) are given by the diagonal of:

\[
Z = P_s \sum_{j=1}^{u} \left( H \hat{F}_P^{(j)} - D^{(j)} \right) \left( H \hat{F}_P^{(j)} - D^{(j)} \right)^H.
\]

As was mentioned above, calculation of the desired precoder requires a good channel estimation and a matrix inversion operation each time the precoder updates. An efficient precoder update scheme, which was suggested in [15], uses the error signal vector transferred back from the
users to the ONU to update the precoder. Assuming that each user knows its direct channel coefficient, the error signal measured by the $i$-th user in the $n$-th received symbol is given by:

$$\varepsilon^i_n = x^i_n - h_{ii}s^i_n. \quad (17)$$

Representing the measured error signal of all users in vector form gives:

$$\varepsilon_n = x_n - Ds_n = (HF_n - D)s_n + v_n. \quad (18)$$

The users send back to the ONU a quantized version of the error signal vector which can be written as:

$$\hat{\varepsilon}_n = D^{-1}\varepsilon_n + w_n = D^{-1}(HF_n - D)s_n + D^{-1}v_n + w_n, \quad (19)$$

where $w_n$ is the error due to quantization which in the following will be assumed to be a statistically independent random variable with zero mean and covariance matrix $E [w_n w_n^H] = \sigma_w^2 I$. A simplified version of [15] was analyzed by Bergel and Leshem [16], where the precoder update equation is given by:

$$F_{n+1} = F_n - \alpha \hat{\varepsilon}_n s_n^H, \quad (20)$$

where $\alpha$ is termed the adaptation step size. We distinguish between two kinds of partial FEXT cancellations. The first is full FEXT cancellation for selected users as was analyzed in [16], where the FEXT due to all cross-talkers of those selected users is cancelled, while for the unselected users there is no FEXT cancellation at all. This type of partial FEXT cancellation was suggested when not applying FEXT cancellation for users with low SNR. The equation for this case is given by setting $\Gamma_j = \Gamma_{\text{max}}$ for all $1 \leq j \leq u$:

$$F_{n+1} = F_n - \alpha \Gamma_{\text{max}} \hat{\varepsilon}_n s_n^H. \quad (21)$$

In [16] the authors derived sufficient conditions that guarantee that the adaptive precoder, (21), converges to the ideal partial precoder $F_P$. It was also shown that cancelling the FEXT for fewer lines can help channels to satisfy the convergence conditions, and makes it possible to increase the step size for faster convergence. In this paper we analyze the general case of partial FEXT cancellation, which require lower implementation complexity. Here, each of the users has its own set of receivers that require FEXT cancellation. The structure of the partial precoding matrix is
determined by the set of \( u \) precoder update sets \( \{G_1, \ldots, G_u\} \). In this case the precoder update equation cannot be written in a matrix form, and we write an update equation per column:

\[
F^{(j)}_{n+1} = F^{(j)}_n - \alpha_n \Gamma_j \hat{\xi}_n s_n^{j*},
\]  

(22)

where \( s_n^{j*} \) is the complex conjugate of \( s_n^{j} \). Note that in this paper we consider also a variable step size, \( \alpha_n \). In practical systems, the use of a variable step size allows a better control on the tradeoff between better FEXT cancellation and faster convergence. In particular, the step size can be increased in system transition times to allow faster convergence, and decreased in steady state to allow better FEXT cancellation. In the following sections we use the variable step size to prove that the partial adaptive precoder, (22), converges to the partial precoder, \( F_P \), when the step size decreases in the correct rate. We also give an approximate steady state analysis of this precoder.\(^2\)

III. CONVERGENCE ANALYSIS

In this section we analyze the performance of the precoder defined in (22). We start by deriving sufficient conditions that ensure convergence. We define the distance between the adaptive precoder and \( F_P \) by:

\[
\Delta_n = F_n - F_P
\]

(23)

and:

\[
W_n = E[\Delta_n \Delta_n^H], \quad \delta_n = \text{tr}\{W_n\}.
\]

(24)

Note that \( \delta_n \) is the Frobenius norm of \( \Delta_n \) and therefore convergence of \( \delta_n \) to zero indicates that every element in \( F_n \) converges in mean squares (MS) to the corresponding element in \( F_P \).

Our main result is given by the following theorem, which provides sufficient conditions for the precoder convergence:

**Theorem 3.1:** If:

\[
\beta_{\max} = \left[ \max_{m \in \mathcal{G}_{\max}} \left( \sum_{i \in \mathcal{G}_{\max}} \frac{|h_{m,i}|^2}{h_{m,m}} \right), \max_{i \in \mathcal{G}_{\max}} \left( \sum_{m \in \mathcal{G}_{\max}, m \neq i} \frac{|h_{m,i}|^2}{h_{m,m}} \right) \right] < 1,
\]

(25)

\(^2\)In this paper we focus on the adaptive precoder update, given the precoder structure. For an analysis of different methods to select the precoder structure, see [17].
then:

A. a sufficient condition for the convergence of the precoder to $F_P$ (i.e., $\lim_{n \to \infty} \delta_n = 0$) is that

$$\alpha_n = \mu \cdot n^{-\theta},$$  \hspace{1cm} (26)

for some $0.5 < \theta < 1$ and $\mu > 0$.

B. For any constant step size the asymptotic difference between $F_n$ and $F_P$ is bounded if:

$$\alpha_n = \alpha < \frac{2}{KP_s (1 + \beta_{\max})},$$  \hspace{1cm} (27)

where $K = u + L - 1$.

C. If the condition of part B is satisfied then the Frobenius norm of the difference between $F_P$ and the actual precoder $F_n$ after sufficiently long time is bounded by:

$$\lim_{n \to \infty} \delta_n \leq \alpha \cdot \text{tr} \left\{ \sum_{j=1}^{u} \left( (D^{-H}D^{-1}) \Gamma_j \left( Z + \sigma_v^2 I + \sigma_w^2 \Gamma_j \right) \right) \right\}$$

$$- \frac{2 - \alpha K P_s (1 + \beta_{\max}^2) - 2 \beta_{\max} |1 - \alpha K P_s|}{2},$$  \hspace{1cm} (28)

where $Z$ is the residual FEXT given in (16).

Note that the conditions that ensure that the distance between $F_n$ and $F_P$ is bounded, are the same as the sufficient conditions, derived in [16], for bounding the distance between the precoder given in (21) and $F_{ZF}$. Also note that (28) shows that the convergence error, $\delta_\infty$, can be made as small as desired using a small enough $\alpha$.

**Proof:** Following definition (23) the difference between each column of the precoder and the corresponding column in $F_P$ is given by:

$$\Delta^{(j)}_n = F^{(j)}_n - F^{(j)}_P.$$  \hspace{1cm} (29)

Subtracting $F^{(j)}_P$ from both sides of (22) and using (19) we get:

$$\Delta^{(j)}_{n+1} = \Delta^{(j)}_n - \alpha_n \Gamma_j \left( D^{-1} \epsilon_n + w_n \right) s_n^{j*}.$$  \hspace{1cm} (30)

The received error signal term, given in (18), can be written using (29) as:

$$\epsilon_n = (HF_n - D)s_n + v_n$$

$$= \sum_{i=1}^{u} \left[ (HF_n^{(i)} - D^{(i)}) s_n^{i} \right] + v_n$$

$$= \sum_{i=1}^{u} \left[ \left( H \left( \Delta^{(i)}_n + F^{(i)}_n \right) - D^{(i)} \right) s_n^{i} \right] + v_n$$

$$= \sum_{i=1}^{u} \left[ H \Delta^{(i)}_n s_n^{i} + \left( H F^{(i)}_P - D^{(i)} \right) s_n^{i} \right] + v_n.$$  \hspace{1cm} (31)
We define $\bar{\Gamma}_j$ to be the complementary matrix of $\Gamma_j$ (i.e., $\Gamma_j + \bar{\Gamma}_j = I$). Using (11), equation (31) becomes:

$$
e_n = \sum_{i=1}^{u} \left[ H \Delta_n^{(i)} s_n^i + (\Gamma_i + \bar{\Gamma}_i) \cdot \left( HF_P^{(i)} - D^{(i)} \right) s_n^i \right] + v_n$$

$$= \sum_{i=1}^{u} \left[ H \Delta_n^{(i)} s_n^i + \bar{\Gamma}_i \cdot \left( HF_P^{(i)} - D^{(i)} \right) s_n^i \right] + v_n. \quad (32)$$

Substituting (32) in (30) we get:

$$\Delta_{n+1}^{(j)} = \Delta_n^{(j)} - \alpha_n \Gamma_j \left( D^{-1} \sum_{i=1}^{u} \left[ H \Delta_n^{(i)} s_n^i + \bar{\Gamma}_i \left( HF_P^{(i)} - D^{(i)} \right) s_n^i \right] \right) s_n^j + \alpha_n \Gamma_j (D^{-1} v_n + w_n) s_n^j. \quad (33)$$

Taking the $j$-th element out of the summation in (33) using $\Gamma_j \cdot \bar{\Gamma}_j = 0$ (note that the multiplication of two diagonal matrices is commutative, therefore: $\Gamma_j D^{-1} = D^{-1} \Gamma_j$), we get:

$$\Delta_{n+1}^{(j)} = \Delta_n^{(j)} - \alpha_n \Gamma_j \left( D^{-1} \sum_{i=1, i \neq j}^{u} \left[ H \Delta_n^{(i)} s_n^i + \bar{\Gamma}_i \left( HF_P^{(i)} - D^{(i)} \right) s_n^i \right] \right) s_n^j + \alpha_n \Gamma_j (D^{-1} v_n + w_n) s_n^j. \quad (34)$$

Next we show that (34) converges in expectation to zero. Then, we complete the proof and show that $\Delta_n$ converges to zero in the mean square sense as well.

**A. Convergence in expectation**

We evaluate the expectation of both sides of (34). Noting that $\Delta_n$ is statistically independent of $s_n$ (since $F_n$ is calculated based on $s_{n-1}$), and that the expectation of the third and fourth terms on the right hand side of (34) is 0, we have:

$$E \left[ \Delta_{n+1}^{(j)} \right] = E \left[ \Delta_n^{(j)} \right] - \alpha_n P_s \Gamma_j D^{-1} HE \left[ \Delta_n^{(j)} \right]. \quad (35)$$

Similarly to (7) we define $\hat{F}_n$ by:

$$F_n = \hat{F}_n + I, \quad (36)$$

where $\hat{F}_n^{(j)}$ has zeros in the elements that do not belong to the set $G_j$, and the precoder error, (29), is equal to the error in the update elements:

$$\Delta_n^{(j)} = F_n^{(j)} - F_P^{(j)} = \hat{F}_n^{(j)} - \hat{F}_P^{(j)}. \quad (37)$$
Note that $\Delta_n^{(j)}$ has zeros in the rows that do not belong to the set $\mathcal{G}_j$. Therefore:

$$\Delta_n^{(j)} = \Gamma_j \Delta_n^{(j)}.$$  \hfill (38)

Defining $\hat{H}_j = \Gamma_j H \Gamma_j$, noting that $\Gamma_j \Gamma_j = \Gamma_j$ and $D^{-1} \Gamma_j = \Gamma_j D^{-1}$, we measure the convergence of (35) to zero by the value of:

$$E \left[ \Delta_n^{(j)} \right]^H E \left[ \Delta_n^{(j)} \right] = E \left[ \Delta_n^{(j)} \right]^H \left( \Gamma_j - \alpha_n P_s D^{-1} \hat{H}_j \right)^H \left( \Gamma_j - \alpha_n P_s D^{-1} \hat{H}_j \right) E \left[ \Delta_n^{(j)} \right].$$ \hfill (39)

Defining: $Q_{j,n} = (\Gamma_j - \alpha_n P_s D^{-1} \hat{H}_j)$ we bound (39):

$$E \left[ \Delta_n^{(j)} \right]^H E \left[ \Delta_n^{(j)} \right] \leq E \left[ \Delta_n^{(j)} \right]^H E \left[ \Delta_n^{(j)} \right] \cdot \rho \left( Q_{j,n} \right) \leq E \left[ \Delta_n^{(j)} \right]^H E \left[ \Delta_n^{(j)} \right] \prod_{k=1}^n \rho \left( Q_{j,k} \right),$$  \hfill (40)

where $\rho \left( A^H A \right)$ is the spectral radius (or maximal eigenvalue) of the matrix $\left( A^H A \right)$, which is bounded by ([18] page 223):

$$\rho \left( A^H A \right) \leq \left\| A^H \right\|_1 \cdot \left\| A \right\|_1 = \left[ \max_i \sum_{j=1}^u | A_{ij} | \right] \left[ \max_j \sum_{i=1}^u | A_{ij} | \right],$$  \hfill (41)

therefore:

$$\prod_{k=1}^n \rho \left( Q_{j,k} \right) \leq \max_j \left( \prod_{k=1}^n \rho \left( Q_{j,k} \right) \right) \leq \max_j \left( \prod_{k=1}^n \| Q_{j,k}^H \|_1 \cdot \left\| Q_{j,k} \right\|_1 \right).$$  \hfill (42)

Testing the elements of the matrix $Q_{j,n}$ we bound (42) by:

$$\max_j \left( \prod_{k=1}^n \left\| Q_{j,k}^H \right\|_1 \cdot \left\| Q_{j,k} \right\|_1 \right) \leq \prod_{k=1}^n (\left| 1 - \alpha_k P_s \right| + \alpha_k P_s \beta_E)^2,$$  \hfill (43)

where:

$$\beta_E = \max_j \left[ \max_{m \in \mathcal{U}_j} \left( \sum_{i \in \mathcal{U}_j, i \neq m} \frac{| h_{m,i} |}{h_{m,m}} \right), \max_{m \in \mathcal{U}_j} \left( \sum_{i \in \mathcal{U}_j, m \neq i} \frac{h_{m,i}}{h_{m,m}} \right) \right].$$  \hfill (44)

Inspecting (43) we can conclude that if there exists $\xi < 1$ so that:

$$\left| 1 - \alpha_n P_s + \alpha_n P_s \beta_E \right| < \xi < 1,$$  \hfill (45)

for all $n$, then the expectation converges to zero as time tends to infinity. The condition in (45) is satisfied if:

$$\beta_E < 1,$$  \hfill (46)
and the step size parameter for all n satisfies:

$$\zeta < \alpha_n < \frac{2}{P_s(1 + \beta_E)},$$

(47)

for some $\zeta > 0$. From the definition of $\beta_E$, (44), one can conclude that the convergence conditions depend on the "worst" cancelling set. Thus, if FEXT cancellation is applied to fewer users, then the channel condition and the condition on $\alpha_n$ can be more relaxed.

B. Mean square error convergence

We now show that the precoder $F_n$ in (22), converges in mean squares to $F_P$ (which completes the proof of 3.1). Following the definitions of $W_n$ and $\delta_n$, given in (24), we now define:

$$W_{j,n} = E[\Delta_n^{(j)} \Delta_n^{(j)H}]$$

(48)

and

$$\delta_{j,n} = \text{tr}\{W_{j,n}\},$$

(49)

noting that $W_n = \sum_{j=1}^u W_{j,n}$ and $\delta_n = \sum_{j=1}^u \delta_{j,n}$. Rewriting (34) as:

$$\Delta_{n+1}^{(j)} = \Delta_n^{(j)} - \alpha_n |s_n^j|^2 \Gamma J D^{-1} H \Delta_n^{(j)}$$

$$- \alpha_n \Gamma J D^{-1} \sum_{i=1, i \neq j}^u H \Delta_n^{(i)} s_n^i s_n^i$$

$$- \alpha_n \Gamma J D^{-1} \sum_{i=1, i \neq j}^u \tilde{G}_i \left( H F_P^{(i)} - D^{(i)} \right) s_n^i s_n^i$$

$$- \alpha_n \Gamma J \left( D^{-1} v_n + w_n \right) s_n^j,$$

(50)

and substituting it into (48) we get:

$$W_{j,n+1} = W_{j,n} - \alpha_n P_s \Gamma J D^{-1} H W_{j,n} - \alpha_n P_s W_{j,n} H^H D^{-H} \Gamma J$$

$$+ \alpha_n^2 LP_s^2 \Gamma J D^{-1} H W_{j,n} H^H D^{-H} \Gamma J$$

$$+ \alpha_n^2 P_s^2 \Gamma J D^{-1} \sum_{i=1, i \neq j}^u H W_{i,n} H^H D^{-H} \Gamma J$$

$$+ \Phi_{j,n} + \alpha_n^2 \Omega_j,$$

(51)

where the four terms in the first two lines of (51) are the outcome of the multiplication of the two terms in the first line of (50) by their conjugates, using $L = E \left[ |s_n^j|^4 \right] / P_s^2$ which ranges
from 1 in QPSK modulation to 2 in Gaussian modulation. The term in the third line of (51) is the outcome of the multiplication of the term in the second line of (50) by its conjugate. The term $\Phi_{j,n}$, defined as:

$$\Phi_{j,n} = \alpha_n^2 D^{-1} \Gamma_j \sum_{i=1}^{u} P_s^2 H E \left[ \Delta_n^{(i)} \right] \left( H F_P^{(i)} - D^{(i)} \right) \bar{\Gamma}_i \Gamma_j D^{-H}$$

$$+ \alpha_n^2 P_s^2 D^{-1} \Gamma_j \sum_{i=1}^{u} \Gamma_i \left( H F_P^{(i)} - D^{(i)} \right) E \left[ \left( \Delta_n^{(i)} \right)^H \right] H^H \Gamma_j D^{-H}$$

(52)

is a matrix that includes the term $E \left[ \Delta_n^{(j)} \right]$ in each of its terms, which results from the multiplications of the second line of (50) by the conjugate of the third line and vice versa. (if conditions (46) and (47) are valid, then $\lim_{n \to \infty} \Phi_{j,n} = 0$). The last term in (51) results from multiplying (separately) each element in the third and fourth lines of (50) by its conjugate value, where:

$$\Omega_j = P_s D^{-1} \Gamma_j \left( Z + \sigma_v^2 I + D \sigma_w^2 D^H \right) \Gamma_j D^{-H}.$$  

(53)

This matrix includes the effect of noise, quantization, and the power of the residual FEXT, Z (given in (16)). This term is constant in time and negatively affects the performance even in the absence of noise. Note that summations (52) and (53) also include the case $i=j$ because: $\Gamma_j \cdot \bar{\Gamma}_j = 0$. Also note that the terms $\Phi_{j,n}$ and $Z$ contain the residual FEXT which is not cancelled even in the ideal partial precoder, $F_P$. This FEXT appears in the unselected terms and therefore: $H F_P^{(i)} - D^{(i)} = \bar{\Gamma}_i \left( H F_P^{(i)} - D^{(i)} \right)$. If $\Gamma_j = \Gamma_{\max}$ for all $j$, as was analyzed in [16], $\Phi_{j,n}$ and the term $\Gamma_{\max} Z \Gamma_{\max}$ in $\Omega_j$, are zeroed, using $\Gamma_{\max} \cdot \bar{\Gamma}_{\max} = 0$.

Summing (51) over $j = 1, \ldots, u$, using $W_n = \sum_{j=1}^{u} W_{j,n}$, the trace of $W_n$ is given by:

$$\delta_{n+1} = \text{tr} \left\{ W_n - \alpha_n P_s \sum_{j=1}^{u} D^{-1} \Gamma_j H W_{j,n} - \alpha_n P_s \sum_{j=1}^{u} W_{j,n} H^H \Gamma_j D^{-H} \right\}$$

$$+ \text{tr} \left\{ \alpha_n^2 (L - 1) P_s^2 D^{-1} \Gamma_j H W_{j,n} H^H \Gamma_j D^{-H} \right\}$$

$$+ \text{tr} \left\{ \alpha_n^2 P_s^2 \sum_{j=1}^{u} D^{-1} \Gamma_j H W_n H^H \Gamma_j D^{-H} \right\}$$

$$+ \text{tr} \left\{ \Phi_n + \alpha_n^2 \Omega \right\},$$

(54)

where $\Phi_n = \sum_{j=1}^{u} \Phi_{j,n}$ and $\Omega = \sum_{j=1}^{u} \Omega_j$. We now upper bound (54) by replacing $\Gamma_j$ throughout
(54) with $\Gamma_{\max}$, using the fact that $W_{j,n}$ is semi positive definite for all $j$:

$$
\delta_{n+1} \leq \text{tr} \left\{ W_n - \alpha_n P_s \sum_{j=1}^{u} D^{-1} \Gamma_{\max} H W_{j,n} - \alpha_n P_s \sum_{j=1}^{u} W_{j,n} H^H \Gamma_{\max} D^{-H} \right\} 
+ \text{tr} \left\{ \alpha_n^2 (L-1) P_s \sum_{j=1}^{u} D^{-1} \Gamma_{\max} H W_{j,n} H^H \Gamma_{\max} D^{-H} \right\} 
+ \text{tr} \left\{ \alpha_n^2 P_s \sum_{j=1}^{u} D^{-1} \Gamma_{\max} H W_n H^H \Gamma_{\max} D^{-H} \right\} 
+ \text{tr} \left\{ \Phi_n + \alpha_n^2 \Omega \right\}.
$$

(55)

Note that the trace of the first line of (54) did not change because $W_{j,n}$ has zeros throughout the columns and rows that do not belong to set $\mathcal{G}_j$ (i.e., $W_{j,n} = \Gamma_j W_{j,n} = W_{j,n} \Gamma_j$). Using again $W_n = \sum_{j=1}^{u} W_{j,n}$ we merge rows two and three in (55), using $K = u + L - 1$, to get:

$$
\delta_{n+1} \leq \text{tr} \left\{ W_n - \alpha_n P_s D^{-1} \Gamma_{\max} H W_n - \alpha_n P_s W_n H^H \Gamma_{\max} D^{-H} \right\} 
+ \text{tr} \left\{ \alpha_n^2 K P_s^2 D^{-1} \Gamma_{\max} H W_n H^H \Gamma_{\max} D^{-H} \right\} 
+ \text{tr} \left\{ \Phi_n + \alpha_n^2 \Omega \right\}.
$$

(56)

Noting that $W_n = \Gamma_{\max} W_n = W_s \Gamma_{\max}$ (because all rows that were selected by any $\Gamma_j$ are also selected by $\Gamma_{\max}$), we define $\hat{H}_{\max} = \Gamma_{\max} H \Gamma_{\max}$, and (56) can be written as:

$$
\delta_{n+1} \leq \text{tr} \left\{ W_n - \alpha_n P_s D^{-1} \hat{H}_{\max} W_n - \alpha_n P_s W_n \hat{H}_{\max} H^H \Gamma_{\max} D^{-H} \right\} 
+ \text{tr} \left\{ \alpha_n^2 K P_s^2 D^{-1} \hat{H}_{\max} W_n \hat{H}_{\max} H^H \Gamma_{\max} D^{-H} \right\} 
+ \text{tr} \left\{ \Phi_n + \alpha_n^2 \Omega \right\}.
$$

(57)

The first and second rows of (57) have the same structure as the bound on the Frobenius norm of the distance between the full updated precoder for selected users, given by (21), and the ZF solution presented in [16]. Repeating the steps presented in [16] we define:

$$
\hat{Q}_n = (\Gamma_{\max} - \alpha_n K P_s D^{-1} \hat{H}_{\max}),
$$

(58)

and rearrange the elements in (57) to get:

$$
\delta_{n+1} \leq \text{tr} \left\{ \left( \frac{1}{K} \hat{Q}_n W_n \hat{Q}_n^H + \frac{K-1}{K} I \right) \right\} + \text{tr} \left\{ \Phi_n + \alpha_n^2 \Omega \right\} 
= \text{tr} \left\{ W_n \left( \frac{1}{K} \hat{Q}_n \hat{Q}_n^H + \frac{K-1}{K} I \right) \right\} + \text{tr} \left\{ \Phi_n + \alpha_n^2 \Omega \right\}.
$$

(59)
Using the spectral radius, (59) is bounded by:

$$\delta_{n+1} \leq \delta_n \cdot \left( \frac{1}{K} \rho \left( \tilde{Q}_n^H \tilde{Q}_n \right) + \frac{K-1}{K} \right) + \text{tr} \left\{ \Phi_n + \alpha_n^2 \Omega \right\},$$

(60)

and the spectral radius is upper bounded by (using (41) again):

$$\rho \left( \tilde{Q}_n^H \tilde{Q}_n \right) \leq \| \tilde{Q}_n^H \|_1 \cdot \| \tilde{Q}_n \|_1 \leq \left( |1 - \alpha_n K P_s| + \alpha_n K P_s \beta_{\text{max}} \right)^2.$$  

(61)

to get (61) into (60) the convergence error is bounded by:

$$\delta_{n+1} \leq \delta_n \cdot \left( \frac{1}{K} \left( |1 - \alpha_n K P_s| + \alpha_n K P_s \beta_{\text{max}} \right)^2 + \frac{K-1}{K} \right) + \text{tr} \left\{ \Phi_n + \alpha_n^2 \Omega \right\}.$$  

(62)

In the constant step size case ($\alpha_n = \alpha$ for all $n$), conditions (25) and (27) ensures that:

$$\left( |1 - \alpha K P_s| + \alpha K P_s \beta_{\text{max}} \right) < 1,$$

(63)

which, together with the requirement of the convergence of $\Phi_n$ guaranty the convergence of (62). Note that conditions (25) and (27) are stricter than the conditions for the convergence in expectation given in (46) and (47); Therefore, when these conditions are met, after long enough convergence time, the term $\Phi_n$ (that includes the term $E[\Delta_n]$), fades ($\lim_{n \to \infty} \text{tr} \{ \Phi_n \} \to 0$).

Substituting the definition of $\Omega$ from (53), the bound on the MS convergence is given by

$$\delta_{\infty} \leq \frac{\alpha^2 K P_s \cdot \text{tr} \left\{ \sum_{j=1}^{n} \left( (D^{-H} D^{-1}) \Gamma_j (Z + \sigma_w^2 I) + \Gamma_j \sigma_w^2 \right) \right\}}{1 - \left( |1 - \alpha K P_s| + \alpha K P_s \beta_{\text{max}} \right)^2}.$$  

(64)

Extending the denominator gives (28), and hence completes the proof of parts B and C of the theorem.

As mentioned in Subsection III-A, we expect that the partial precoder (22) will allow convergence even in channels in which the full update precoder (21), does not converge. Such better convergence conditions can indeed be derived, by continuing from equation (54) without replacing $\Gamma_j$ with $\Gamma_{\text{max}}$. But, the resulting condition, although less restrictive, is much more complicated, and hence not useful for practical scenarios. In this paper we restricted ourselves to the more convenient convergence conditions given in the theorem, which are satisfied in most practical channels.

To complete the proof of the theorem we restrict the channel and set the step size as required by part A of the theorem: $\beta_{\text{max}} < 1$ and $\alpha_n = \mu \cdot n^{-\theta}$, where $\mu > 0$ and $0.5 < \theta < 1$ are
constants. As the step size decreases in time we can find \( \ell > 0 \) such that for all \( n \geq \ell \):

\[
\alpha_n K P_s < 1, \tag{65}
\]

and

\[
\frac{\mu \cdot P_s (1 - \beta_{\text{max}})}{n^\theta} > \frac{1}{n} \tag{66}
\]

(note that \( \theta < 1 \)). From (61) we have for \( n \geq \ell \):

\[
\rho (Q_n^H Q_n) \leq (1 - \alpha_n K P_s (1 - \beta_{\text{max}}))^2 \leq 1 - \alpha_n K P_s (1 - \beta_{\text{max}}), \tag{67}
\]

and substituting into (60) gives:

\[
\delta_{n+1} \leq \delta_n (1 - \alpha_n \cdot c) + \alpha_n^2 \gamma, \tag{68}
\]

where: \( c = P_s (1 - \beta_{\text{max}}) \) and \( \gamma = \text{tr} \{ |\Phi_\ell| / \alpha_\ell^2 + \Omega \} \) are both constant in time (note that \( \frac{\text{tr} \{ |\Phi_\ell| \}}{\alpha_\ell^2} \geq \frac{\text{tr} \{ |\Phi_n| \}}{\alpha_n^2} \) for \( n \geq \ell \)). Writing (68) recursively starting at \( n = \ell \) we get:

\[
\delta_{n+1} \leq \delta_\ell \prod_{k=\ell}^n (1 - \alpha_k \cdot c) + \gamma \cdot \sum_{k=\ell}^n \alpha_k^2 \prod_{m=k+1}^n (1 - \alpha_m \cdot c). \tag{69}
\]

Substituting the step size, (26), gives:

\[
\delta_{n+1} \leq \delta_\ell \prod_{k=\ell}^n \left(1 - \frac{\mu \cdot c}{k^\theta}\right) + \mu^2 \cdot \gamma \cdot \sum_{k=\ell}^n \frac{1}{k^{2\theta}} \prod_{m=k+1}^n \left(1 - \frac{\mu \cdot c}{m^\theta}\right). \tag{70}
\]

Using (66):

\[
\delta_{n+1} \leq \delta_\ell \prod_{k=\ell}^n \left(1 - \frac{1}{k}\right) + \mu^2 \cdot \gamma \cdot \sum_{k=\ell}^n \frac{1}{k^{2\theta}} \prod_{m=k+1}^n \left(1 - \frac{1}{m}\right). \tag{71}
\]

Noticing that \( \prod_{k=\ell}^n (1 - \frac{1}{k}) = \frac{\ell - 1}{n} \) and \( \prod_{m=k+1}^n (1 - \frac{1}{m}) = \frac{k}{n} \), we write (71) as:

\[
\delta_{n+1} \leq \delta_\ell \frac{\ell - 1}{n} + \mu^2 \cdot \gamma \cdot \frac{1}{n} \sum_{k=\ell}^n \frac{k}{k^{2\theta}}. \tag{72}
\]

For \( \theta > 0.5 : \lim_{n \to \infty} \delta_n \to 0 \), which completes the proof of the theorem.
IV. STEADY STATE ANALYSIS

After ensuring the precoder convergence, we test the steady state error for the case of a fixed step size $\alpha$. Testing the covariance matrix of the received error signal:

$$R_{\varepsilon_n} = E [\varepsilon_n \varepsilon_n^H] ,$$  \hspace{1cm} (73)

we evaluate the steady state error as time goes to infinity. Substituting (32) into (73), using $E[\Delta_n^{(j)}] \rightarrow 0$ and the statistical independence of $\Delta_n$ and $s_n$ (since $F_n$ is calculated based on $s_{n-1}$) we have:

$$R_{\varepsilon_\infty} = \left[\sum_{i=1}^{u} P_s H W_j H^H + P_s \bar{\Gamma}_i \left(H F_p^{(i)} - D^{(i)}\right) \left(H F_p^{(i)} - D^{(i)}\right)^H \bar{\Gamma}_i \right] + \sigma_v^2 I$$

$$= P_s H W H^H + Z + \sigma_v^2 I ,$$  \hspace{1cm} (74)

where $Z$ is defined in (16), $W_i = \lim_{n \rightarrow \infty} W_{i,n}$, and $W = \sum_{i} W_i$ is the steady state value of the precoder error covariance matrix (note that the multiplication cross term disappeared because $\bar{\Gamma}_i \Delta_n^{(i)} = 0$). To evaluate (74) we first need to evaluate $W = \lim_{n \rightarrow \infty} W_n$. Inspecting (51), taking time to infinity (so $\lim_{n \rightarrow \infty} \Phi_{i,n} \rightarrow 0$) the steady state error covariance matrix per column must satisfy:

$$\alpha^2 \Omega_j = \alpha P_s \Gamma_j D^{-1} H W_j + \alpha P_s W_j H^H D^{-H} \Gamma_j$$

$$- \alpha^2 (L - 1) P_s^2 \Gamma_j D^{-1} H W_j H^H D^{-H} \Gamma_j$$

$$- \alpha^2 P_s^2 \Gamma_j D^{-1} H W H^H D^{-H} \Gamma_j .$$  \hspace{1cm} (75)

We make two approximations in the right side of (75). The first is removing $\Gamma_j$ from the last term, the second (and more significant one) is assuming $D^{-1} H \approx I$. Summing (75) over $j = 1,..,u$ , using $W_j = \Gamma_j W_j = \bar{W}_j \Gamma_j$ and $K = u + L - 1$ we get:

$$W \cdot (2 \alpha P_s - \alpha^2 K P_s^2) \approx \alpha^2 \Omega .$$  \hspace{1cm} (76)

Substituting the definition of $\Omega$ from (53), $W$ in the steady state can be approximated by:

$$W \approx \frac{\alpha}{(2 - \alpha K P_s)} \sum_{j=1}^{u} D^{-1} \Gamma_j (Z + \sigma_v^2 I + D \sigma_v^2 D^H) \Gamma_j D^{-H} .$$  \hspace{1cm} (77)

Substituting (77) into (74) we get:

$$R_{\varepsilon_\infty} \approx \frac{\alpha P_s}{(2 - \alpha K P_s)} \left( H D^{-1} \sum_{j=1}^{u} (\Gamma_j (Z + \sigma_v^2 I + D \sigma_v^2 D^H) \Gamma_j) D^{-H} H^H \right) + Z + \sigma_v^2 I .$$  \hspace{1cm} (78)
Using again the approximation $\mathbf{HD}^{-1} \approx \mathbf{I}$:

$$
\mathbf{R}_{\varepsilon,\infty} \approx \frac{\alpha P_s}{(2 - \alpha KP_s)} \cdot \left( \sum_{j=1}^{u} \Gamma_j \left( \mathbf{Z} + \sigma^2_v + \mathbf{D}\sigma^2_w\mathbf{D}^H \right) \Gamma_j \right) + \mathbf{Z} + \sigma^2_v \cdot \mathbf{I}.
$$

(79)

In order to obtain a more insightful expression, we consider the trace of (79), and making one last approximation, we assume that the sets $\mathcal{G}_j = \{g^1_j \ldots g^r_j\}$ for each $j$ are selected independently. Therefore, for a large $u$, defining the empiric average set size $r = \frac{1}{u} \sum_{j=1}^{u} |\mathcal{G}_j|$, we have: $\sum_{j=1}^{u} \Gamma_j \approx r \cdot \mathbf{I}$.\(^3\) Using $\text{tr}\{\mathbf{AB}\} = \text{tr}\{\mathbf{BA}\}$ and $\Gamma_j \cdot \Gamma_j = \Gamma_j$, the sum of the steady state error of all users is approximated by:

$$
\text{tr}\left\{\mathbf{R}_{\varepsilon,\infty}\right\} \approx \left( \frac{\alpha P_s r}{(2 - \alpha KP_s)} + 1 \right) \cdot \text{tr} \left\{ \mathbf{Z} + \sigma^2_v \mathbf{I} \right\} + \frac{\alpha P_s r \sigma^2_w}{(2 - \alpha KP_s)} \text{tr}\left\{ \mathbf{D}\mathbf{D}^H \right\}.
$$

(80)

In an ideal partial FEXT cancellation, the error signal covariance is given by $\mathbf{Z} + \sigma^2_v \mathbf{I}$, where $\mathbf{Z}$ contains the error due to the non-cancelled element FEXT terms. The error increase is due to the use of the iterative precoder and can be determined by $\alpha$ and by the number of users to be cancelled. Note that for full cancellation, $r = u$ and $\mathbf{Z} = 0$, so that (80) is the same as the result derived in [16]. Although the approximations made in the derivation of (80) are fairly heuristic, the resulting approximation is surprisingly accurate as shown in the following section.

V. SIMULATION RESULTS

To demonstrate the results derived in the previous sections we conducted several simulations using 28 channels measured by France Telecom\(^4\). In the following simulations we chose the active elements to be the $28 \cdot r$ strongest elements in the channel matrix. The transmitted PSD was set to -60dBm/Hz, the noise PSD is -140dBm/Hz and the step size $\alpha$ was chosen to be half of its maximal value calculated by (27) for all cases.\(^5\) These simulations do not apply error signal quantization ($\sigma_w = 0$).

\(^3\)Alternatively, one can consider a “fair” partial FEXT cancellation scheme, which cancels the same number of interferers to each receiver. For such a scheme the approximation is replaced by an equality: $\sum_{j=1}^{u} \Gamma_j = r \cdot \mathbf{I}$.

\(^4\)The authors would like to thank M. Ouzzif, R. Tarafi, H. Marriott and F. Gauthier of France Telecom R&ID, who conducted the VDSL channel measurements as partners in the U-BROAD project.

\(^5\)In practice the value of the channel parameter $\beta_{\max}$ is often unknown. In such case, a practical approach can take the worst case value $\beta_{\max} = 1$, and use: $\alpha = \frac{\text{const}}{KP_s}$. 

Figure 1 depicts the mean square error convergence of the adaptive precoder $F_n$ to the ideal partial precoder $F_P$ as a function of the number of symbols transmitted for a distance of 300 meters in a frequency of 14.25 MHZ ($\beta_{\text{max}} = 0.3576$). The convergence is measured by $\delta_n$ defined in (24). The figure shows the empirical evaluation of $\delta_n$ based on a Monte Carlo simulation of 30 systems for various values of $r$. The figure also shows the bound defined in (62). It can be seen that as $r$ decreases, the precoder convergence is faster, but the precoder converges further away from $F_P$ due to the higher residual FEXT. One can see that although the convergence bound is not tight it well describes the precoder convergence.

Figure 2 shows the signal to noise plus interference ratio (SINR) averaged over all users as a function of time for the same case (300 meters, 14.25 MHz). For reference, the figure also shows the average SINR using the ideal partial precoder $F_p$. It can be seen that for all values of $r$, the SINR converges to a value which is very close to the SINR obtained with an ideal partial precoder. Note that using smaller values of $\alpha$ will allow the SINR to be as close as desired to the ideal value at the price of slower convergence. Extending the simulation to the whole bandwidth (30MHz), Figure 3 shows the average capacity achieved from all users as a function of time at a distance of 300 meters. The figure shows the curves for different values of $r$ and the capacity achieved using $F_p$ for each case. The results show similar behavior as the results in Figure 2 for the SINR convergence.

Figure 4 demonstrates the accuracy of the approximation given in (80) for the sum of all users’ steady state square errors. Again we use channel measurements corresponding to a distance of 300 meters at a frequency of 14.25 MHz. The figure shows the curves of the sum of all users’ square errors as function of time for different values of $r$. The figure also shows by markers the corresponding approximation for each case. It can be seen that the approximation gives an almost exact prediction in all cases. Note that (80) was derived using the assumption that the FEXT cancellation sets are independent and uniformly distributed. The accuracy of (80) indicates that this assumption gives a good representation of system performance.

To study the amount of increase in noise and FEXT due to the iterative scheme and its distribution among the users, Figure 5 depicts the histogram of all users’ steady state SINR loss, using the same channel measurements as before (300m, 14.25MHz) when cancelling 20 cross-talkers per user. The dotted line indicates the approximation derived in (80). The standard deviation from the predicted users’ performance is about 0.5 dB. This is also a typical result for
different frequencies and different numbers of cross-talkers canceled per user. Thus, this simple steady state approximation provides a useful and robust tool for system design and analysis.

VI. CONCLUSION

In this paper we presented and analyzed an adaptive precoder for partial FEXT cancellation. Partial FEXT cancellation is very important because it can overcome major practical constraints (e.g., availability of appropriate end-user equipment or limited system complexity). We derived sufficient conditions that guarantee the convergence of the precoder to the ideal partial FEXT precoder presented by Cendrillon et al. [17]. The analysis also provides bounds that enable the right choice of parameters and provides prediction on the expected performance. The results are supported by numerical simulations which show that the precoder converges very closely to the ideal precoder presented in [17] and that partial FEXT cancellation also accelerates the convergence.

REFERENCES


Fig. 1. Convergence of the adaptive precoder to the ideal precoder in mean squares as a function of the number of symbols transmitted, at a distance of 300m and a frequency of 14.25MHz (averaged over 30 systems). Also shown is the upper bound on the convergence defined by (62).
Fig. 2. Average SINR (of all 28 users) at a distance of 300m and frequency of 14.25MHz, using the adaptive precoder and the ideal precoder, as a function of the number of symbols transmitted.
Fig. 3. Average capacity (of all 28 users) using the adaptive precoder and the ideal precoder, as a function of the number symbols transmitted.
Fig. 4. Sum of all users’ square error over time, using channel measurements at a distance of 300m over a frequency of 14.25MHz. Also shown is the approximation given by (80).
Fig. 5. Sum of all users’ square error in steady state, derived by using channel measurements at a distance of 300m over a frequency of 14.25MHz and cancelling 20 cross-talkers per user (r=20). Also shown is (dotted line) the approximation given by (80).